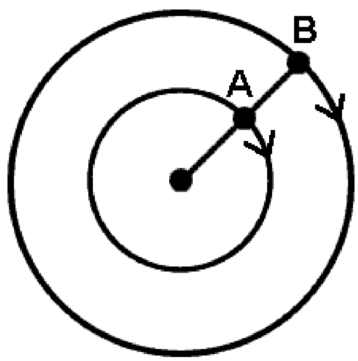
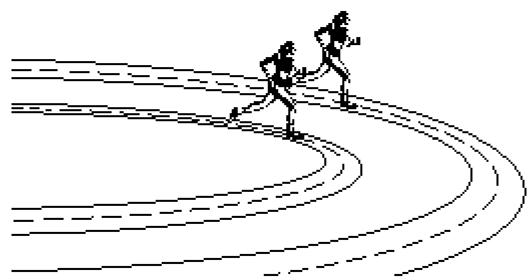
Pre-AP Physics Notes Ch 7-8
Circular Motion

I. Rotational and Circular Motion

- **Rotational motion** refers to the motion of a body or system that spins about an axis. The ***axis of rotation*** is the line about which the rotation occurs. **Circular motion** refers to the motion of a particular point on an object that is undergoing rotational motion. Because the ***direction*** of motion is continually changing, it is difficult to describe the motion of a point using only linear quantities; therefore, angular measurements are typically used because *all points on a rigid rotating object, except the points on the axis, move through the same angular displacement during the same time interval.* Angular motion is measured in units of ***radians*** **(1 rev = 2π rad = 360^o).** All points on a rigid rotating object have the same angular speed, but not the same linear (tangential) speed. The farther a point is from the axis of rotation, the faster the point is moving. As shown below, points A and B have the same angular speed, but point B has the larger tangential speed.





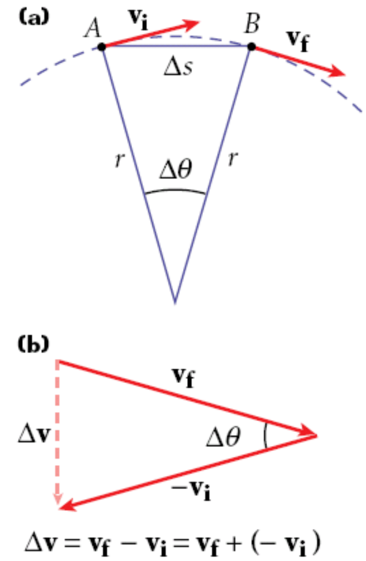
- **Uniform circular motion** refers to an object *moving in a circle at constant speed*. Speed of an object undergoing uniform circular motion can be calculated by distance/time. For one revolution the distance the point travels is the circumference of the circular path. The time it takes for the point to make one revolution is known as period, and since period is inversely related to frequency

  $v=\Delta x/t=2\pi r/T=2\pi rf$

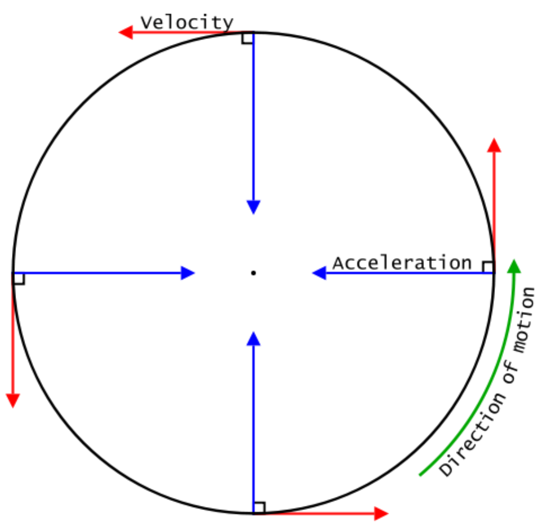
  - **Period (T)** – time required to complete one revolution; units are seconds
  - **Frequency (f)** – cycles per second; units are s^{-1} or Hertz (Hz); **T=1/f**

- ***Example 1.*** A child on a merry-go-round is moving with a speed of 1.35 m/s when 1.20 m from the center of the merry-go-round. Determine the time it takes for the child to make one revolution.

II. Centripetal acceleration



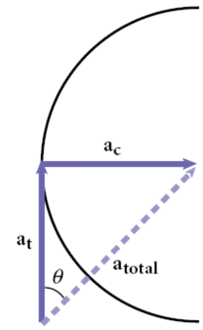
- An object that moves in a circle is accelerating even if its speed is constant. For an object undergoing uniform circular motion, the *magnitude* of the velocity (speed) remains constant, but the *direction* of the velocity is continually changing. Since acceleration is defined as the rate of change in velocity, *a change in direction of v constitutes acceleration* just as does a change in magnitude. Therefore, *an object moving in a circle is continuously accelerating, even when the speed remains constant.* This acceleration is called ***centripetal*** (***center seeking) acceleration*** and is *always* directed toward the ***CENTER*** of the circular path. For an object moving in a circle of radius *r* with constant speed *v* the magnitude of the centripetal acceleration is calculated by $\boldsymbol{a_c = v^2/r}$.
  - Centripetal acceleration depends on *v* & *r*. The greater the *v*, the faster the velocity changes direction so the larger the acceleration. The larger the radius, the less rapidly the velocity changes direction so the smaller the acceleration.
  - As shown in the diagram below, the velocity vector is always ***perpendicular*** to the centripetal acceleration. This is because the centripetal acceleration vector always points toward the center of the circle**,** but the velocity vector always points in the direction of motion (*tangent to the circular path*).



$$a_c = \frac{v^2}{r}$$

- ***Example 2.*** A jet plane traveling at 500 m/s pulls out of a dive by moving in an arc of radius 6.00 km. What is the plane's acceleration in *g*'s?

Centripetal vs. tangential acceleration



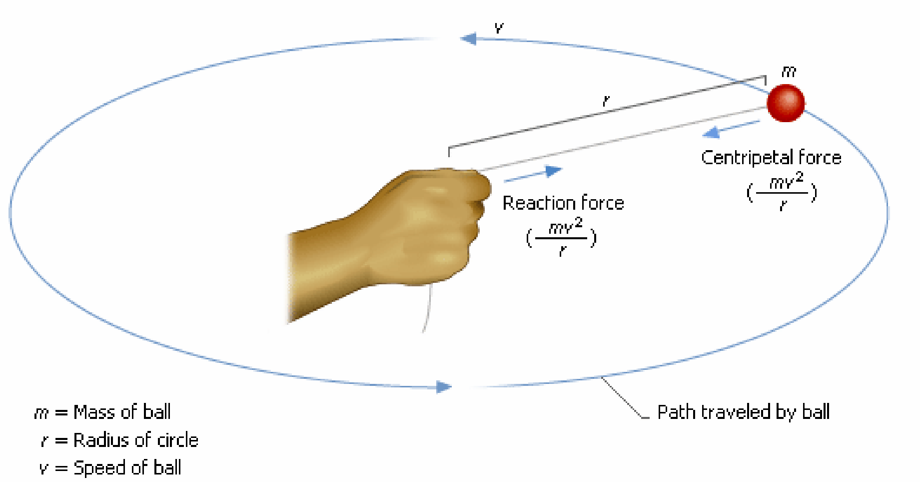
- Centripetal acceleration is experienced by any object undergoing circular motion and is always directed towards the center of the circular path. Centripetal acceleration changes direction of an object, *not* speed. ***Tangential acceleration*** occurs when *speed* of the object is changing and, as its name implies, is tangent to the circular path. *Tangential and centripetal acceleration are perpendicular.*

III. Causes of circular motion

- According to Newton's Second Law *($\Sigma F=ma$)*, an object that is accelerating must have a net force acting upon it. If an object is undergoing uniform circular motion, the net force is sometimes called the centripetal force. Careful, there is no centripetal force – it simply refers to the net force causing the centripetal acceleration. The actual force causing the centripetal acceleration is determined from the free-body diagram (tension, gravity, friction, normal force, etc).

- Since $\Sigma F=ma$ and $a_c=v^2/r$, the magnitude of the centripetal force equals mv^2/r or, written together, $\Sigma F_c=mv^2/r$. The direction of the centripetal force is the same as the centripetal acceleration (toward the ***center*** of the circular path). If this net force were not applied, the object would obey Newton's first law and fly off in a straight line ***tangent*** to the circular path.

- You have probably heard of **centrifugal (center fleeing) forces.** ***Centrifugal forces do NOT exist. There is no outward force!*** Ever swung an object on a string above your head? The misconception comes from "feeling" a pull on your hand from the string. This is simply Newton's 3rd law in reaction to the inward force you are putting on the string to keep the object moving in a circle. If you let go and there was a centrifugal force acting, then the object would fly ***OUTward*** when the string was released. ***This does NOT happen***. The object flies off ***tangentially*** to the circular path.



- ***Example 3.*** A 1,000 kg car rounds a curve on a flat road of radius 50m at a speed of 50 km/h. Will the car make the turn, or will it skid, if; a) the pavement is dry and $\mu_s = 0.60$? b) the pavement is icy and $\mu_s = 0.25$? Is the result independent of the mass of the car?

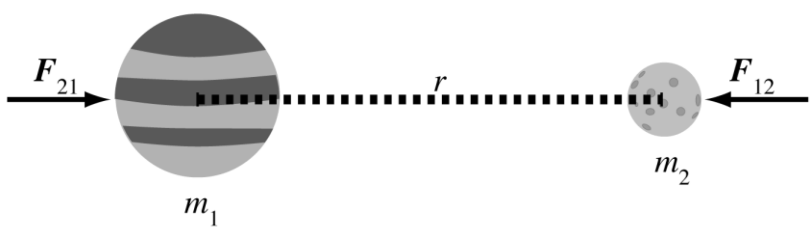
- ***Example 4.*** Tarzan (mass = 78.0 kg) swings from rest on a 30.0 m long vine of negligible mass which is initially inclined at an angle of 37.0^o with the vertical. Calculate Tarzan's speed and the tension in the vine at the bottom of the swing.

- ***Example 5.*** A 1,200 kg sports car moving at 20 m/s crosses the rounded top of a hill (radius = 115 m). Calculate the normal force on the car. Determine the minimum speed the car would have to have in order to just come of the ground at the top of the hill.

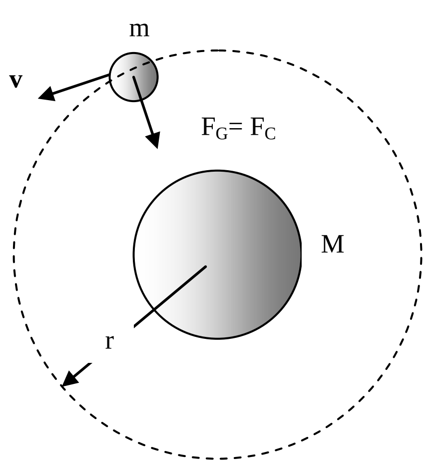
IV. Satellites in Circular Orbits

- You should remember from chapter 4 that every particle in the universe exerts an attractive force on every other particle. The magnitude of the force is proportional to the product of the two masses and inversely to the square of the distance between the two objects ***(inverse square law).***

$$F = G\frac{m_1 m_2}{r^2}$$



- As a satellite orbits the earth, it is pulled toward the earth with a gravitational force which is acting as a centripetal force. The inertia of the satellite causes it to tend to follow a straight-line path, but the gravitational force pulls it toward the center of the orbit.



- If a satellite of mass *m* moves in a circular orbit around a planet of mass *M*, we can set the centripetal force equal to the gravitational force and solve for the speed of the satellite orbiting at a particular distance *r*:

$$F_c = F_G$$
$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$
$$v = \sqrt{\frac{GM}{r}}$$

- ***Example 6.*** Many satellites circle the earth in orbits that are approximately twice the radius of the earth. What would be the speed of a satellite orbiting at $2R_e$?

V. Rotational equilibrium and torque

- **Torque** is a quantity that measures the ability of a force to rotate an object around some axis. Torque depends upon the component of force *perpendicular* to the axis of rotation and the distance of the force from the axis of rotation. A force applied parallel to the axis will not produce torque.

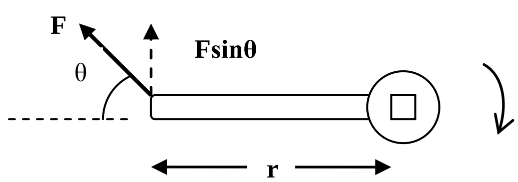
  τ = Fr (sin θ)

  τ = torque

  F (sin θ) = perpendicular component of the force

  r = length of the lever arm

  - **Lever arm –** (moment arm) the perpendicular distance from the axis of rotation to a line drawn along the direction of the force

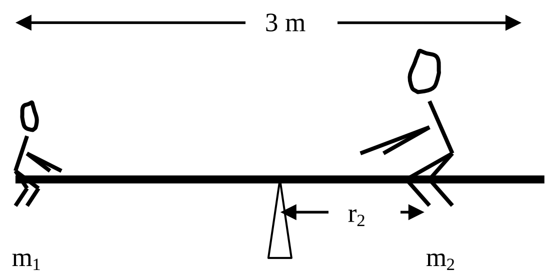


- ***Example 7.*** If the torque required to loosen a nut on the wheel of a car has a magnitude of 40.0 N m, what minimum force must be exerted by a mechanic at the end of a 30.0 cm wrench to loosen the nut?

- Torque can either be positive or negative depending on the direction the force tends to rotate the object; torque is *positive if the rotation is counterclockwise and negative if the rotation is clockwise.* If an object is in ***rotational equilibrium****, the net torque on the object is zero* (sum of positive and negative torques is zero).

This can be very useful in determining the magnitudes of forces acting on an object in equilibrium, as can be seen in the following examples.

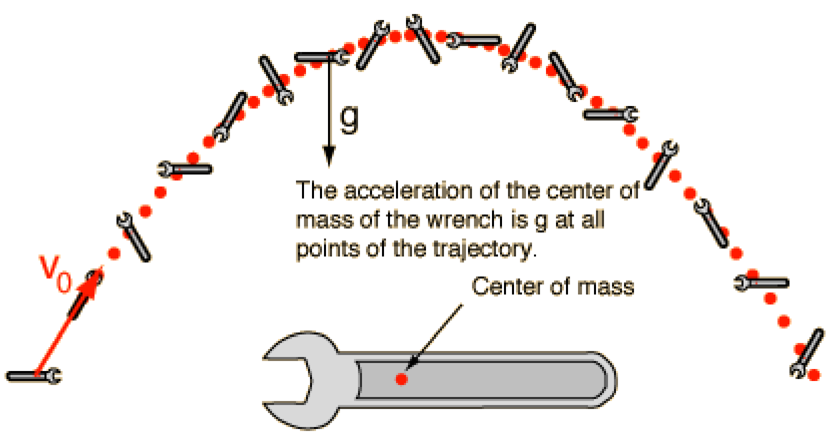
- ***Example 8.*** Two children sit on a see-saw which is 3 m long and pivoted on an axis at its center. The first child has a mass m_1 of 25 kg and sits at the left end of the see-saw, while the second child has a mass m_2 of 50 kg and sits somewhere on the see-saw to the right of the axis. At what distance r_2 from the axis should the second child sit to keep the see-saw horizontal?



- ***Example 9.*** A 700.0 N window washer is standing on a uniform scaffold supported by a vertical rope at each end. The scaffold weighs 200.0 N and is 3.00 m long. What is the tension in each rope when the window washer stands 1.00 m from one end?

VI. Center of Mass

- **Center of mass** is the point at which all the mass of the body can be considered to be concentrated. Center of mass for a rigid object is located along the line for which it will balance. As shown below, when gravity is the only force acting on a rotating object, it will rotate around its center of mass (technically that point would be the object's center of gravity, but for this book you can treat center of mass and center of gravity as equivalent terms).



VII. Difference between mass (translational inertia) and moment of inertia (rotational inertia)

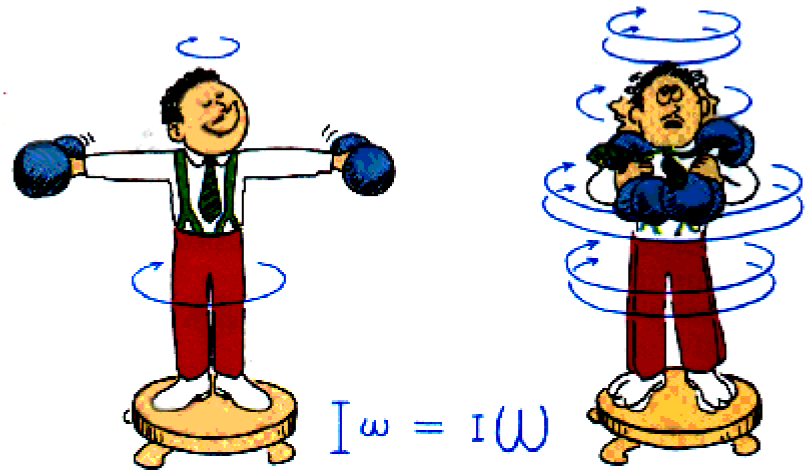
- **Mass** is a measure of the amount of matter of an object or simply the measure of inertia (resistance to changes in translational motion) for an object that is not rotating. Remember that mass is an intrinsic property of an object that does not change as long as matter is not lost or gained. **Moment of inertia** is the measure of inertia that resists changes in *rotational* motion. But unlike mass, moment of inertia is ***not*** an intrinsic property of an object, *since moment of inertia depends on an object's mass, distribution of mass, and axis of rotation. **Moment of inertia is the*** **rotational** ***analog of mass.***
- For point masses the moment of inertia can be calculated by $I = \Sigma mr^2$. The box below gives formulas for finding the moments of inertia for some common objects. For this class you will not have to calculate any moments of inertia but you will need to know that if two rotating objects have the same mass that the object that has more mass farther from the axis will have a larger moment of inertia. For example, a rolling ring will have a larger moment of inertia than a rolling disk of the same mass.

Table 8-1 The moment of inertia for a few shapes

Shape	Moment of inertia	Shape	Moment of inertia
thin hoop about symmetry axis	MR^2	thin rod about perpendicular axis through center	$\frac{1}{12}M\ell^2$
thin hoop about diameter	$\frac{1}{2}MR^2$	thin rod about perpendicular axis through end	$\frac{1}{3}M\ell^2$
point mass about axis	MR^2	solid sphere about diameter	$\frac{2}{5}MR^2$
disk or cylinder about symmetry axis	$\frac{1}{2}MR^2$	thin spherical shell about diameter	$\frac{2}{3}MR^2$

VIII. Conservation of angular momentum

- Rotating objects posses ***angular momentum*** which is the product of the moment of inertia times the angular speed. In Ch 6 you learned that linear momentum is always conserved as long as you include all internal forces ($\Sigma F_{external} = 0$). The same is true for angular momentum. Angular momentum is always conserved as long as the net external torque is zero, but unlike mass, moment of inertia can easily change. If moment of inertia decreases, angular speed increases. If moment of inertia increases, angular speed decreases. So, why does the spinning skater below speed up as he pulls his arms inward?



IX. Rotational kinetic energy and conservation of mechanical energy

- Spinning objects have ***rotational kinetic energy*** which is a separate quantity from translational kinetic energy. Changes in rotational kinetic energy must be considered when analyzing conservation of mechanical energy. As with angular momentum, rotational kinetic energy depends upon moment of inertia. So, if you start a solid disk and a thin ring with the same mass and diameter from rest at the top of a hill (see diagram below), which object will reach the bottom of the hill first and with the greatest speed assuming each rolls without slipping? Then, reverse the question. If each disk were rolling with the same speed at the bottom of the hill, which would travel farther up the hill assuming both roll without slipping?


Pre-AP Homework/Practice Problems:
Ch 7 #'s 37,38,39,43,47,48,52
Ch 8 #'s 9,20,46,50