COURSE TITLE: Pre-Calculus (Honors)

COURSE DESCRIPTION: Pre-calculus (Honors) is an immediate preparation for calculus. A deeper appreciation of the function concept is stressed and includes exponential, logarithmic, polar and parametric representations. This course includes units on vector spaces. The topics are similar to those studied in the regular Pre-Calculus class but are treated from a more theoretical point of view. This is a college preparatory class.

Prerequisite: Placement based on Math Department recommendation only.

# COURSE REQUIREMENT/REQUIRED MATERIALS:

- A. Text: PreCalculus with limits a Graphing Approach 6<sup>th</sup> edition, Larson. 2012.
- B. Folder.
- C. Pencils and red pens.
- D. Graphing calculator (TI-83 or TI-84).
- E. Notebook, graph paper.
- F. Scientific calculator.

# COURSE OBJECTIVES/STUDENT OUTCOMES:

A. Students will understand and apply basic algebra skills.

Upon completing this goal the student will be able to:

- \* simplify absolute value expressions
- $^{\star}$  express absolute value equations on a number line and as a distance
- \* solve absolute value equations
- \* simplify expressions containing negative and rational exponents
- \* simplify compound fractions
- \* evaluate expressions using a graphing calculator
- \* use a graphing calculator to graph absolute value inequalities

B. Students will discern the differences and similarities among functions and relations.

- \* identify a function as a type of a relation
- \* determine the domain and range of functions and relations from: a table of values, a graph, an equation, a mapping display
- \* use functional notation to evaluate and graph functions
- $^{\star}$  combine functions using: addition, subtraction, multiplication, division, and composition
- \* identify functions as decreasing, increasing and constant
- \* test for even and odd functions
- \* identify continuous and discontinuous functions
- $^{\star}$  use the vertical line test to determine if a graph represents a function
- \* determine if a function is one to one
- \* determine the inverse of a one to one function both graphically and algebraically

C. Students will understand concepts of graphing equations with two variables.

Upon completing this goal the student will be able to:

- \* use graphing calculators to verify the graphs of the functions
- \* graph linear functions
- \* determine the slope, x-intercept, y-intercept of a line
- \* determine the equation of a line given data for a line
- \* determine the distance between two points on a line
- $^{\star}$  determine the midpoint of a segment when given the coordinates of the endpoints
- \* determine the symmetry with respect to the X axis, Y axis, origin, Y = X line when given the equation and/or graph of a relation
- \* find the X and Y intercepts of a graph of an equation
- \* graph "piece-wise" functions
- \* sketch the following basic functions:
  - a) Square root
- b) Cube root
- c) Quadratic

d) Cubic

- e) Absolute Value f) Polynomial
- \* sketch graphs of the following functions with respect to Y = F(X)
  - Y = F(X) + C, a vertical shift
  - Y = F(X + C), a horizontal shift
  - Y = -F(X), reflection in the X axis
  - Y = F(-X), reflection in the Y axis
- D. Students will acquire an understanding of quadratic functions.

Upon completing this goal the student will be able to:

- \* graph any quadratic function
- \* solve max/min problems
- \* solve quadratic equations
- \* translate graphs of quadratic functions
- \* use a graphing calculator to graph and solve problems using quadratic functions and equations
- E. Students will develop an understanding of the nature of polynomial functions.

- \* apply the leading coefficient test to determine the end behavior of the graph of the polynomial function
- $^{\star}$  use synthetic division to find roots of the equation
- $^{\star}$  use the factor and remainder theorems to help solve polynomial equations
- \* write polynomial equations, given the roots of the equation
- \* determine possible rational roots
- \* find rational roots and solve equations
- $^{\star}$  find the upper and lower bound for real roots
- \* determine if an equation does not have rational roots
- $\mbox{\ensuremath{^{\ast}}}$  use the conjugate roots theorem to find roots (including complex roots) of polynomial equations
- \* use Descartes' Rule of Signs to obtain information about the roots of polynomial functions
- \* approximate irrational roots

- \* use previously learned techniques to sketch graphs of polynomial functions
- $^{\star}$  use the technique of approximating the values of the function in the vicinity of each x intercept to sketch the graph
- \* use the graphing calculator to verify the graphs of the functions and find roots
- F. Students will develop an understanding of the nature of rational functions.

Upon completing this goal the student will be able to:

- \* identify rational functions
- \* find vertical and horizontal asymptotes
- $^{\star}$  sketch the graphs of rational functions, showing intercepts and asymptotes
- \* use the graphing calculator to verify the graphs of the functions and solve problems
- G. Students will gain an understanding of exponential and logarithmic functions.

Upon completing this goal the student will be able to:

- \* sketch the graphs of exponential functions, specifying domain, range, intercept(s), and the asymptote along with other properties
- \* evaluate exponential expressions
- \* solve exponential equations
- \* use the graphing calculator to solve half-life, growth, intensity, logistics, interest problems
- \* sketch the graphs of logarithmic functions, specifying domain, range, intercept(s) and asymptote
- \* evaluate logarithmic expressions using properties of logs
- \* solve logarithmic equations
- H. Students will experience the concepts of conic sections.

Upon completing this goal the student will be able to:

- \* graph the conics
- \* determine the equations of the conics when given various data
- \* identify the conics from its equation
- \* solve problems using conic concepts
- $\bullet\,$  use the graphing calculator to verify graphs and as a tool to solve problems
- I. Students will develop an understanding of the trigonometric functions based on the wrapping function concept.

- $^{\star}$  change radian measure of angles to degree measure and vice versa
- \* define the six trig functions based upon points on the unit circle and numbers on the number line
- \* evaluate trig functions with respect to special radian and degree measures using the wrapping functions
- \* distinguish even and odd trig functions
- \* evaluate trig functions using right triangles

- \* apply basic right triangle trig relationships to verify identities
- \* solve problems using right triangle trig
- \* apply algebraic skills to trigonometric situations
- J. Students will develop skills in graphing trig functions, solving trig equations and solving trig work problems

Upon completing this goal the student will be able to:

- \* graph trig functions
- \* graph the inverses of the trig functions
- $^{\star}$  apply the addition of ordinates principle to graph the sum or difference of two functions
- \* write equations of trig functions when given the graph
- \* solve trig equations
- \* apply sum and difference formulas
- \* apply multiple angle formulas
- $^{\star}$  apply product to sum formulas and sum to product formulas
- \* apply power reducing formulas
- \* apply negative angle formulas
- \* use the graphing calculator to graph trig functions and solve trig problems
- \* verify identities
- K. Students will gain an understanding of sequences and series.

Upon completing this goal the student will be able to:

- $^{\star}$  distinguish sequences which are arithmetic, geometric, or neither
- \* find the nth term of arithmetic, geometric, and other sequence
- \* find the sum of n terms of a given sequence
- $^{\star}$  distinguish between sequences that are convergent or divergent
- L. Students will develop skills in using parametric equations.

Upon completing this goal the student will be able to:

- $^{\star}$  write the corresponding rectangular equation by eliminating the parameter
- \* sketch the curve represented by the parametric equations
- $^{\star}$  apply parametric equations to solve problems
- \* use parametric equations with the graphing calculator
- M. Students will understand the polar coordinate system.

- \* convert points in rectangular form to polar form and vice versa
- \* convert rectangular equations to polar form and vice versa
- \* graph circles, cardioids, limacons, roses, lemniscates, spirals, and other graphs which readily apply to the polar system of graphing
- \* use the graphing calculator to graph polar equations

### COURSE OUTLINE:

- I. Basic Algebra review
  - A. Real number system
  - B. Exponents and Radicals
  - C. Polynomials
    - 1. Special products
    - 2. Factoring
  - D. Fractional expressions
  - E. Equations
    - 1. Linear
    - 2. Quadratic
  - F. Inequalities
- II. Functions and graphs
  - A. Graphs of equations
    - 1. Intercepts
    - 2. Symmetry tests
      - a) X axis
      - b) Y axis
      - c) Origin
      - d) the line: y = x
    - 3. Transformations
      - a) Rigid
        - (1) Reflection in x axis
        - (2) Reflection in y axis
        - (3) Horizontal translation
        - (4) Vertical translation
      - b) Non Rigid
  - B. Functions
    - 1. Graphs
    - 2. Combinations of functions
    - 3. Inverse functions
  - C. Variation and mathematical models
  - D. Applications using the graphing calculator
- III. Conic Sections
  - A. Graphs
    - 1. Circle
    - 2. Parabola
    - 3. Ellipse
    - 4. Hyperbola
  - B. Equations
    - 1. Circle
    - 2. Parabola
    - 3. Ellipse
    - 4. Hyperbola
  - C. Applications using the graphing calculator
- IV. Polynomial and Rational Functions
  - A. Polynomial division and synthetic division
  - B. Real Zeros
  - C. Complex numbers
  - D. Complex zeros
  - E. Graphs of polynomial functions
  - F. Graphs of rational functions
  - G. Applications using the graphing calculator

- V. Exponential and Logarithmic Functions
  - A. Exponential Functions
  - B. Logarithmic Functions
  - C. Properties of logarithms
  - D. Solving exponential and logarithmic equations
  - E. Applications using the graphing calculator

# VI. Trigonometry

- A. Radian and degree measure
- B. Trig. functions and the unit circle
- C. Trig. functions of an acute angle
- D. Trig. functions of any angle
- E. Graphs of six trig. functions
- F. Graphing by addition of ordinates
- G. Damped trig. graphs
- H. Inverse trig. functions
- I. Applications using the graphing calculator

# VII. Analytic Trigonometry

- A. Trig. identities
- B. Solving trig. equations
- C. Sum and Difference formulas
- D. Multiple angle formulas
- E. Product Sum formulas

# VII. Sequences and Series

- A. Arithmetic sequences
  - 1. nth term
  - 2. Sum of n terms
- B. Geometric sequences and series
  - 1. nth terms
  - 2. Sum of n terms
  - 3. Infinite sums
- C. Other sequences
  - 1. nth term
  - 2. Sum of n terms

# IX. Analytic Geometry Topics

- A. Polar Coordinates
- B. Graphs of polar equations
- C. Polar equations of the conics
- D. Plane curves and parametic equations
- E. Applications using the graphing calculator