



Student Study Guide

5.5 The Quadratic Formula

Objectives

- Use the quadratic formula to find real roots of quadratic equations.
- Use the roots of a quadratic equation to locate the axis of symmetry of a parabola.

Glossary Terms

quadratic formula

Rules and Properties

Quadratic Formula

If $ax^2 + bx + c = 0$ and $a \neq 0$, then the solutions, or roots, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Axis of Symmetry of a Parabola

If $ax^2 + bx + c = 0$, where $a \neq 0$, then the equation for the axis of symmetry of the parabola is $x = -\frac{b}{2a}$.

Key Skills

Use the quadratic formula to find the real roots of quadratic equations.

Solve $x^2 + 5x - 1 = 0$.

$a = 1, b = 5, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{29}}{2}$$

$x \approx 0.19$ or $x \approx -5.19$

The coordinates of the vertex of the graph of $f(x) = ax^2 + bx + c$ are $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Exercises

Use the quadratic formula to solve each equation. Round answers to the nearest tenth.

1. $x^2 - 3x - 7 = 0$

$\frac{3 \pm \sqrt{57}}{2}$

2. $4x^2 - 23x - 6 = 0$

$6, -1/4$

3. $x^2 + 9x = 3$

$\frac{-9 \pm \sqrt{93}}{2}$

4. $2x^2 + 18 = 15x$

$6, \frac{3}{2}$

5. $2(x^2 + x) = 5$

$\frac{-1 \pm \sqrt{11}}{2}$

6. $x^2 = 7x + 15$

$\frac{7 \pm \sqrt{109}}{2}$

7. $8 = x^2 + x$

$\frac{-1 \pm \sqrt{33}}{2}$

8. $12x^2 + x = 2$

$\frac{-1 \pm \sqrt{97}}{24}$

For each quadratic function, write the equation for the axis of symmetry, and find the coordinates of the vertex.

9. $f(x) = x^2 + 12x + 20$

$x = -\frac{b}{2a}$
 $(-6, -16) \quad x = -6$

10. $g(x) = x^2 - 11x + 30$

$(5.5, -25)$

11. $f(x) = x^2 - 7x + 2$

$(3.5, -10.25)$

$x = \frac{-(-11)}{2(1)} = \frac{11}{2}$

$x = \frac{11}{2}$



Objectives

- Classify and find all roots of a quadratic equation.
- Graph and perform operations on complex numbers.

Glossary Terms

absolute value of a complex number complex plane discriminant double root
 rationalizing the denominator imaginary axis real axis imaginary number

Rules and Properties

Solutions of a Quadratic Equation

Let $ax^2 + bx + c = 0$, where $a \neq 0$.

- If $b^2 - 4ac > 0$, then the quadratic equation has 2 distinct real solutions.
- If $b^2 - 4ac = 0$, then the equation has 1 real solution, a double root.
- If $b^2 - 4ac < 0$, then the equation has 0 real solutions. \rightarrow 2 complex solutions (imaginary)

Key Skills

Find and classify all roots of a quadratic equation.

Solve $x^2 + 4 = 0$. $a = 1, b = 0, c = 4 \rightarrow b^2 - 4ac = 0 - 4(4) = -16$

The discriminant is negative so, the solutions are complex: $x = \frac{-0 \pm \sqrt{-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$

Graph and perform operations on complex numbers.

$|3 + 5i| = \sqrt{3^2 + 5^2} = \sqrt{34}$ $\frac{2+i}{3+i} = \frac{(2+i)(3-i)}{(3+i)(3-i)} = \frac{7+i}{10}$

$(6+i)(1+i) = 6+i+6i+i^2 = 5+7i$

To graph a number on the complex plane, graph the real part using the horizontal axis and the imaginary part using the vertical axis.

Exercises

Find the discriminant first!

Determine the number of real solutions. Then solve, giving the answers in the form $a + bi$.

1. $x^2 - 2x + 11 = 0$ $-40, 2 \text{ complex } 1 \pm i\sqrt{10}$

2. $2x^2 + 3x + 12 = 0$ $-87, 2 \text{ complex } -3 \pm \sqrt{87}$

3. $x^2 + x + 1 = 0$ $-3, 2 \text{ complex } \frac{-1 \pm i\sqrt{3}}{2}$

4. $x^2 - 3x + 10 = 0$ $-31, 2 \text{ complex } \frac{3 \pm i\sqrt{31}}{2}$

Simplify each expression.

5. $(11 + 3i) + (7 - 4i) = 18 - i$

6. $(3 + 8i) - (7 - 2i) = -4 + 10i$

7. $(14 - 3i)(2 + 5i) = 43 + 64i$

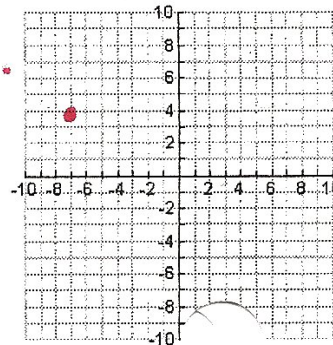
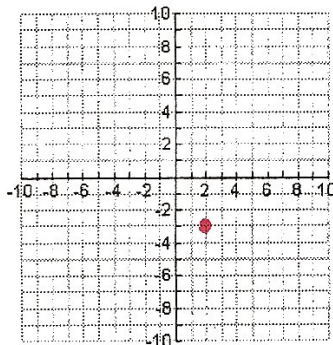
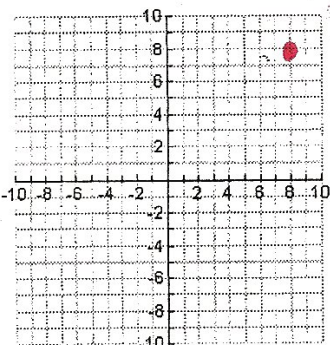
8. $\frac{2 - 7i}{5 + i} = \frac{3 - 37i}{26}$

Evaluate and graph each absolute value on your own paper.

9. $|8 + 8i| = 8\sqrt{2}$

10. $|2 - 3i| = \sqrt{13}$

11. $|-7 + 4i| = \sqrt{65}$





Student Study Guide

5.8 Solving Quadratic Inequalities

Objectives

- Write, solve, and graph a quadratic inequality in one variable.
- Write, solve, and graph a quadratic inequality in two variables.

Glossary Terms

quadratic inequalities in two variables

Key Skills

Solve and graph quadratic inequalities in one variable.

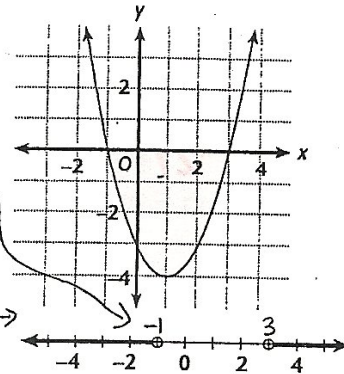
$$x^2 - 2x - 3 > 0$$

↑
Shades
← →

Factor: $(x-3)(x+1) = 0$

$$\begin{aligned} x-3 &= 0 & x+1 &= 0 \\ x &= 3 & x &= -1 \end{aligned}$$

These are the boundary pts



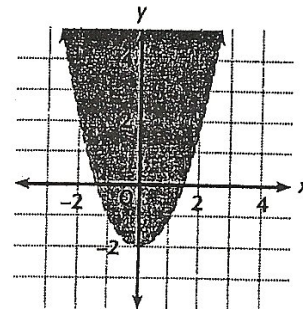
Remember:

greater than $>$ shades ← →
less than $<$ shades ○ ○

Solve and graph quadratic inequalities in two variables.

$$y > x^2 - 2$$

Graph $y = x^2 - 2$ and test a point, such as $(0, 0)$.
Since $0 > 0 - 2$, shade the region containing $(0, 0)$.



Exercises

Solve each quadratic inequality. Using your own paper, graph the solution on a number line.

1. $4x^2 - 28x + 49 > 0$ 2. $2x^2 - 15x + 7 < 0$ 3. $x^2 + 9x + 20 > 0$ 4. $2x^2 \leq 13x$

omit

5. $x^2 + 3x \geq 10$ 6. $10x < x^2 + 9$ 7. $3x^2 + 7x - 6 \leq 0$

Graph each quadratic inequality on your own paper.

8. $y > 2(x-1)^2$ 9. $y < x^2 - 6x + 3$ 10. $y \geq x^2 + 3x - 2$ 11. $y \geq -x^2 + 5x + 3$

8) $(x-9)(x-1)$

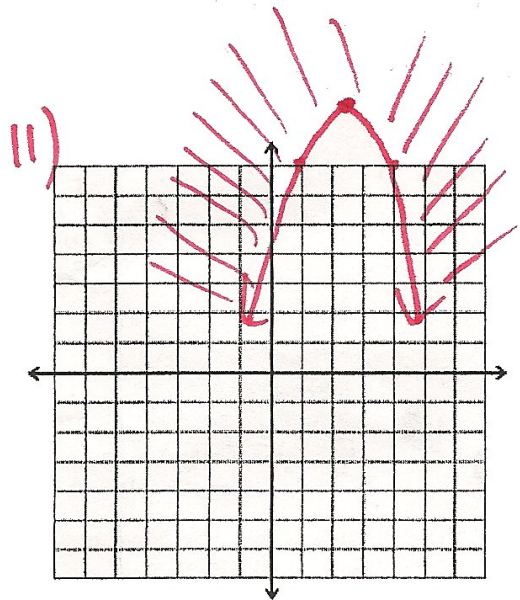
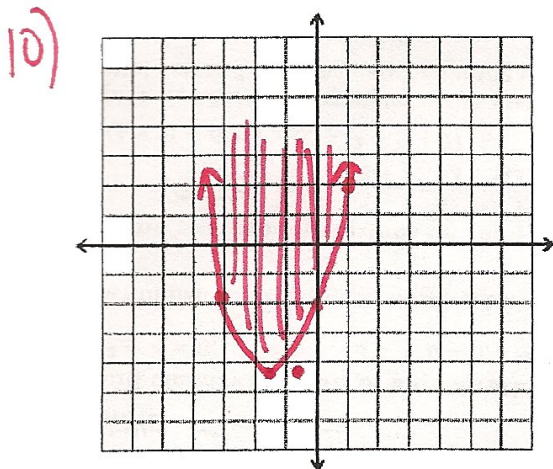
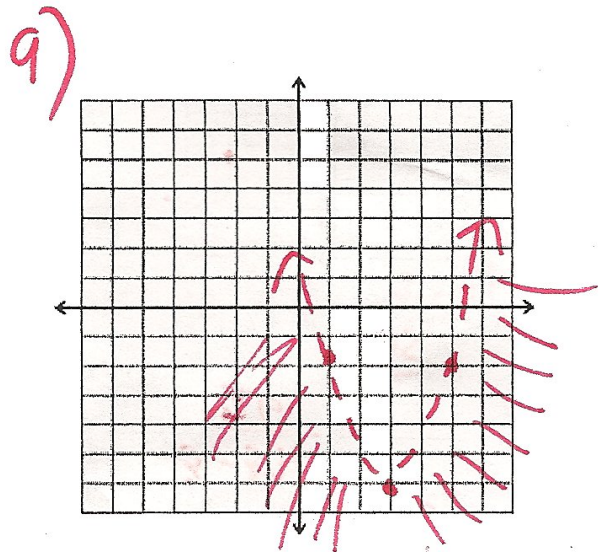
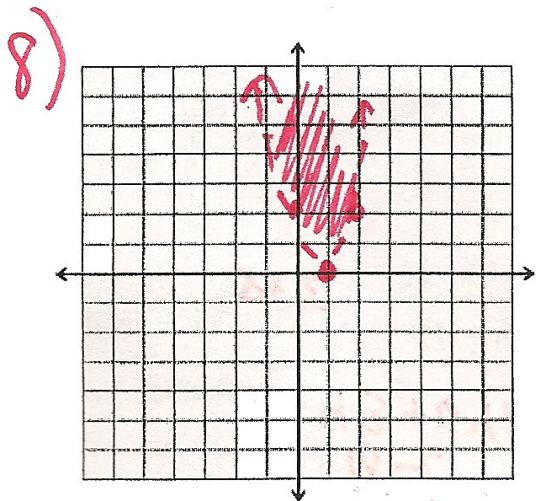
$$0 < x^2 - 10x + 9$$

$$0 = (x-9)(x-1)$$

$$x-9=0 \quad x-1=0$$

$$x=9, \quad x=1$$



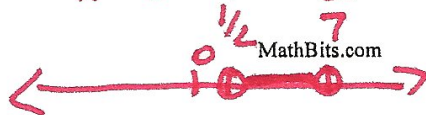


#1-7
1) omit

2) $(2x-14)(2x-1)$

$(x-7)(2x-1)=0$

$x=7 \quad x=1/2$



3) $x^2+9x+20 > 0$

$(x+4)(x+5)$

$x=-4 \quad x=-5$



4) $2x^2-13x \leq 0$

$x(2x-13)=0$

$x=0 \quad 2x-13=0$
 $2x=13$
 $x=13/2$
 6.5



5) $x^2+3x-10 \geq 0$

$(x+5)(x-2)=0$

$x=-5 \quad x=2$



7) $3x^2+7x-6 \leq 0$

$(3x+9)(3x-2)=0$

$(x+3)(3x-2)=0$

$x=-3 \quad x=2/3$

