

Name _____
Block _____

Date _____
Unit 2 Algebra

Unit 2: PROIR KNOWLEDGE

Everything you need to know about FRACTIONS

A fraction bar is another way of writing a division symbol.

For example: $4 \div 2 = \frac{4}{2}$

Operations with Fractions

Adding and Subtracting

You need common denominator

For example:

1. $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

2. $\frac{2}{6} - \frac{4}{6} = \frac{-2}{6} = -\frac{1}{3}$

3. $\frac{1}{5} + \frac{2}{3} = \frac{3}{15} + \frac{10}{15} = \frac{13}{15}$

4. $\frac{1}{4} - \frac{-3}{6} = \frac{6}{24} - \frac{-12}{24} = \frac{18}{24} = \frac{9}{12} = \frac{3}{4}$

5. $2\frac{1}{3} + \frac{3}{4} = \frac{7}{3} + \frac{3}{4} = \frac{28}{12} + \frac{9}{12} = \frac{37}{12}$

Multiplying

To multiply fractions, we multiply

Top: Top, Bottom: Bottom

Sometimes, we can

cross-cancel by reducing numbers that are across from each other.

For example:

1. $\frac{3}{2} \cdot \frac{2}{6} = \frac{1}{4}$

2. $\frac{3}{1} \cdot \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

3. $\frac{1}{2} \cdot \frac{-2}{5} = \frac{-1}{5}$

Dividing

To divide fractions, we

Keep, Switch, flip

For example:

1. $\frac{3}{4} \div \frac{2}{6} = \frac{3}{4} \cdot \frac{6}{2} = \frac{9}{4}$

2. $3 \div \frac{1}{6} = \frac{3}{1} \cdot \frac{6}{1} = \frac{18}{1} = 18$

3. $\frac{1}{2} \cdot 6 = \frac{1}{2} \cdot \frac{6}{1} = \frac{6}{2} = 3$

4. $2\frac{1}{4} \div \frac{-3}{5} = \frac{9}{4} \cdot \frac{5}{-3} = \frac{45}{-12} = \frac{-45}{12}$

Proportions

When 2 fractions are equal its called a proportion.

Here, we solve an equation for x.

We use proportions to solve an equation for a missing variable.

1. $\frac{12}{20} = \frac{x}{100} = \frac{20x}{20} = \frac{1200}{20} \quad x = 60$

2. $\frac{2x}{3} = \frac{80}{60}$

Equations with Fractions

We use all of this PROIR KNOWLEDGE to solve equations with fractions.

1. $5 = \frac{2}{3}x + 2$

$\frac{3}{3} \cdot 3 = \frac{3}{3} \cdot \frac{2}{3}x$

$\frac{9}{2} = x$

2. $\frac{2}{5}x - \frac{1}{2} = 6\frac{1}{6}$

$\frac{2}{5}x$

Solving Equations

(Chapter 2 in textbook)

Equations with Two Steps

To solve two-step equations: "Undo" by + or -, then \times or \div . (Reverse the problem)

Example problems:

1. Solve $3x + 4 = 15$

$$\begin{array}{r} -4 \quad -4 \\ 3x = 11 \\ \frac{3x}{3} = \frac{11}{3} \\ x = \frac{11}{3} \end{array}$$

Check!

$$\begin{array}{r} 3\left(\frac{11}{3}\right) + 4 = 15 \\ 11 + 4 = 15 \\ 15 = 15 \checkmark \end{array}$$

2. Solve $-2a + 5 = 18$

$$\begin{array}{r} -5 \quad -5 \\ -2a = 13 \\ \frac{-2a}{-2} = \frac{13}{-2} \\ a = \frac{13}{-2} \end{array}$$

Check!

$$\begin{array}{r} -2\left(\frac{13}{-2}\right) + 5 = 18 \\ 13 + 5 = 18 \\ 18 = 18 \checkmark \end{array}$$

3. Solve $-\frac{4}{7}k - 9 = 1$

$$\begin{array}{r} +9 \quad +9 \\ -\frac{4}{7}k = 10 \\ \frac{-4}{7}k = 10 \cdot \frac{7}{7} \\ \frac{-4}{7}k = 70 \\ \frac{-4}{7}k \cdot \frac{7}{-4} = \frac{70}{-4} \\ k = \frac{70}{-4} = -\frac{35}{2} \end{array}$$

Check!

$$\begin{array}{r} -\frac{4}{7}\left(-\frac{35}{2}\right) - 9 = 1 \\ 10 - 9 = 1 \\ 1 = 1 \checkmark \end{array}$$

4. Solve $30 = 2b - 40 = k = \frac{35}{2}$

When you have a fraction, multiply by the reciprocal!

Multi-Step Equations

To solve multi-step equations, we follow the same procedure as two-step equations with the following five steps:

1. Simplify each side of the equation.
2. Move all variables to the same side.
3. UNDO the equation by adding or subtracting, then Multiplying or Dividing.
4. Box your answer and check!

Example Problems:

1) $6a + 5a = -11$

$$\begin{array}{r} 11a = -11 \\ \frac{11a}{11} = \frac{-11}{11} \\ a = -1 \end{array}$$

Check!

$$\begin{array}{r} 6(-1) + 5(-1) = -11 \\ -6 + 5 = -1 \\ -1 = -1 \checkmark \end{array}$$

2) $-6n - 2n = 16$

$$\begin{array}{r} -8n = 16 \\ \frac{-8n}{-8} = \frac{16}{-8} \\ n = -2 \end{array}$$

3) $4x + 6 + 3 = 17$

4) $0 = -5n - 2n$

Simplify first!

$$\begin{aligned}
 18 &= 3(3x - 6) \\
 18 &= 9x - 18 \\
 +18 & \quad +18 \\
 \hline
 36 &= 9x \\
 \frac{36}{9} &= \frac{9x}{9} \\
 4 &= x
 \end{aligned}$$

check:

$$\begin{aligned}
 18 &= 3(3(4) - 6) \\
 18 &= 3(6) \\
 18 &= 18 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 37 &= -3 + 5(x + 6) \\
 37 &= -3 + 5x + 30 \\
 37 &= 27 + 5x \\
 -27 & \quad -27 \\
 \hline
 10 &= 5x \\
 \frac{10}{5} &= \frac{5x}{5} \quad 2 = x
 \end{aligned}$$

check:

$$\begin{aligned}
 37 &= -3 + 5(2) + 6 \\
 37 &= -3 + 10 + 30 \\
 37 &= 7 + 30 \\
 37 &= 37 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8x - 2 &= -9 + 7x \\
 -7x & \quad -7x \\
 \hline
 x - 2 &= -9 \\
 +2 & \quad +2 \\
 \hline
 x &= -7
 \end{aligned}$$

check:

$$\begin{aligned}
 8(-7) - 2 &= -9 + 7(-7) \\
 -56 - 2 &= -9 - 49 \\
 -58 &= -58 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8(4k - 4) &= -5k - 32 \\
 32k - 32 &= -5k - 32 \\
 +32 & \quad +32 \\
 \hline
 32k &= -9k - 0 \\
 +5k & \quad +5k \\
 \hline
 37k &= 0 \\
 \frac{37k}{37} &= \frac{0}{37} \\
 k &= 0
 \end{aligned}$$

check:

$$\begin{aligned}
 8(4(0) - 4) &= -5(0) - 32 \\
 0 - 32 &= 0 - 32 \\
 -32 &= -32 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 -11 - 5a &= 6(5a + 4) \\
 -11 - 5a &= 30a + 24 \\
 +5a & \quad +5a \\
 \hline
 -11 &= 35a + 24 \\
 -24 & \quad -24 \\
 \hline
 -35 &= 35a \\
 \frac{-35}{35} &= \frac{35a}{35} \\
 -1 &= a
 \end{aligned}$$

check:

$$\begin{aligned}
 -11 - 5(-1) &= 6(5(-1) + 4) \\
 -11 + 5 &= 6(-1) \\
 -6 &= -6 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 5(2x + 6) &= -4(-5 - 2x) + 3x \\
 10x + 30 &= 20 + 8x + 3x \\
 10x + 30 &= 20 + 11x \\
 -10x & \quad -10x \\
 \hline
 30 &= 20 + x \\
 -20 & \quad -20 \\
 \hline
 10 &= x
 \end{aligned}$$

check:

$$\begin{aligned}
 5(2(10) + 6) &= -4(-5 - 2(10)) + 3(10) \\
 5(26) &= -4(-25) + 3(10) \\
 130 &= 100 + 30 \\
 130 &= 130 \checkmark
 \end{aligned}$$

Using Equations to Solve Word Problems

For each of the following problems,

- Define a variable. (write a let statement)
- Write an equation.
- Solve it!
- Clearly show your answer.
- Check!

1. Eric opens a savings account with \$50. Each week after that he deposits \$8. In how many weeks will he have saved \$450?

let $x = \# \text{ of weeks}$

let

$$\begin{array}{r} 50 + 8x = 450 \\ -50 \quad -50 \\ \hline 8x = 400 \\ \div 8 \quad \div 8 \\ \hline x = 50 \end{array}$$

50 weeks

check!

$$\begin{array}{r} 50 + 8(50) = 450 \\ 50 + 400 = 450 \\ 450 = 450 \checkmark \end{array}$$

2. A soccer field is 75 yards shorter than 3 times its width. Its perimeter is 370 yards. Find the length and width.

let $x = \text{width}$
let $3x - 75 = \text{length}$
 $p = 2l + 2w$

$$\begin{array}{r} 3p = 2l + 2w \\ 370 = 2(3x - 75) + 2(x) \\ 370 = 6x - 150 + 2x \\ 370 = 8x - 150 \\ +150 \quad +150 \\ \hline 520 = 8x \\ \div 8 \quad \div 8 \\ \hline 65 = x \end{array}$$

Answer: width = 65 yards
length = 120 yards

$$\begin{array}{r} 3x - 75 \\ 3(65) - 75 \\ 195 - 75 \\ \hline 120 \text{ yds} \end{array}$$

3. One side of a triangle is 3 cm more than twice as long as the shortest side. The third side is 11 cm longer than the shortest side. The perimeter of the triangle is 102 cm. Find the length of each side.

let $x = \text{shortest side}$
let $2x + 3 = \text{one side}$
let $x + 11 = \text{another side}$

$$\begin{array}{r} x + 2x + 3 + x + 11 = 102 \\ 4x + 14 = 102 \\ -14 \quad -14 \\ \hline 4x = 88 \\ \div 4 \quad \div 4 \\ \hline x = 22 \end{array}$$

Answers:
shortest side = 22 cm
one side = $2(22) + 3 = 47$
another side = $22 + 11 = 33$ cm

4. Find 2 consecutive integers whose sum is -47.

let $x = 1^{\text{st}}$ consecutive integer
let $x + 1 = \text{next consecutive integer}$

$$\begin{array}{r} x + x + 1 = -47 \\ 2x + 1 = -47 \\ -1 \quad -1 \\ \hline 2x = -48 \\ \div 2 \quad \div 2 \\ \hline x = -24 \end{array}$$

* first integer = -24
* next integer = -24 + 1 = -23
check:
 $-24 + -23 = -47$
 $-47 = -47 \checkmark$

5. Find two consecutive even integers whose sum is 46.

let $x = \text{first even } \mathbb{Z}$
let $x + 2 = \text{second even } \mathbb{Z}$

$$\begin{array}{r} x + x + 2 = 46 \\ 2x + 2 = 46 \\ -2 \quad -2 \\ \hline 2x = 44 \\ \div 2 \quad \div 2 \\ \hline x = 22 \end{array}$$

* first even $\mathbb{Z} = \span style="border: 1px solid black; padding: 2px;">22
* next even $\mathbb{Z} = 22 + 2 = \span style="border: 1px solid black; padding: 2px;">24$$

6. Find two consecutive odd integers whose sum is 40.

let $x = \text{first odd } \mathbb{Z}$
let $x + 2 = \text{next odd } \mathbb{Z}$

I ♥ You

