

Matrix Operations

What is a Matrix?

❄ **MATRIX**: A rectangular arrangement of numbers in rows and columns.

❄ The **ORDER** of a matrix is the number of the rows and columns.

❄ The **ENTRIES** are the numbers in the matrix.

❄ This order of this matrix is a 2 x 3.

columns ↓

$$\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

rows →

What is the order?

$$\begin{bmatrix} 8 & -1 & 3 \\ 0 & 0 & 2 \\ 10 & 4 & -3 \end{bmatrix}$$

3×3 (or square matrix)

$$[9 \quad -5 \quad 7 \quad 0]$$

1×4 (Also called a column matrix)

$$\begin{bmatrix} -2 & 0 & 4 & 6 & 3 \\ 1 & 1 & -5 & -9 & 8 \\ 7 & 3 & 2 & 7 & 6 \end{bmatrix}$$

3×5

$$\begin{bmatrix} -9 \\ 7 \\ 0 \\ 6 \end{bmatrix}$$

4×1

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

2×2 (or square matrix)

(Also called a row matrix)

Adding Two Matrices

- ▶ To add two matrices, they must have the same order. To add, you simply add corresponding entries.

$$\begin{bmatrix} 5 & -3 \\ -3 & 4 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 3 & 0 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 5 + (-2) & -3 + 1 \\ -3 + 3 & 4 + 0 \\ 0 + 4 & 7 + (-3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ 0 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & -1 & 3 \\ -5 & 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 7 & 5 & 2 \\ 5 & 3 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+(-1) & 0+7 & -1+5 & 3+2 \\ -5+5 & 4+3 & 2+3 & 9+(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 7 & 4 & 5 \\ 0 & 7 & 5 & 7 \end{bmatrix}$$

Subtracting Two Matrices

- ▶ To subtract two matrices, they must have the same order. You simply subtract corresponding entries.

$$\begin{bmatrix} 9 & -2 & 4 \\ 5 & 0 & 6 \\ 1 & 3 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 7 \\ 1 & 5 & -4 \\ -2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 9-4 & -2-0 & 4-7 \\ 5-1 & 0-5 & 6-(-4) \\ 1-(-2) & 3-3 & 8-2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -2 & -3 \\ 4 & -5 & 10 \\ 3 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 3 \\ 8 & 0 & -7 \\ 1 & 5 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 8 \\ 3 & -1 & 1 \\ -4 & 2 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & -4-1 & 3-8 \\ 8-3 & 0-(-1) & -7-1 \\ 1-(-4) & 5-2 & 0-7 \end{bmatrix} = \begin{bmatrix} 2 & -5 & -5 \\ 5 & 1 & -8 \\ 5 & 3 & -7 \end{bmatrix}$$

Multiplying a Matrix by a Scalar

- ▶ In matrix algebra, a real number is often called a **SCALAR**. To multiply a matrix by a scalar, you multiply each entry in the matrix by that scalar.

$$4 \begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 4(-2) & 4(0) \\ 4(4) & 4(-1) \end{bmatrix}$$
$$= \begin{bmatrix} -8 & 0 \\ 16 & -4 \end{bmatrix}$$

$$-2\left(\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \end{bmatrix}\right)$$

$$= -2\left(\begin{bmatrix} 1-4 & -2+5 \\ 0+6 & 3+(-8) \end{bmatrix}\right)$$

$$= -2 \begin{bmatrix} -3 & 3 \\ 6 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -2(-3) & -2(3) \\ -2(6) & -2(-5) \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -12 & 10 \end{bmatrix}$$