

Pre-AP Physics Notes: Ch 6
Momentum and Collisions

Momentum and its relationship to force

- Momentum describes an object's motion. Linear momentum is the product of its mass and velocity ($p=mv$) and is symbolized by "p". Momentum is a vector quantity that has the same direction as the velocity.

$$p=mv \quad \text{units are } \text{kg} \cdot \text{m/s} = \text{N} \cdot \text{s}$$

- A net force is required to change momentum. Newton's second law states that an unbalanced (net) force acting on a mass will accelerate the mass in the direction of the force. Another way of saying this is that a net force acting on a mass will cause the mass to change its momentum. We can rearrange the equation for Newton's second law to emphasize the change in momentum:

$$\mathbf{F}_{net} = m\mathbf{a} = m\left(\frac{\Delta\mathbf{v}}{\Delta t}\right)$$

Rearranging this equation by dividing both sides by Δt gives

$$\mathbf{F}\Delta t = m\Delta\mathbf{v} = m\mathbf{v}_f - m\mathbf{v}_i$$

The left side of the equation ($F\Delta t$) is called the impulse, and the right side is the change in momentum. This equation reflects the impulse-momentum theorem, and in words can be stated "a force acting on a mass during a time causes the mass to change its momentum". The force F in this equation is the average force acting over the time interval.

- Impulse-momentum theorem ($F\Delta t=\Delta p$)
 - The impulse ($F\Delta t$) of a force is the product of the average force and the time interval during which the force acts, which is also equal to Δp . Impulse is a vector quantity and has the same direction as the average net force. Impulse is very useful when dealing with forces that act over a short time and/or time varying forces—hitting a baseball with a bat, for instance (graph below).

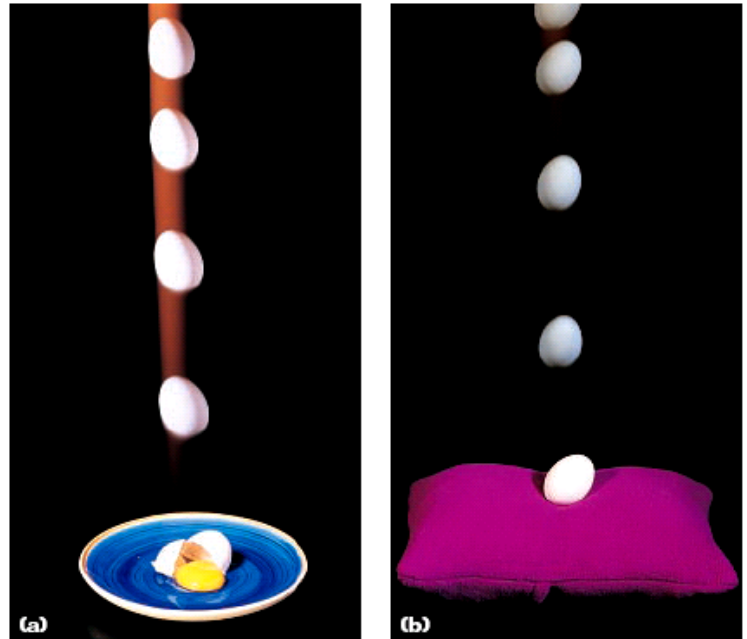
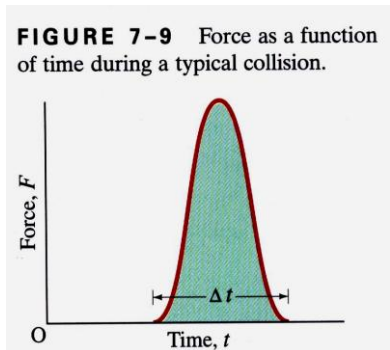


Figure 6-5
A large force exerted over a short time (a) causes the same change in the egg's momentum as a small force exerted over a longer time (b).

- A change in momentum over a longer time requires less force. The same impulse (same Δp) can be given to an object by a smaller force, F , if applied over a greater time interval, as long as $F\Delta t$ remains the constant. This is the reason why an egg dropped on a pillow does not break while an egg dropped from the same height on the plate does break.

- Example 1: A 0.40 kg soccer ball approaches a player horizontally with a velocity of 18.0 m/s. The player strikes the ball and causes it to move in the opposite direction with a velocity of 22.0 m/s. (a) Calculate the magnitude of the impulse delivered to the ball by the player. (b) If the player's foot is in contact with the ball for 0.120 s, find the magnitude of the average net force delivered to the ball by the player.

Law of conservation of momentum

- A net force is required to change an object's momentum (Newton's 2nd law). You probably remember from Newton's 3rd law that forces always occur in pairs (action-reaction), so when two objects interact the forces they apply on each other must be equal in magnitude and opposite in direction. Since the forces are of equal magnitude and they act over the same amount of time, the impulse on each object must also be of equal magnitude. Since each object experiences the same magnitude of impulse, the magnitude of the change in momentum of each object must also be the same. This is the basis for the law of conservation of momentum.
- The law of conservation of momentum states that the total momentum of all objects interacting with one another remains constant regardless of the nature of the forces between the objects. The law of conservation of momentum is particularly useful when dealing with situations where the forces are not constant such as collisions, explosions, or rocket propulsions—a form of a controlled explosion.

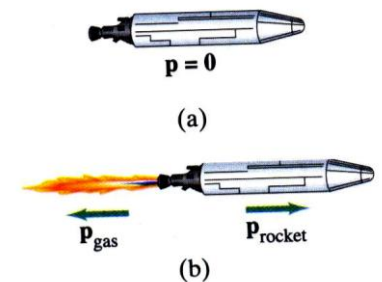


FIGURE 7-6 (a) A rocket, containing fuel, at rest in some reference frame. (b) In the same reference frame, the rocket fires and gases are expelled at high speed out the rear. The total vector momentum, $\mathbf{p}_{\text{gas}} + \mathbf{p}_{\text{rocket}}$, remains zero.

For example, look at figure 7-6. Before the rocket is fired, $p_{\text{total}} = 0$. As fuel burns, p_{total} remains unchanged ($p_{\text{total}} = 0$). The backward p of the expelled gas is just balanced by the forward p gained by the rocket. Thus, rockets can accelerate in empty space.

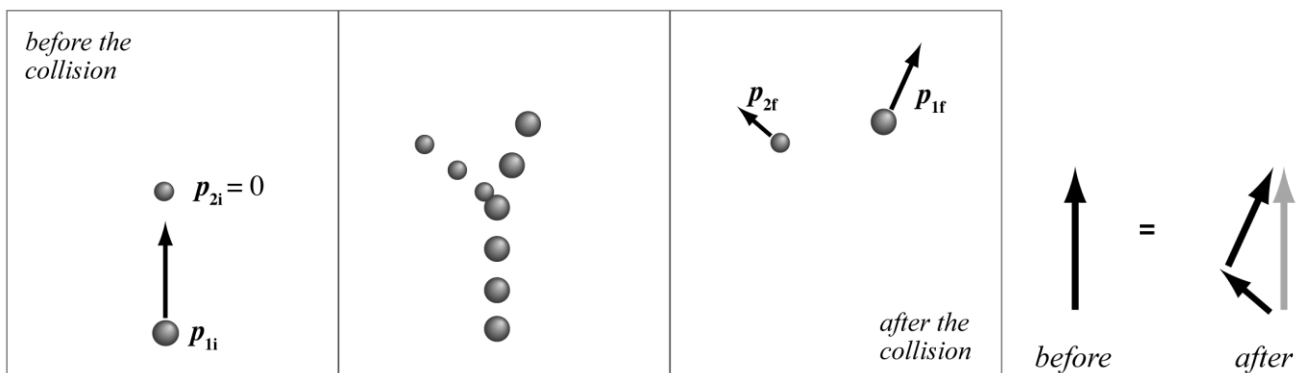
- Note that when applying the law of conservation of momentum the objects must be isolated. In an isolated system the only forces present are those between the objects of the system (internal forces—action-reaction pairs). The net external force must be zero ($\Sigma F_{\text{external}} = 0$). For example, you may think that momentum is not conserved for a falling rock since its speed and momentum are increasing as it falls; however, you must take into account that the earth is accelerating upward towards the rock. Momentum is conserved for the earth-rock system.

Collisions are separated into two categories based on kinetic energy conservation

1. Elastic collision -- both momentum and kinetic energy are conserved
2. Inelastic collision – momentum is conserved but not kinetic energy
 - A perfectly (or totally) inelastic collision is a special type of inelastic collision in which the two objects stick together and move as one object after the collision.

Type of collision	Diagram	What happens	Conserved quantity
perfectly inelastic		The two objects stick together after the collision so that their final velocities are the same.	momentum
elastic		The two objects bounce after the collision so that they move separately.	momentum kinetic energy
inelastic		The two objects deform during the collision so that the total kinetic energy decreases, but the objects move separately after the collision.	momentum

- Since momentum is a vector quantity, if a collision is two dimensional (glancing collision), you have to analyze the x and y independently. Momentum is conserved in both dimensions. Look at the diagram below. A ball moving north experiences a glancing collision with a ball at rest. Since the horizontal component of the initial momentum is zero, the total final momentum in the horizontal must also be zero. This means both the horizontal components are equal and oppositely directed. The sum of their vertical components must equal the initial balls momentum. The diagram below right shows how the vector sum is indeed equal to the initial momentum.



Steps for solving problems using conservation of linear momentum

- Decide which objects are included in the system. Sketch the situation.
- Make sure that the system is isolated (only internal forces present; $\Sigma F_{\text{external}} = 0$) or that the interactions are sufficiently short and intense that you can ignore external forces. If the system is not isolated, you would have to include the other objects causing the forces or use a different method (Newton's laws, conservation of energy, etc.) to solve the problem.
- Set the total final momentum equal to the total initial momentum. Solve for unknown quantities. Remember that momentum is a vector quantity so you must analyze x and y independently.

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$$

- Example 2: A 76.0 kg boater, initially at rest in a stationary 45.0 kg boat, steps out of the boat and onto the dock. If the boater moves out of the boat with a velocity of 2.5 m/s to the right, what is the final velocity of the boat?

- Example 3: A 1850 kg luxury sedan stopped at a traffic light is struck from the rear by a compact car with a mass of 975 kg. The two cars become entangled as a result of the collision. If the compact car was moving at a velocity of 22.0 m/s to the north before the collision, what is the velocity of the entangled mass after the collision?

- Example 4: Two clay balls collide head-on in a perfectly inelastic collision. The first ball has a mass of 0.500 kg and an initial velocity of 4.00 m/s to the right. The mass of the second ball is 0.250 kg, and it has an initial velocity of 3.00 m/s to the left. What is the final velocity of the composite ball of clay after the collision? What is the decrease in kinetic energy during the collision?

- Example 5: A 0.15 kg toy car moving to the right at 20.0 cm/s collides with a 0.20 kg car moving to the left at 30.0 cm/s. After the collision, the 0.15 kg car moves with a velocity of 37.1 cm/s to the left. Find the velocity of the 0.20 kg car after the collision.