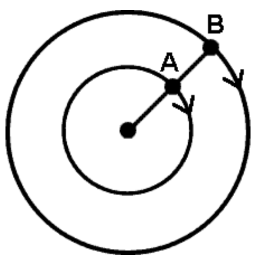
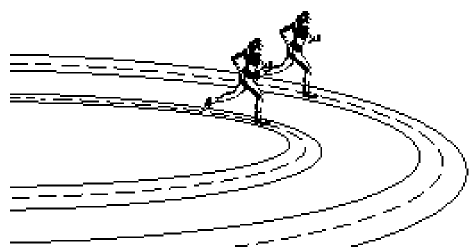
Physics Notes Unit 6- Circular Motion and Gravitation, Equilibrium, and Torque

I. Rotational and Circular Motion

- **Rotational motion** refers to the motion of a body or system that spins about an axis. The ***axis of rotation*** is the line about which the rotation occurs.
- **Circular motion** refers to the motion of a particular point on an object that is undergoing rotational motion. Because the ***direction*** of motion is continually changing, it is difficult to describe the motion of a point using only linear quantities; therefore, angular measurements are typically used because *all points on a rigid rotating object, except the points on the rotation axis, move through the same angular displacement during the same time interval.* Angular displacement is measured in units of ***radians*** **(1 rev = 2π rad = 360°) but we will not use radians in calculations.** Instead we will use the circumference as distance and for velocity it will be in m/s.
- All points on a rigid rotating object have the same angular speed, but not the same linear (tangential) speed. The farther a point is from the axis of rotation, the faster the point is moving. As shown below, points A and B have the same angular speed, but point B has the larger tangential (or linear) speed.





Uniform circular motion **refers to an object *moving in a circle at constant <u>speed</u>* … BUT it <u>cannot</u> have a constant <u>velocity</u> while traveling in a circle!!**

To find the linear (or tangential) speed of an object traveling in a circle: Use the first formula for speed: $v = \frac{\Delta x}{t}$ but the distance or Δx in the case of something going around a circular path would be the circumference $C = 2\pi r$, this gives you the speed by $v = \frac{2\pi r}{T}$, where *T* is the Period or time for one revolution or cycle.

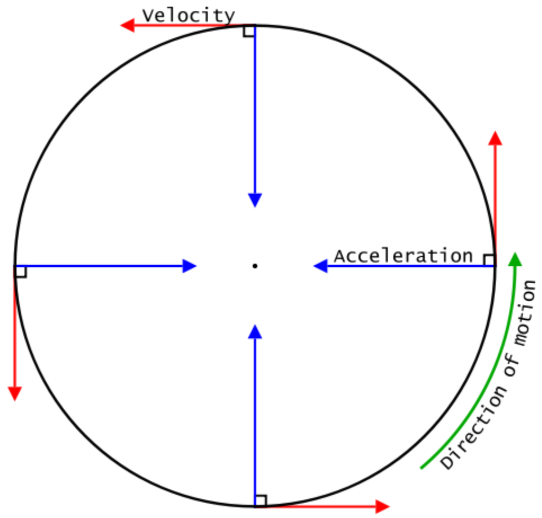
II. Centripetal acceleration

- An object that moves in a circle at ***constant speed***, v, is said to experience ***uniform circular motion.*** The ***magnitude*** of the velocity remains constant, but the ***direction*** of the velocity is continuously changing. Since acceleration is defined as the rate of change in velocity and velocity includes speed and direction, ***a change in direction of v is an acceleration***. Therefore, ***an object moving in a circle is continuously accelerating, even when the speed remains constant.*** This acceleration is called ***centripetal acceleration.***
- **Centripetal acceleration** – "***center seeking";*** an object moving in a circle of radius **r** with constant speed **v** has an acceleration ***directed toward the center of the circle*** and whose magnitude can be calculated by

$$a_c = \frac{v^2}{r}$$

- This acceleration depends on **v & r**; the greater **v**, the faster the velocity changes direction; the larger the radius, the less rapidly the velocity changes direction

- ***The centripetal acceleration vector points toward the center of the circle, BUT* the velocity vector always points in the direction of the motion (tangent to the circular path).** In uniform circular motion, Velocity and centripetal acceleration are always perpendicular to each other.

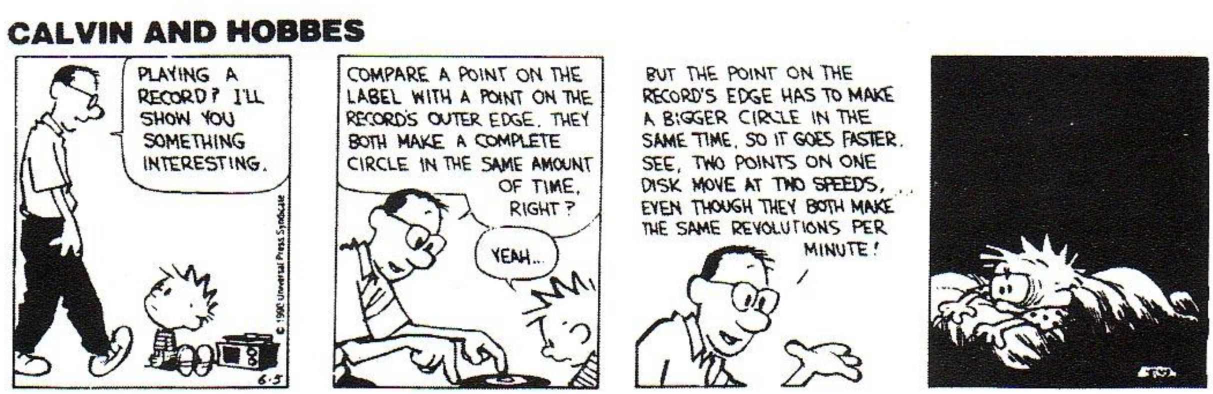


$$a_c = \frac{v^2}{r}$$

Link to: Centripetal Force and acceleration on bowling ball
http://paer.rutgers.edu/PT3/experiment.php?topicid=5&exptid=56

- Speed of an object undergoing uniform circular motion can be calculated by distance/time. For one revolution the distance the point travels is the circumference of the circular path. The time it takes for the point to make one revolution is known as period (T), and since period is inversely related to frequency (f) the following is true: $v = \frac{\Delta x}{t} = \frac{2\pi r}{T}$

- Period (T) – time required to complete one revolution; units are seconds



III. Forces or Causes of circular motion

- According to Newton's Second Law (ΣF = ma), an object that is accelerating must have a net force acting upon it; if an object is undergoing uniform circular motion, we call that net force the centripetal force. Careful, there is no **special or fundamental** centripetal force – there is no special category of force here – it is simply a possible net force. **The net force causing the centripetal acceleration can be any of the following:**

  tension in a string or rubber band…
  gravity…
  friction…
  a normal or support force!

- So according to Newton's Second Law:

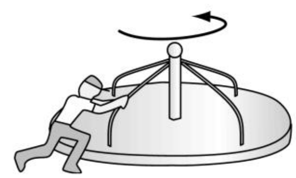
  $F_{net} = ma$ and if the acceleration is in a circular path…it is a centripetal acceleration or "a_c"

$$a_c = \frac{v^2}{r}$$

therefore the acceleration and *the net force are directed toward the center of the circular path.*

If this net force were not applied, the object would obey Newton's first law and fly off in a straight line ***tangent*** to the circular path. **This center-seeking force is called a centripetal force and is** $F_c = ma_c = \dfrac{mv^2}{r}$

Link to ball trapped in circular motion in a ring until the ring is removed:
http://paer.rutgers.edu/PT3/experiment.php?topicid=5&exptid=57

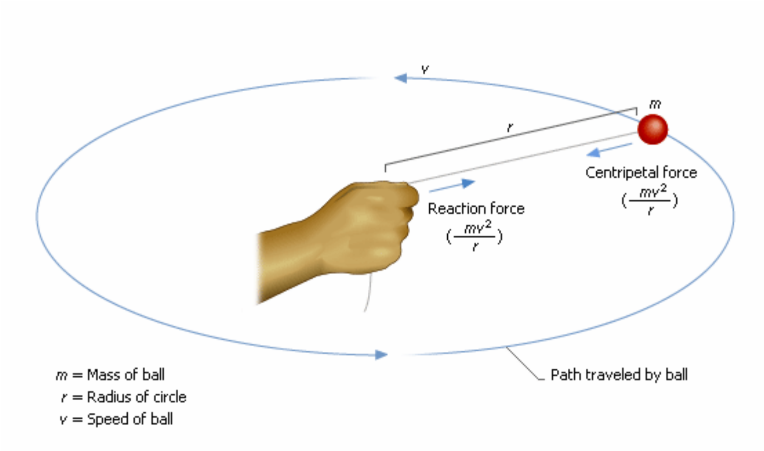


Example 1. A 30kg child on a merry-go-round is moving with a speed of 1.35 m/s when 1.20 m from the center of the merry-go-round. Determine the time it takes for the child to make one revolution. Then calculate the centripetal acceleration and net force on the child.

The equation for ANY force causing circular motion can be set equal to the formula for centripetal force F_c in order to solve problems! Friction force can be equal to F_c, gravity can be equal to the F_c and so forth.

Example 2. A 1,000 kg car rounds a curve on a flat road of radius 50m at a speed of 50 km/h. Will the car make the turn, or will it skid, if; a) the pavement is dry and $\mu_s = 0.60$? b) the pavement is icy and $\mu_s = 0.25$? Does the result depend on the mass of the car?

- You have probably heard of **centrifugal (center fleeing) forces.** ***Centrifugal forces do NOT exist. There is <u>no outward force</u> on the object that is rotating!*** REMEMBER: Centri-<u>F</u>UGAL = <u>F</u>AKE FORCE! Ever swung an object on a string above your head? The misconception comes from "feeling" a pull on your hand from the string. This is simply Newton's 3rd law in reaction to the inward force you are putting on the string to keep the object moving in a circle. **(Hand pulls inward on Ball & Ball pulls outward on Hand BUT there is no outward force on the ball.)** If you let go and there was a centrifugal FAKE force acting, then the object would fly straight ***OUTward*** from the center when the string was released. ***This does NOT happen.*** The object flies off ***tangentially*** to the circular path.



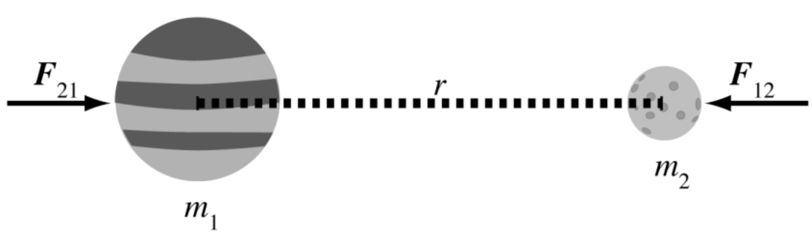
Example 3. A pilot is flying a small plane at 30.0 m/s in a circular path with a radius of 100.0 m. If a force of 635 N is needed to maintain the pilot's circular motion, what is the pilot's mass?

Example 4. A jet plane traveling at 500 m/s pulls out of a dive by moving in an arc of radius 6.00 km. What is the plane's acceleration in *g's*?

IV. Newton's law of universal gravitation [worksheet on gravitation conceptual math]

- Every particle in the universe exerts an attractive force on every other particle
- Gravitational force is proportional to the product of the two masses and inversely proportional to the distance between two masses squared (follows ***inverse square law***)
- Gravitational force acts between the ***center of mass*** of the two objects (see diagram below)

$$F = G\frac{m_1 m_2}{r^2}$$



Where F is the Force of gravity between two objects, G = $6.67x10^{-11}\frac{N \bullet m^2}{kg^2}$ G is the gravitational constant, m in the formula (not in the units of G) is mass, and r is the distance between the two objects. These weird units for G are what's required to solve for the gravitational force in the original equation.

Example 5

Find the distance between a 0.300 kg billiard ball and a 0.400 kg billiard ball if the magnitude of the gravitational force is 8.92×10^{-11} N.

Also be prepared for conceptual math problems about gravity. For example:

If the original force due to gravity between two objects is 30N, then what would be the force between them if they each had double the mass?

b. Same scenario, but this time they are moved to 3 times as far away from each other?

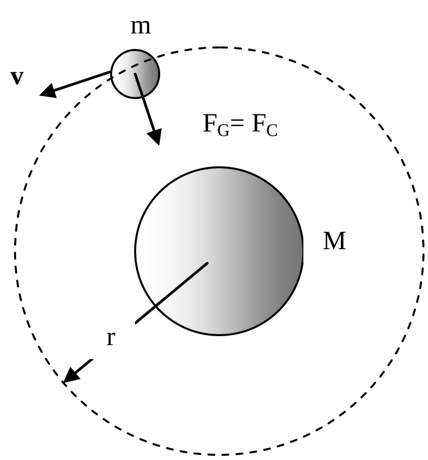
c. How about 1/3 as far from each other?

d. This time one of them has double the mass and they are moved to ½ the original distance from one another.

V. Satellites in Circular Orbits

Gravity can be the cause of a centripetal acceleration!

- As a satellite orbits the earth, it is pulled toward the earth with a gravitational force which is acting as a centripetal force. <u>The inertia of the satellite causes it to tend to follow a straight-line path, but the centripetal gravitational force pulls it toward the center of the orbit.</u>



- If a satellite of mass *m* moves in a circular orbit around a planet of mass *M*, we can set the centripetal force equal to the gravitational force and solve for the speed of the satellite orbiting at a particular distance.

$$F_c = F_G$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

KEEP in mind: You can only use this equation **to find velocity when gravity is supplying the centripetal force!!**

Example 6

Many satellites circle the earth in orbits that are approximately twice the radius of the earth. What would be the speed of a satellite orbiting at **$2R_e$**? **Mass of Earth: 6.0×10^{24} kg**
Radius of Earth: 6.37×10^{6} m

Example 7

The Moon is held in its orbit by the force of gravity between the Earth and the moon.
Calculate the speed of the moon in its orbit if: the distance center to center of Earth to the moon is 3.84×10^{8} m. The mass of earth is 5.97×10^{24} kg and the mass of the moon is 7.32×10^{22} kg.

Ch 8 Torque and Equilibrium

VI. The magnitude of torque

- **Torque** – a force that tends to cause rotation. Torque depends upon the component of force perpendicular to the lever arm and the lever arm distance is measured from the axis of rotation to the point where the force is applied. A force applied parallel to the axis will not produce torque.
- ***Another way to say this: Torque is equal to the perpendicular force times the lever arm distance.***

$$\tau = Fd \quad \text{or} \quad \tau = F_{\perp} r$$

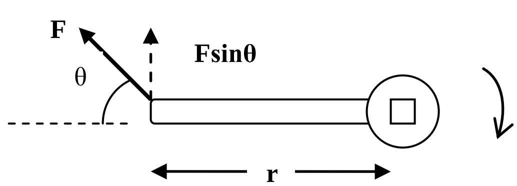
or if the force is not applied in a perpendicular manner to the lever arm $\tau = Fd (\sin \theta)$

τ = torque (symbol is called "Tau")

$F_{\perp}$ = **perpendicular component of force if at an angle [calculated as F ($\sin \theta$)]**

r or d = **length of the lever arm**

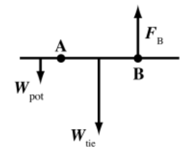
- **Lever arm** – (also called "moment" arm) is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force
- Torque can either be positive or negative depending on the direction the force tends to rotate the object; positive if the rotation is counterclockwise and negative it the rotation is clockwise



Example 8

If the torque required to loosen a nut on the wheel of a car has a magnitude of 40.0 N m, what minimum force must be exerted by a mechanic at the end of a 30.0 cm wrench to loosen the nut?

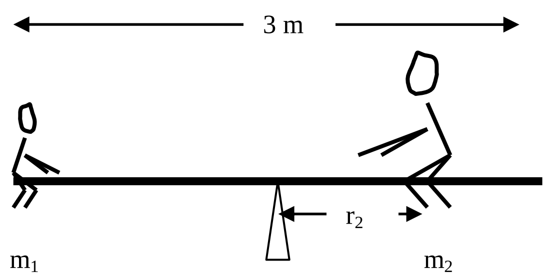
Rotational equilibrium: ***If the sum of all the torques acting on an object is equal to zero, the object will not rotate.*** If the CW torques and the CCW torques acting on an object are balanced so the net torque is zero, the object will be in **rotational equilibrium**.
For example in the case of a see saw: If the person that is twice as heavy as the other person sits half as far away from the point of rotation (fulcrum or axis of rotation) the torques will be balanced and the see saw will not accelerate in either direction…it is in **equilibrium**.



Expect to calculate unknown distances or masses/forces using the idea of balanced torques (like in the "keeping in balance" lab!) To do such calculations it is necessary to construct a **"lever diagram" like the one shown here.**

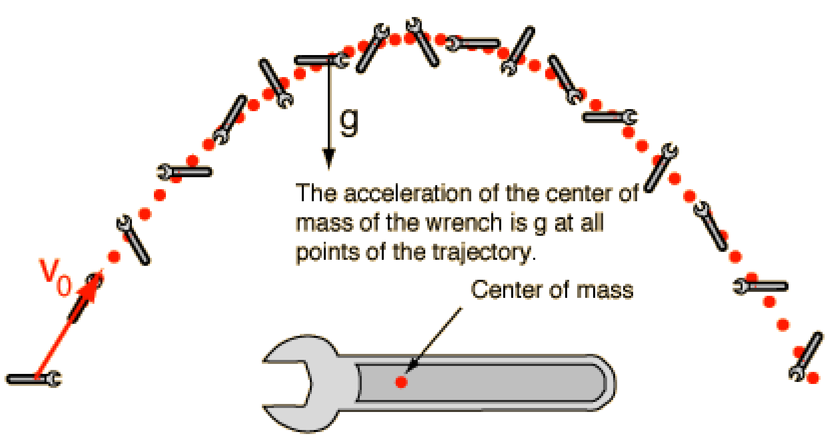
If an object is in ***rotational equilibrium****, the counter-clockwise torques and the clockwise torques must be balanced. The net torque on the object is zero* (sum of positive and negative torques is zero). This can be very useful in determining the magnitudes of forces acting on an object in equilibrium, as can be seen in the following examples.

Example 9. Two children sit on a see-saw which is 3 m long and pivoted on an axis at its center. The first child has a mass m_1 of 25 kg and sits at the left end of the see-saw, while the second child has a mass m_2 of 50 kg and sits somewhere on the see-saw to the right of the axis. At what distance r_2 from the axis should the second child sit to keep the see-saw horizontal?



VII. Center of mass – (also called **Center of Gravity or CG** for our purposes) the point at which all the mass of the body can be considered to be concentrated when analyzing motion.

If gravity is the only force acting on a rotating object (as in the diagram below), it will rotate around its center of mass (technically that point would be the object's center of gravity**, but for this course you can treat center of mass and center of gravity as equivalent terms)**



Example 10:
The center of gravity of a certain meterstick (mass = 145 g) is at the 48cm mark and there is a 62g mass hanging at the 40 cm mark. The fulcrum of this system is at the 80cm mark. Calculate where you would have **to place** a 550g mass to produce equilibrium. Draw a lever diagram of the situation before you calculate.

VIII. Moment of Inertia and rotational motion

Difference between mass (or inertia) and moment of inertia (or rotational inertia)

- **Mass** – measure of the amount of matter of an object or simply the measure of inertia of an object. That is the resistance an object has to changes in motion; **MASS** <u>**is an *intrinsic***</u> ***property of an object that does not change as long as matter is not lost or gained.***

- **Moment of inertia** – measure of inertia that resists changes in rotational motion; **unlike mass, moment of inertia is** <u>**not an intrinsic property of an object**</u>, ***since moment of inertia depends on more than just an object's mass. It depends on the distribution of mass (or where the mass is located relative to the axis of rotation) also.***

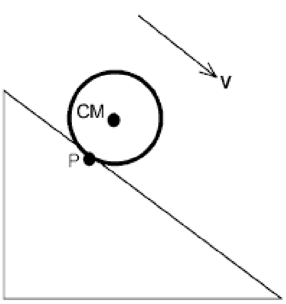
- *Moment of inertia is the* rotational *analog of mass – an object with a large moment of inertia would be more resistant to changing its rotational motion.*

The moments of inertia (I) for several shapes are:

I for Disk = $1/2mr^2$ I for hoop = mr^2 I for solid ball = $2/5mr^2$ m is mass

Questions: If they all had the same mass, which of these (disk, hoop or ball) would be hardest to start to rotate?
Which would be easiest?
In what order would these reach the bottom of a hill when released from the top at the same time?

Gravity exerts a force on the center of gravity (CG) of an object. If the CG of the object is not over its base of support, the object will not be "supported" and it will topple or roll over. A torque is created by the lever arm distance between the point of support and the place where the weight vector would intersect the ramp or support surface. (show Torque and Pregnancy)



The rotational inertia and torque work together to give some interesting results when comparing objects rolling down an incline.
Demo: ring versus solid disk, and weighted ring

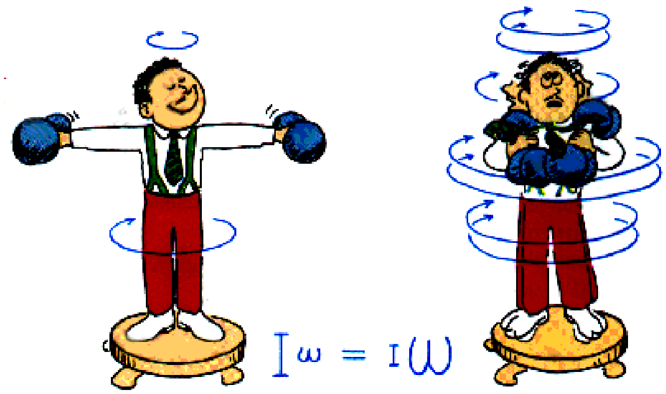
Link to race of solid snow packed bottle versus water filled bottle.
http://paer.rutgers.edu/PT3/experiment.php?topicid=5&exptid=85

IX. Rotating objects and Angular Momentum

Law of <u>conservation</u> of angular momentum - If no external torques act on a rotating system, **the <u>object's angular momentum will not change</u>**! **The product of rotational inertia times rotational speed is called angular momentum.**

The formula is L=Iω, where **L** is angular momentum, **I** is rotational inertia (or moment of inertia), and **ω** is rotational speed.

The object can change its rotational inertia (or moment of inertia) and when it does so, its rotational speed will change in such a way that the product of the two will remain the same.
As you can tell, if "**I**" **increases then "ω" must decrease in order for L to remain constant and conserved. I and ω are inversely proportional.**



Demo: weights and the rotating platform
How could you double the rotational inertia?
When you do that, what happens to the rotational speed?
What happens to the rotational speed if you were to cut the rotational inertia to 1/3 the original?
Why do ice skaters put their foot out to the side and spread their arms out when they want to stop spinning so fast?

Physics homework in textbook:
Ch. 7 p.269-271: **Parts 1-2: #'s 4, 13, 14, 16-19, and 25**
Part 3: #'s 28, 31, 33-37, 39, and 48
Ch. 8 p.305-308: **#'s 2, 3, 6, 9, 14, 18, 30-31, and 34**