Sec 9.1 (Day 2): Manipulation of Power Series

Last class we looked at several functions and wrote a series to represent them. This time we are going to work the other direction. Write a function that is represented by the series below and find the interval of convergence.

$$\sum_{n=0}^{\infty} 3^n x^n$$

$$\sum_{n=0}^{\infty} \left(\frac{-1}{4}\right)^n (x-2)^n$$

Do you remember the series that represents $f(x) = \dfrac{1}{1+x^2}$?

Can you use the series above to write a new series to represent $f(x) = \tan^{-1}(x)$?

We can manipulate series the same way we manipulate functions. The following is a list of techniques we can use:

- Recognize it as the sum of a geometric power series.
- Add/Subtract two power series.
- Multiply/Divide a power series by a variable and/or a constant.
- Differentiate or integrate a power series.
- Substitute into a known series. (we will look at this later)

Use manipulation techniques to express the following as a power series.

$$f(x) = \frac{x}{1+x^2}$$

Use manipulation techniques to express the following as a power series.

$$f(x) = \frac{-2x}{\left(1+x^2\right)^2}$$

Use manipulation techniques to express the following as a power series.

$$f(x) = \ln(1+x)$$

Use the power series for $f(x) = \frac{1}{1-x}$ to find the following.

Express $g(x) = f'(x)$ as a power series. Give the first four nonzero terms and the general term. What is the interval of convergence?

Identify $g(x)$

Is it possible to find $g(0.5)$ by using the power series you found? If so, find the value by using the power series and then comparing it to the actual value.

Is it possible to find $g(-1.5)$ by using the power series you found? If so, find the value by using the power series and then comparing it to the actual value.

Use the power series for $f(x)=\dfrac{1}{1-x}$ to find the following.

Express $h(x)=\int_0^x f(t)dt$ as a power series if.

Give the first four nonzero terms and the general term. What is the interval of convergence? Identify $h(x)$