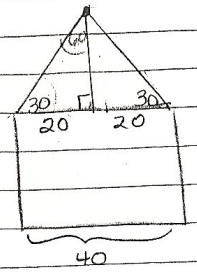
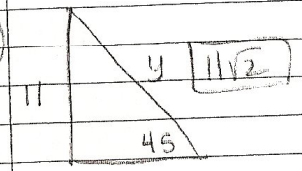
F09 A16: Final Exam Review

1) 

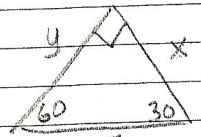
30 60 90
X X√3 2X
20

X√3 = 20
X = 20√3 / √3√3
X = 20√3 / 3
≈ 11.5

2) 

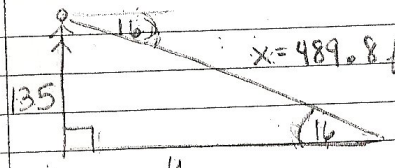
45 45 90
X X X√2
11√2

y = 11√2

3) 

30 60 90
X X√3 2X
5/2 5√3/2 2X = 5
X = 5/2

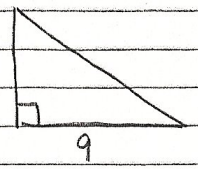
X = 5√3/2 y = 5/2

4) 

x = 489.8 ft

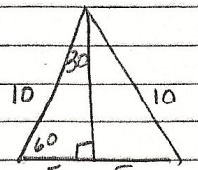
sin 16° = 135/x
x sin 16 = 135
x = 135 / sin 16
x = 489.8 ft

tan 16 = 135/y
y tan 16 = 135
y = 135 / tan 16 = 470.8

5) 

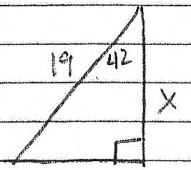
$6^2 + 9^2 = c^2$
$36 + 81 = c^2$
$117 = c^2$
$10.8 \approx c$

Perimeter = 6 + 9 + 10.8 = 25.8 cm

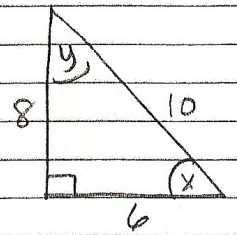
6) 

altitude: $5\sqrt{3}$

$A = \frac{1}{2}(10)(5\sqrt{3})$
$A = 5(5\sqrt{3})$
$A = 25\sqrt{3}$ cm²

7) 

$\cos 42 = \frac{x}{19}$
$19(\cos 42) = x$
$14.1 \approx x$

8) 

$\sin x = \frac{8}{10}$
$\sin^{-1}(8/10) \approx 53.1°$

$\sin y = \frac{6}{10}$
$\sin^{-1}(6/10) \approx 36.9°$

or: $90 - 53.1 \approx 36.9$

9) $(x-5)^2+(y-10)^2=49$

center: $(5,10)$ $\quad r: \sqrt{49}=7$

10) Solve: $\begin{cases}(x-1)^2+(y-1)^2=4\\ x+y=4 \Rightarrow y=4-x\end{cases}$

(I) $(x-1)^2+(4-x-1)^2=4$

$(x-1)^2+(-x+3)^2=4$

$(x-1)(x-1)+(-x+3)(-x+3)=4$

$x^2-2x+1+x^2-6x+9=4$

$\frac{2x^2-8x+10}{2}=\frac{4}{2}$

$x^2-4x+5=2$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3, \quad x=1$

Let $x=3$	Let $x=1$
$y=4-x$	$y=4-x$
$y=4-3$	$y=4-1$
$y=1$	$y=3$

$(3,1)$ $\qquad$ $(1,3)$

You can also solve this using your calculator!

11) $(-2,7) \quad r=\sqrt{5}$

$(x--2)^2+(y-7)^2=(\sqrt{5})^2$

$(x+2)^2+(y-7)^2=5$

12) Solve: $\begin{cases}(x+1)^2+(y+1)^2=4\\ x^2+y^2=4\end{cases}$

Change to $y=$ form + use graphing calculator to solve:

$(y+1)^2=4-(x+1)^2$

$y+1=\pm\sqrt{4-(x+1)^2}$

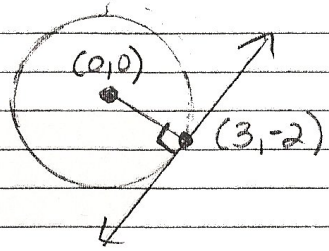
$y=-1\pm\sqrt{4-(x+1)^2}$

$y^2=4-x^2$

$y=\pm\sqrt{4-x^2}$

Solutions: $(.8,-1.9)$ $\quad (-1.8,.8)$

13) $x^2+y^2=13 \quad (3,-2)$



(I) $m=\frac{-2-0}{3-0}=-\frac{2}{3}$

(II) use negative rec. slope

(III) $y=mx+b$

$-2=\frac{3}{2}(3)+b$

$-2=\frac{9}{2}+b$

$-6.5=b$

Equation of Tangent Line

$y=\frac{3}{2}x-6.5$

15) distance: $\sqrt{(3--2)^2+(8-2)^2+(1-1)^2} = \sqrt{61}$

mid pt: $\left(\frac{3+-2}{2}, \frac{8+2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 5, 1\right)$

14) Find radius: $\frac{-5-3}{-2-2} = \frac{-8}{-4} = 2$

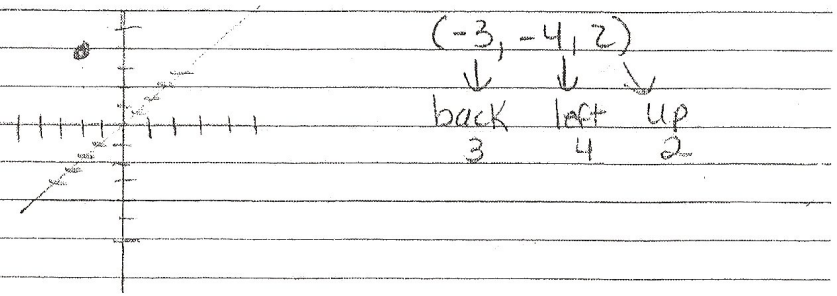
Center $(2,3)$ $r = 4\sqrt{5}$

$(x-2)^2+(y-3)^2 = (4\sqrt{5})^2$

$\boxed{(x-2)^2+(y-3)^2 = 80}$

16) You would start at the origin $(0,0,0)$ and go forward 2, left 7, up 10.

17)



$(-3, -4, 2)$

back 3, left 4, up 2

18) $-4x+3y+2z+12=0$

$-4x+3y+2z=-12$

x intercept	y intercept	z intercept
$-4x=-12$	$3y=-12$	$2z=-12$
$x=3$	$y=-4$	$z=-6$
$\boxed{(3,0,0)}$	$\boxed{(0,-4,0)}$	$\boxed{(0,0,-6)}$

19) $\sqrt{(-7-0)^2+(5-0)^2+(10-0)^2}$

$\sqrt{49+25+10} = \sqrt{84} = \sqrt{2\cdot 42}$

$\sqrt{2\cdot 2\cdot 3\cdot 7}$

$\boxed{2\sqrt{21}}$

20) a) 7 edges
b) 5 vertices
c) 4 paths
d) 2 paths

21) Use Calculator: $\begin{bmatrix} -5 & -12 \\ 36 & 32 \end{bmatrix}$

22) $\det(C) = 43$

23) $D^{-1} = \begin{bmatrix} -1/4 & 0 \\ -5/6 & -1/3 \end{bmatrix}$

24) $B \times C = \begin{bmatrix} -32 & 34 & -12 \\ -57 & 37 & -26 \end{bmatrix}$

25) $-2x+1=13$ $\quad$ $3y+2=14$

$-2x=12$ $\quad$ $3y=12$

$x=-6$ $\quad$ $y=4$

26) $\begin{cases} 2x+3y+4z=7 \\ -x+5y+2z=6 \\ 3x+6y+0z=3 \end{cases}$

$$\underset{A}{\begin{bmatrix} 2 & 3 & 4 \\ -1 & 5 & 2 \\ 3 & 6 & 0 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{B}{\begin{bmatrix} 7 \\ 6 \\ 3 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -.2 \\ .6 \\ 1.4 \end{bmatrix}$$

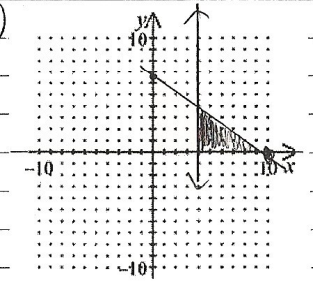
$x = -.2 \quad y = .6 \quad z = 1.4$

27) a) Let x = # of small dollhouses Let y = # of large dollhouses

Constraints: $x \geq 4$, $y \geq 0$, $4x+6y \leq 40$

Objective function: $P = 12x + 20y$

b)



c) Corner points: (10,0) (4,4) (4,0)

$P = 12(10) + 20(0) = \$120$

$P = 12(4) + 20(4) = \$128$

$P = 12(4) + 20(0) = \$48$

d) He should make 4 small + 4 large for a max. profit of \$128.00

28) $\left(\frac{4a^3b^{-3}}{a^{-2}b^2}\right)^{-2} = \frac{4^{-2}a^{-6}b^6}{a^4b^{-4}} = \frac{1}{4^2}a^{-10}b^{10} =$

$$\boxed{\frac{b^{10}}{16}}$$

29) $\left(\frac{3z}{x^{-4}}\right)^2 \left(\frac{3x^{-12}yz^{-3}}{2xy^7}\right)^{-3} =$

$$\frac{3^2z^2}{x^{-8}} \cdot \frac{3^{-3}x^{36}y^{-3}z^9}{2^{-3}x^{-3}y^{-21}} =$$

$$\frac{3^{-1}x^{36}y^{-3}z^{11}}{2^{-3}x^{-11}y^{-21}} = \frac{2^3x^{47}y^{18}z^{11}}{3} =$$

$$\boxed{\frac{8x^{47}y^{18}z^{11}}{3}}$$

30) $(4a^3b^2)(5a^4b^7)^2$

$4a^3b^2 \cdot 5^2a^8b^{14} = \boxed{100a^{11}b^{16}}$

31) $(16)^{-3/4} = \left(\frac{1}{16}\right)^{3/4} = \left(\sqrt[4]{\frac{1}{16}}\right)^3 = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

32) $\underbrace{(25a^3b^9c^{-10}d^4e^2f^{-2}g^{-1})^0}(4a^4b^2)$

$1 \cdot 4a^4b^2 = \boxed{4a^4b^2}$

33) $(49)^{1/2} = \sqrt{49} = 7$

34) $f(x) = \frac{2}{3}x + 3$

$y = \frac{2}{3}x + 3$

$x = \frac{2}{3}y + 3$

$x - 3 = \frac{2}{3}y$

$\frac{3}{2}(x-3) = y$

$\frac{3}{2}x - \frac{9}{2} = f^{-1}(x)$

35) $f(x) = \frac{x-2}{6}$

$y = \frac{x-2}{6}$

$x = \frac{y-2}{6}$

$6x = y - 2$

$6x - 2 = y$

$6x - 2 = f^{-1}(x)$

36) Given: $\{(3,4)(-2,7)(1,9)(5,9)\}$

a) yes b) D: $\{-2, 1, 3, 5\}$
R: $\{4, 7, 9\}$

c) $\{(4,3)(7,-2)(9,1)(9,5)\}$ d) No

37) $f(x) = x + 3$ $g(x) = 3x^2 - 2$

a) $f(g(x))$
$f(3x^2-2)$
$3x^2 - 2 + 3$
$3x^2 + 1$

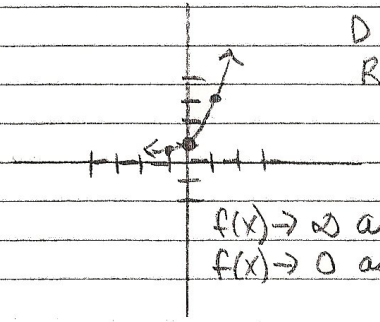
b) $g(f(x))$
$g(x+3)$
$3(x+3)^2 - 2$
$3(x+3)(x+3) - 2$
$(3x+9)(x+3) - 2$
$3x^2 + 9x + 9x + 27 - 2$
$3x^2 + 18x + 25$

c) $f(g(-3))$
$f(3(-3)^2 - 2)$
$f(3(9) - 2)$
$f(25)$
$25 + 3$
28

d) $g(f(5))$
$g(5+3)$
$g(8)$
$3(8)^2 - 2$
$3(64) - 2$
190

38) a) $f(x) = 3^x$

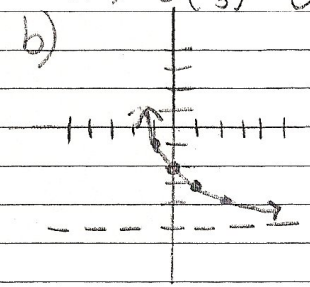
x	y
0	1
1	3
-1	1/3



D: $\mathbb{R}$
R: $\{y \mid y > 0\}$

$f(x) \to \infty$ as $x \to \infty$
$f(x) \to 0$ as $x \to -\infty$

39) b) $f(x) = 3\left(\frac{2}{3}\right)^x - 5$



x	y
0	-2
1	-3
-1	$-1\frac{1}{2}$
2	$-1\frac{1}{3}$

D: $\mathbb{R}$

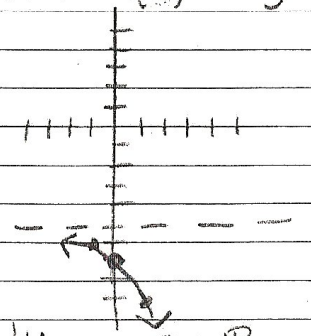
R: $\{y \mid y > -5\}$

$f(x) \to -5$ as $x \to \infty$

$f(x) \to \infty$ as $x \to -\infty$

V.S by a factor of 3
moves down 5

c) $f(x) = -(2)^{x+1} - 5$



x	y
0	-7
-1	-6
1	-9
2	-13

D: $\mathbb{R}$

R: $\{y \mid y < -5\}$

$f(x) \to -\infty$ as $x \to \infty$

$f(x) \to -5$ as $x \to -\infty$

reflects down,
moves left 1, down 5

40) $25^{x-4} = 5^{3x+1}$

$(5^2)^{x-4} = 5^{3x+1}$

$2(x-4) = 3x+1$

$2x - 8 = 3x + 1$

$-9 = x$

41) $250(1.15)^3 = 380.2$ mg

42) a) $450{,}000(1.023)^5 \approx 520{,}000$

b) 15 years

43) $A = 5000\left(1 + \frac{.065}{2}\right)^{(2 \cdot 12)}$

$= \$10{,}772.87$