



1. What variables or unknowns are involved?
2. What quantity is to be maximized or minimized and how do I express that quantity in terms of my unknowns?
3. What constraints do I have? How can I express those constraints in terms of my unknowns? (In linear programming problems, this step results in a set of linear inequalities.)

## Example - potter making cups and plates

A potter is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses  $\frac{3}{4}$  lb. of clay and each plate uses one lb. of clay. She has 20 hours available for making the cups and plates and has 250 lbs. of clay on hand. She makes a profit of \$2 on each cup and \$1.50 on each plate. How many cups and how many plates should she make in order to maximize her profit?

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$$(6 \text{ min./cup})(x \text{ cups}) + (3 \text{ min./plate})(y \text{ plates}) \leq (20 \text{ hrs.})(60 \text{ min./hr.}) \text{ or}$$

$$6x + 3y \leq 1200$$

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$(3/4 \text{ lb. of clay/cup})(x \text{ cups}) +$   
 $(1 \text{ lb. of clay/plate})(y \text{ plates}) \leq 250 \text{ lbs. of clay}$

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**Non-negative constraints:**  $x \geq 0, y \geq 0$

## Summary:

$x$  = number of cups the potter makes

$y$  = number of plates the potter makes

The potter wants to maximize profit

$$P = \$2x + \$1.50y$$

## Constraints:

$$6x + 3y \leq 1200$$

$$.75x + y \leq 250$$

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Step 2

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Step 3

## Farmer planting corn and soybeans

A farmer has a 320 acre farm on which she plants two crops: corn and soybeans. For each acre of corn planted, her expenses are \$50 and for each acre of soybeans planted, her expenses are \$100. Each acre of corn requires 100 bushels of storage and yields a profit of \$60; each acre of soybeans requires 40 bushels of storage and yields a profit of \$90. If the total amount of storage space available is 19,200 bushels and the farmer has only \$20,000 on hand, how many acres of each crop should she plant in order to maximize her profit? What will her profit be if she follows this strategy?

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**\$ constraint:**  $50x + 100y \leq 20,000$

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**\$ constraint:**  $50x + 100y \leq 20,000$

**storage constraint:**  $100x + 40y \leq 19,200$

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**\$ constraint:**  $50x + 100y \leq 20,000$

**storage constraint:**  $100x + 40y \leq 19,200$

**non-negative constraints:**  $x \geq 0, y \geq 0$

## Example - Aluminum and Copper Wire

A plant makes aluminum and copper wire. Each pound of aluminum wire requires 5 kwh of electricity and  $\frac{1}{4}$  hr. of labor. Each pound of copper wire requires 2 kwh of electricity and  $\frac{1}{2}$  hr. of labor. Production of copper wire is restricted by the fact that raw materials are available to produce at most 60 lbs./day. Electricity is limited to 500 kwh/day and labor to 40 person-hrs./day. If the profit from aluminum wire is \$.25/lb. and the profit from copper is \$.40/lb., **how much of each should be produced to maximize profit and what is the maximum profit?**

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**Constraints:**

$$y \leq 60$$

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**Constraints:**

$$y \leq 60$$

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**Constraints:**

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$$5x + 2y \leq 500$$

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$$x \geq 0$$

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## Sofa Factories

A company makes two types of sofas, regular and long, at two locations, one in Hickory and one in Lenoir. The plant in Hickory has a daily operating budget of \$45,000 and can produce at most 300 sofas daily in any combination. It costs \$150 to make a regular sofa and \$200 to make a long sofa at the Hickory plant. The Lenoir plant has a daily operating budget of \$36,000, can produce at most 250 sofas daily in any combination and makes a regular sofa for \$135 and a long sofa for \$180. The company wants to limit production to a maximum of 250 regular sofas and 350 long sofas each day. If the company makes a profit of \$50 on each regular sofa and \$70 on each long sofa, how many of each type should be made at each plant in order to maximize profit? What is the maximum profit?

# Sofa Factories

x = regular sofas made in Hickory

y = long sofas made in Hickory

z = regular sofas made in Lenoir

w = long sofas made in Lenoir

**\$ constraint at Hickory:**  $150x + 200y \leq 45,000$

**Hickory sofa limit:**  $x + y \leq 300$

**\$ constraint at Lenoir:**  $135z + 180w \leq 36,000$

**Lenoir sofa limit:**  $z + w \leq 250$

**regular sofa limit:**  $x + z \leq 250$

**long sofa limit:**  $y + w \leq 350$

**non-neg:**  $x \geq 0, y \geq 0, z \geq 0, w \geq 0$

**Maximize profit**  $P = 50x + 70y + 50z + 70w$