

Problems Solved!

An Algebra Study Guide

Part I: *Elementary Algebra*

- ✓ *Over 850 Solved Problems*
- ✓ *Useful Tips and Explanations*
- ✓ *7 Sample Chapter Exams*
- ✓ *Sample Midterm and Final Exam*

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<http://infinity.cos.edu/algebra>





Algebra Study Guide

A Solutions Oriented Approach

Preface:

This study guide is designed to supplement your current textbook. Your text is well written in a patient style and the appropriate sections should be read before each class meeting.

With that said, *Problems Solved* is a solutions oriented approach to Algebra. I wanted to show students what steps and presentations to use when submitting work. When solving problems we need to communicate our thought process by showing steps wherever possible. The question I always ask, and you should ask yourself too, is “if I gave my work to a friend would he be able to actually read it and follow the steps?” Often times I receive homework assignments with numbers and calculations strewn about the paper with the answer circled somewhere in the middle. Despite the common misconception, the correct answers in Mathematics are very important but are only half the story. The ability to communicate correct solutions is just as important. This guide shows what steps to include when working Algebra problems.

In addition, I hope you will benefit from the sample exams. Take time to try the problems without looking at the answers or solutions; do not be afraid to get them wrong. Trust me, the real learning occurs when you figure out *why* you are solving problems incorrectly!

I hope that you find this guide useful and that it makes your experience in Algebra more enjoyable. If you have any suggestions or comments feel free to email me.

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Algebra Study Guide

A Solutions Oriented Approach

Part I: Elementary Algebra

This study guide was created to show students what written solutions in Algebra should look like. Often times we do not know exactly what steps to show or even how to present our answers. To use this guide simply flip to the appropriate section and scan for similar problems to those assigned by the instructor. You will not find the exact same problem but it is likely that you can find one very similar. When solving your particular assigned problems you should submit solutions that have the same basic steps presented in this guide.

*If you are not sure what a solution should look like
then this manual is for you!*



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If your math skills are rusty then this is the place to start. In this section we will review some of the basics of arithmetic. Mathematics is a skill that is easily lost over time. It definitely helps to warm up before jumping into Algebra. Spend some extra time in the Distributive Property section. It is one of our more important properties that will be used often.

PreAlgebra Review

*1.1 ... Real Numbers and their Operations**1.2 ... Fractions**1.3 ... Percents**1.4 ... Order of Operations**1.5 ... Evaluating Variable Expressions**1.6 ... Distributive Property**Sample Exam**Sample Exam Solutions*

For the most part, Beginning Algebra will focus on real numbers. A number is real if it can be written as a fraction or a decimal.

Natural Numbers – subset of real numbers $\{1, 2, 3, 4, 5, \dots\}$

Whole Numbers – add zero to the list of Natural Numbers $\{0, 1, 2, 3, 4, 5, \dots\}$

Integers – include all negative natural numbers $\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

Rational Numbers – any number of the form $\frac{a}{b}$ where a and b are integers.

Irrational Numbers – numbers that cannot be written as a ratio such as $\sqrt{5}$ or π .

When first learning how to subtract real numbers it is useful to think of this operation as adding the opposite. For example, $5 - 2$ can be thought of as adding the opposite of two to five, $5 - 2 = 5 + (-2) = 3$. This may seem cumbersome, but sometimes it can help in understanding problems where you wish to subtract a negative number.

Subtract $5 - (-2)$	
$ \begin{aligned} 5 - (-2) &= 5 + 2 && \leftarrow \\ &= 7 \end{aligned} $	<i>The opposite of -2 is +2.</i>

When adding two negative numbers the result will be negative. If you spend \$12.00 for a pair of shoes and \$25.00 for a bag you will owe \$37.00 at checkout, $-12 - 25 = -37$.

A. Add and Subtract

$7 + 12 - 5$
$ \begin{aligned} 7 + 12 - 5 \\ = 19 - 5 \\ = 14 \end{aligned} $

$-3 - 9 + 5$
$ \begin{aligned} -3 - 9 + 5 \\ = -12 + 5 \\ = -7 \end{aligned} $

$-4 - 5 + (-3)$
$ \begin{aligned} -4 - 5 + (-3) \\ = -9 + (-3) \\ = -12 \end{aligned} $

$-2 + (-7) - 4$
$ \begin{aligned} -2 + (-7) - 4 \\ = -9 - 4 \\ = -13 \end{aligned} $

$4 - (-12) + (-8)$
$ \begin{aligned} 4 - (-12) + (-8) \\ = 4 + 12 + (-8) \\ = 16 + (-8) \\ = 8 \end{aligned} $

$20 - (-17) + (-10) - 6$
$ \begin{aligned} 20 - (-17) + (-10) - 6 \\ = 20 + 17 - 10 - 6 \\ = 37 - 10 - 6 \\ = 27 - 6 \\ = 21 \end{aligned} $

Multiplying Real Numbers(Negative) x (Positive) = (Negative) Example: $(-5)(6) = -30$ (Negative) x (Negative) = (Positive) Example: $(-5)(-6) = 30$

When multiplying two negative numbers the result will be positive. The rules for division are the same. Be careful when simplifying $5(-6)$, the operation here is multiplication **NOT** subtraction so $5(-6) = -30$.

B. Multiply or Divide

$-2(-10)$
$-2(-10)$
$= 20$

$-5 \cdot 12$
$-5 \cdot 12$
$= -60$

$0(-100)$
$0(-100)$
$= 0$

$5 \cdot 2(-3)(-5)$
$5 \cdot 2(-3)(-5)$
$= 10(-3)(-5)$
$= -30(-5)$
$= 150$

$(-1)(-1)(-1)(-1)$
$(-1)(-1)(-1)(-1)$
$= 1 \cdot (-1)(-1)$
$= -1 \cdot (-1)$
$= 1$

$-6(-3)(-1)$
$-6(-3)(-1)$
$= 18(-1)$
$= -18$

$54 \div (-2)$
$54 \div (-2)$
$= -27$

$-100 \div (-25)$
$-100 \div (-25)$
$= 4$

$-15 \div 3$
$-15 \div 3$
$= -5$

Multiplication by zero is zero, but division is not so straight forward. Dividing by zero is undefined. What happens when you try to divide by zero on a calculator?

Zero and Division

$$\frac{0}{a} = 0 \div a = 0 \text{ where } a \neq 0 \quad \text{and} \quad \frac{a}{0} = a \div 0 \text{ is undefined}$$

$0 \div (-15)$
$0 \div (-15)$
$= 0$

$4 \div 0$
$4 \div 0$
undefined

$0 \div 0$
$0 \div 0$
undefined

Study Tip: The first week of class is a good time to explore the campus. Check out the Student Union, library and tutorial center, keeping an eye out for a nice quiet spot that is conducive to studying and completing homework.

Do not do your homework in front of the TV!



Absolute Value – the distance between 0 and the real number a on the number line, denoted $|a|$.

The absolute value of any real number will always be positive since it is a distance.

C. Simplify

$ -15 $
$ -15 $
$= 15$

$ 100 $
$ 100 $
$= 100$

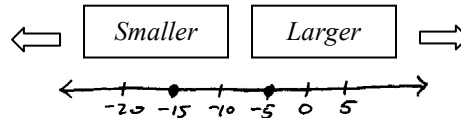
$ -1000 $
$ -1000 $
$= 1000$

$? = 5$
$? = 5$
$ -5 = 5 \text{ or } 5 = 5$
two answers $\{-5, 5\}$

$3 \cdot ? = 12$
$3 ? = 12$
$3 -4 \quad 3 4 $
$= 3 \cdot 4 \quad = 3 \cdot 4$
$= 12 \quad = 12$
two answers $\{-4, 4\}$

$? = -3$
$? = -3$
No Solution

When comparing real numbers, the larger number will always lie to the right of the other on a number line. It is clear that 15 is greater than 5, but it might not be so clear to see that -5 is greater than -15.



Inequalities

$\leq$ “less than or equal to”	$<$ “less than”
$\geq$ “greater than or equal to”	$>$ “greater than”

D. Write the appropriate symbol, either $<$ or $>$

$2 \underline{\hspace{1cm}} 12$
$2 \underline{<} 12$
$2 < 12$

$-2 \underline{\hspace{1cm}} -12$
$-2 \underline{>} -12$
$-2 > -12$

$-48 \underline{\hspace{1cm}} 0$
$-48 \underline{<} 0$
$-48 < 0$

E. List 3 integers satisfying the statement

Less than -5
$? < -5$
$\{-20, -15, -10\}$

Greater than -2
$? > -2$
$\{-1, 0, 1\}$

Between -2 and 7
$-2 < ? < 7$
$\{-1, 0, 5\}$

Answers may vary.

Squares and Square Roots

To **square a number** means to multiply that number times itself. For example:

$$5^2 = 5 \cdot 5 = 25$$

The number 5 is called the **base** and the integer 2 is called the **exponent**. Another way to read the above example is “5 raised to the second power.” Here are some more examples.

F. Simplify

$(-3)^2$	$(1.2)^2$	7^2
$(-3)^2 = (-3)(-3) = 9$	$(1.2)^2 = (1.2)(1.2) = 1.44$	$7^2 = 7 \cdot 7 = 49$
0^2	-4^2	$(-4)^2$
$0^2 = 0 \cdot 0 = 0$	$-4^2 = -4 \cdot 4 = -16$	$(-4)^2 = (-4)(-4) = 16$

It is important to point out the last two solved problems, what is different?

The square root can be thought of as the opposite operation. In other words, if we want the square root of 16 the question is “what squared gives 16?” Actually there are two answers to this question 4 and -4 because:

$$4^2 = 16 \quad \text{and} \quad (-4)^2 = 16$$

So there is a technical distinction here, when we are asked for the square root of numbers in the course we mean the **principal (nonnegative) square root**.

G. Simplify

$\sqrt{64}$	$\sqrt{100}$	$\sqrt{1}$
$\sqrt{64} = 8$ because $8^2 = 64$	$\sqrt{100} = 10$ because $10^2 = 100$	$\sqrt{1} = 1$ because $1^2 = 1$
$\sqrt{0}$	$\sqrt{50}$	$\sqrt{17}$
$\sqrt{0} = 0$ because $0^2 = 0$	$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$ ≈ 7.07	$\sqrt{17} \approx 4.123$ on a calculator

We will talk more about square roots and nth roots in Intermediate Algebra but for now we might find this property useful $\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$ when A and B are positive and real.

Fractions can be one of the biggest barriers to beginning algebra students. Also referred to as rational numbers, fractions are simply real numbers that can be written as a quotient, or ratio, of two integers.

Fraction – A ratio, $\frac{a}{b}$ or $\frac{\text{numerator}}{\text{denominator}}$, where a and b are integers.

Equivalent fractions can be expressed with different numerators and denominators. For example $\frac{4}{8} = \frac{1}{2}$. If you eat 4 out of 8 slices of pizza, that is the same as eating one-half of the pie. Usually we will be required to reduce fractions to lowest terms. Fractions in lowest terms have no common factors in the numerator and denominator other than 1.

$$\begin{array}{c} 60 \\ \swarrow \searrow \\ 6 \quad 10 \\ \swarrow \searrow \swarrow \searrow \\ (2)(3) (2)(5) \end{array}$$

Simplify $\frac{20}{60}$ to lowest terms (Reduce)

$$\frac{20}{60} = \frac{1 \cdot \cancel{2} \cdot \cancel{2} \cdot 5}{1 \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot 5}$$

Cancel the common factors $\Rightarrow$

$$= \frac{1 \cdot \cancel{2} \cdot \cancel{2} \cdot 5}{1 \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{5}} = \frac{1}{3}$$

Identify the
common factors

$$\begin{array}{c} 20 \\ \swarrow \searrow \\ (2) 10 \\ \swarrow \searrow \\ (2) (5) \end{array}$$

Use **factor trees** to find the prime factors.

An alternative way to reduce is to identify the **greatest common factor** (GCF) of the numerator and denominator, then divide both by that number.

$$\frac{20 \div 20}{60 \div 20} = \frac{1}{3}$$

A. Reduce

$\frac{30}{70}$
$\frac{30 \div 10}{70 \div 10} = \frac{3}{7}$

$\frac{105}{300}$
$\frac{105}{300} = \frac{3 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5} = \frac{7}{20}$

$\frac{78}{546}$
$\frac{78}{546} = \frac{2 \cdot 3 \cdot 13}{2 \cdot 3 \cdot 7 \cdot 13} = \frac{1}{7}$

An **improper fraction** is one where the numerator is larger than the denominator. To convert an improper fraction to a mixed number, simply divide. The quotient is the whole number part and the remainder is the new numerator.

$$\begin{array}{r} 4 \\ 5 \overline{) 23} \\ \underline{-20} \\ 3 \end{array}$$

$$\frac{23}{5} = 4\frac{3}{5}$$

$\frac{135}{30}$
$\frac{135}{30} = \frac{3 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5} = \frac{9}{2}$ $= 4\frac{1}{2}$

$\frac{600}{30}$
$\frac{600 \div 30}{30 \div 30} = \frac{20}{1} = 20$

$\frac{165}{45}$
$\frac{165}{45} = \frac{3 \cdot 5 \cdot 11}{3 \cdot 3 \cdot 5} = \frac{11}{3}$ $= 3\frac{2}{3}$

Multiplying Fractions – Simply multiply numerator and denominator $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Dividing Fractions – Multiply by the reciprocal of the divisor $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

When multiplying we should look for terms to cancel before we actually multiply the numerators and denominators. This will eliminate the need to reduce the end result.

B. Multiply or Divide

$$\frac{2}{3} \cdot \frac{5}{7}$$

$$\frac{\cancel{2} \rightarrow 5}{3 \rightarrow 7} = \frac{10}{21}$$

$$\frac{1}{5} \cdot \frac{4}{8}$$

$$\frac{1}{5} \cdot \frac{4}{8} = \frac{4 \div 4}{40 \div 4} = \frac{1}{10}$$

$$\frac{1}{2} \cdot \frac{1}{3}$$

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\frac{5}{7} \cdot \frac{49}{10}$$

$$\frac{\cancel{5}^7}{7} \cdot \frac{7\cancel{49}^7}{\cancel{10}_2} = \frac{7}{2} = 3\frac{1}{2}$$

$$\frac{2}{3} \cdot \frac{9}{12}$$

$$\frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{9}^3}{\cancel{12}_4} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{6}{14} \cdot \frac{21}{12}$$

$$\frac{\cancel{6}}{\cancel{14}_2} \cdot \frac{\cancel{21}^3}{\cancel{12}_4} = \frac{3}{4}$$

$$\frac{1}{2} \div \frac{2}{3}$$

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

$$\frac{5}{9} \div \frac{1}{3}$$

$$\frac{5}{9} \div \frac{1}{3} = \frac{5}{\cancel{9}_3} \cdot \frac{\cancel{3}}{1} = \frac{5}{3}$$

$$\frac{1}{2} \div 2$$

$$\frac{1}{2} \div 2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Convert mixed numbers to improper fractions **before** you multiply or divide. To do this multiply the denominator with the whole number then add the numerator, this will be the new numerator.

$$\begin{array}{l} \text{Add} \\ 3\frac{5}{7} = \frac{7 \cdot 3 + 5}{7} = \frac{26}{7} \\ \text{multiply} \end{array}$$

$$2\frac{1}{2} \div \frac{5}{3}$$

$$2\frac{1}{2} \div \frac{5}{3} = \frac{\cancel{4}}{2} \cdot \frac{3}{\cancel{5}} = \frac{3}{2}$$

$$4\frac{2}{3} \div 3\frac{1}{2}$$

$$4\frac{2}{3} \div 3\frac{1}{2} = \frac{14}{3} \div \frac{7}{2}$$

$$= \frac{14}{3} \cdot \frac{2}{7}$$

$$= \frac{4}{3} = 1\frac{1}{3}$$

$$5 \div 2\frac{3}{5}$$

$$5 \div 2\frac{3}{5} = \frac{5}{1} \div \frac{13}{5}$$

$$= \frac{5}{1} \cdot \frac{5}{13}$$

$$= \frac{25}{13} = 1\frac{12}{13}$$

Sometimes mixed numbers are confused with multiplication. Be sure to remember that $4\frac{2}{3}$ **does not imply multiplication**, but in fact $4\frac{2}{3} = 4 + \frac{2}{3}$, which is actually addition.

The difficulty with fractions usually comes from addition and subtraction because these operations require a common denominator. If I were to say that I ate two pieces of pizza you would want to know what size each piece was. To say I ate 2 pieces out of a pizza cut into 4 big slices is not the same as eating 2 pieces of a pizza cut into 8 slices!

Adding Fractions – with a common denominator

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Subtracting Fractions – with a common denominator

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

C. Add or Subtract

$$\frac{3}{5} + \frac{1}{5}$$

$$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$$

$$\frac{17}{20} - \frac{5}{20}$$

$$\frac{17}{20} - \frac{5}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}$$

$$\frac{5}{7} - 2\frac{1}{7}$$

$$\frac{5}{7} - 2\frac{1}{7} = \frac{5}{7} - \frac{15}{7} = \frac{-10}{7} = -1\frac{3}{7}$$

Simply add or subtract the numerators if the denominator is the same. Although, most of the problems that you are likely to encounter will have different denominators, so you will have to find the **equivalent fractions** with a common denominator before you can add or subtract.

Determine the common denominator: **LCM(15, 10) = 30**

$$\begin{aligned} \frac{7}{15} - \frac{3}{10} &= \frac{7 \cdot 2}{15 \cdot 2} - \frac{3 \cdot 3}{10 \cdot 3} \\ &= \frac{14}{30} - \frac{9}{30} \\ &= \frac{5}{30} \\ &= \frac{1}{6} \end{aligned}$$

Multiply numerator **and** denominator by what you need to get **equivalent fractions** with a common denominator.

Always reduce.

D. Add or Subtract

$$\frac{1}{5} + \frac{2}{3}$$

$$\begin{aligned} \frac{1}{5} + \frac{2}{3} &= \frac{1 \cdot 3}{5 \cdot 3} + \frac{2 \cdot 5}{3 \cdot 5} \\ &= \frac{3}{15} + \frac{10}{15} \\ &= \frac{13}{15} \end{aligned}$$

$$\frac{1}{2} + \frac{1}{3}$$

$$\begin{aligned} \frac{1}{2} + \frac{1}{3} &= \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} \\ &= \frac{3}{6} + \frac{2}{6} \\ &= \frac{5}{6} \end{aligned}$$

$$\frac{2}{7} - \frac{3}{5}$$

$$\begin{aligned} \frac{2}{7} - \frac{3}{5} &= \frac{2 \cdot 5}{7 \cdot 5} - \frac{3 \cdot 7}{5 \cdot 7} \\ &= \frac{10}{35} - \frac{21}{35} \\ &= \frac{-11}{35} \end{aligned}$$

$$\frac{3}{10} + \frac{2}{14}$$

$$\begin{aligned} \frac{3}{10} + \frac{2}{14} &= \frac{3 \cdot 7}{10 \cdot 7} + \frac{2 \cdot 5}{14 \cdot 5} \\ &= \frac{21}{70} + \frac{10}{70} \\ &= \frac{31}{70} \end{aligned}$$

$$\frac{2}{30} + \frac{5}{21}$$

$$\begin{aligned} \frac{2}{30} + \frac{5}{21} &= \frac{2 \cdot 7}{30 \cdot 7} + \frac{5 \cdot 10}{21 \cdot 10} \\ &= \frac{14}{210} + \frac{50}{210} \\ &= \frac{64 \div 2}{210 \div 2} = \frac{32}{105} \end{aligned}$$

$$\frac{3}{18} - \frac{1}{24}$$

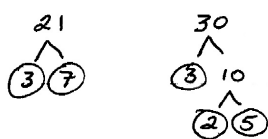
$$\begin{aligned} \frac{3}{18} - \frac{1}{24} &= \frac{3 \cdot 4}{18 \cdot 4} - \frac{1 \cdot 3}{24 \cdot 3} \\ &= \frac{12}{72} - \frac{3}{72} \\ &= \frac{9}{72} = \frac{1}{8} \end{aligned}$$

Least Common Multiple LCM(A, B) – The smallest multiple of both A and B. (The smallest integer that both A and B can divide into evenly.)

When the integers A and B have no common factors it is easy to find the LCM, just multiply them together. For example $\text{LCM}(3, 5) = 15$ and $\text{LCM}(10, 21) = 210$.

When the integers A and B have common factors it is usually more difficult to determine the LCM. One way to do it is to look at the prime factors of each. Identify the common factors then multiply them by the remaining factors to obtain the LCM.

Find the LCM(21, 30)



$$21 = 3 \cdot 7$$

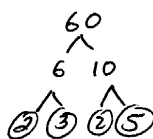
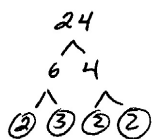
$$30 = 2 \cdot 3 \cdot 5$$

Identify the common factors.

$$\text{LCM}(21, 30) = 3 \cdot 2 \cdot 5 \cdot 7 = 210$$

Multiply the common factor by the remaining factors.

Find the LCM(24, 60)



$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

$$\text{LCM}(24, 60) = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 5 = 120$$

E. Typical Word Problems

Rental Truck – A board that is $5\frac{1}{4}$ feet long needs to be cut into 7 pieces of equal length. What will be length of each piece be?

Division problem

$$5\frac{1}{4} \div 7 = \frac{21}{4} \div \frac{7}{1} = \frac{21}{4} \cdot \frac{1}{7} = \frac{3}{4}$$

Each board will be $\frac{3}{4}$ ft long.

Recipe – A muffin recipe calls for $1\frac{2}{3}$ cups of flour to make 4 muffins. How many cups of flower does the recipe require if you wish to make 16 muffins?

Multiplication problem

To make 16 muffins we need 4 batches of 4 muffins.

Each batch of 4 muffins requires $1\frac{2}{3}$ cups of flour.

So we need $1\frac{2}{3} \cdot 4 = \frac{5}{3} \cdot \frac{4}{1} = \frac{20}{3} = 6\frac{2}{3}$ cups of flour.

Percents are an important part of our everyday life and will show up often in our study of Algebra. We will have to know how to convert percents to real numbers and back again.

Percent (%) – The representation of a number as a part of one hundred, $N\% = \frac{N}{100}$.

Using the definition you can convert percentages to fractions very easily, $31\% = \frac{31}{100}$, but often we will need the decimal equivalent, in which case you simply divide, $\frac{31}{100} = 0.31$. Notice that we could get the same result by moving the decimal over 2 places to the left.

Convert 187% to a decimal.	
$\underbrace{187\%}_{\text{underline}} = 1.87$	← Move the decimal two places to the left.

A. Rewrite as a decimal

89%
$\underbrace{89\%}_{\text{underline}} = .89$

0.5%
$\underbrace{0.5\%}_{\text{underline}} = .005$

215%
$\underbrace{215\%}_{\text{underline}} = 2.15$

$1\frac{1}{2}\%$
$1\frac{1}{2}\% = 1.5\% = .015$

$50\frac{3}{4}\%$
$50\frac{3}{4}\% = 50.75\% = .5075$

23.62%
$23.62\% = .2362$

500%
$500\% = 5$

0%
$0\% = .00 = 0$

$20\frac{1}{3}\%$
$20\frac{1}{3}\% = 20.33\bar{3} = .20\bar{3}$

B. Rewrite as fractions

50%
$50\% = \frac{50 \div 50}{100 \div 50} = \frac{1}{2}$

32%
$32\% = \frac{32 \div 4}{100 \div 4} = \frac{8}{25}$

400%
$400\% = \frac{400}{100} = 4$

$5\frac{5}{7}\%$	$33\frac{1}{3}\%$	$3\frac{7}{31}\%$
$5\frac{5}{7}\% = \frac{40}{7}\%$ $= \frac{40}{7} \div 100$ $= \frac{40}{7} \cdot \frac{1}{100}$ $= \frac{2}{5}$	$33\frac{1}{3}\% = \frac{100}{3}\%$ $= \frac{100}{3} \div 100$ $= \frac{100}{3} \cdot \frac{1}{100}$ $= \frac{1}{3}$	$3\frac{7}{31}\% = \frac{100}{31}\%$ $= \frac{100}{31} \div 100$ $= \frac{100}{31} \cdot \frac{1}{100}$ $= \frac{1}{31}$

To convert a decimal to a percent we can move the decimal to the right two places. One way to remember this is by the following illustration. $\begin{matrix} D & P \\ \curvearrowright & \curvearrowright \end{matrix}$

C. Rewrite as a percent

0.07	0.97	2.34
$.07 = 7\%$	$.97 = 97\%$	$2.34 = 234\%$
$\frac{1}{5}$	$\frac{25}{8}$	$3\frac{1}{4}$
$\frac{1}{5} = .20 = 20\%$	$\frac{25}{8} = 3.125 = 312.5\%$	$3\frac{1}{4} = 3.25 = 325\%$
$\frac{17}{50}$	$\frac{2}{3}$	3
$\frac{17}{50} \cdot \frac{2}{2} = \frac{34}{100} = 34\%$	$\frac{2}{3} \cdot 100\% = \frac{200}{3}\%$ $= 66\frac{2}{3}\%$	$3 \cdot 100\% = 300\%$

D. Typical Word Problems

Tax – A calculator is advertised at \$12.99 plus $7\frac{1}{2}\%$ tax. If you include tax what will the total be?

$$\begin{aligned} \text{Tax rate is } 7\frac{1}{2}\% &= 7.5\% = .075 \\ \text{Amount of tax} &= .075 \cdot 12.99 = \$.97 \\ \text{Total} &= 12.99 + .97 = \$13.96 \end{aligned}$$

Tip – After a nice dinner out, the bill came to \$35.75. For good service you wish to leave a 15% tip, what will the total be including the tip?

$$\begin{aligned} \text{Tip rate is } 15\% &= .15 \\ \text{Amount of Tip} &= .15 \cdot 35.75 = \$5.36 \\ \text{Total} &= 35.75 + 5.36 = \$41.11 \end{aligned}$$

When several operations are to be applied within a calculation we must follow a specific order to ensure a single correct result.

Order of Operations

1. Perform all calculations within the innermost **Parenthesis** first.
2. Then evaluate **Exponent** expressions.
3. Next do the **Multiplication and Division** from left to right.
4. Finally work all **Addition and Subtraction** from left to right last.

Caution: Please do not dismiss the fact that multiplication and division should be worked from **left to right**. Many standardized exams will test us on this fact. The following example illustrates the problem.

Correct Order of Operations	
	$2^3 - 8 \div 4 \cdot 2 + 7$
	$= 8 - \cancel{8 \div 4} 2 + 7$
$\Rightarrow$	$= 8 - 2 \cdot 2 + 7$
	$= 8 - 4 + 7$
	$= 4 + 7$
	$= 11 \quad \checkmark \text{ Correct!}$

Division first
because we
are working
left to right!

Incorrect Order of Operations	
	$2^3 - 8 \div 4 \cdot 2 + 7$
	$= 8 - 8 \div \cancel{4 \cdot 2} + 7$
	$= 8 - 8 \div 8 + 7$
	$= 8 - 1 + 7$
	$= 7 + 7$
	$= 14 \quad \times \text{ Incorrect!}$

Multiplication
first will lead to
an incorrect
answer!

A. Simplify.

$3 + 2 \cdot 3$
$3 + 2 \cdot 3$
$= 3 + 6$
$= 9$

$4 - 5 \cdot 3$
$4 - 5 \cdot 3$
$= 4 - 15$
$= -11$

$2 + 3(-2) - 7$
$2 + 3(-2) - 7$
$= 2 - 6 - 7$
$= -4 - 7$
$= -11$

$8 \div 2 - 3 \cdot 2$
$8 \div 2 - 3 \cdot 2$
$= 4 - 6$
$= -2$

$3 + 6^2 \div 12$
$3 + 6^2 \div 12$
$= 3 + 36 \div 12$
$= 3 + 3$
$= 6$

$-2 - 3^2 + (-2)^2$
$-2 - 3^2 + (-2)^2$
$= -2 - 9 + 4$
$= -11 + 4$
$= -7$

Remember

$$-3^2 = -3 \cdot 3 = -9$$

$$(-2)^2 = (-2)(-2) = 4$$

$$\begin{aligned}
 &(-5)^2 - 2(5)^2 \div 10 \\
 &(-5)^2 - 2(5)^2 \div 10 \\
 &= 25 - 2 \cdot 25 \div 10 \\
 &= 25 - 50 \div 10 \\
 &= 25 - 5 \\
 &= 20
 \end{aligned}$$

$$\begin{aligned}
 &-3(4-7)+2 \\
 &-3(4-7)+2 \\
 &= -3(-3)+2 \\
 &= 9+2 \\
 &= 11
 \end{aligned}$$

$$\begin{aligned}
 &(-2+7)^2 - 10^2 \\
 &(-2+7)^2 - 10^2 \\
 &= (5)^2 - 10^2 \\
 &= 25 - 100 \\
 &= -75
 \end{aligned}$$

$$\begin{aligned}
 &10 - 7(3+2) + 7^2 \\
 &10 - 7(3+2) + 7^2 \\
 &= 10 - 7(5) + 49 \\
 &= 10 - 35 + 49 \\
 &= -25 + 49 \\
 &= 24
 \end{aligned}$$

$$\begin{aligned}
 &-7 - 3(4 - 2 \cdot 8) \\
 &-7 - 3(4 - 2 \cdot 8) \\
 &= -7 - 3(4 - 16) \\
 &= -7 - 3(-12) \\
 &= -7 + 36 \\
 &= 29
 \end{aligned}$$

$$\begin{aligned}
 &2 - 7(3^2 - 3 + 4 \cdot 3) \\
 &2 - 7(3^2 - 3 + 4 \cdot 3) \\
 &= 2 - 7(9 - 3 + 12) \leftarrow \\
 &= 2 - 7(18) \\
 &= 2 - 126 \\
 &= -124
 \end{aligned}$$

Work within the parenthesis first.

Order of Operation problems get a bit more tedious when fractions are involved. Remember that when adding or subtracting fractions we need to first find the equivalent fractions with a common denominator. Multiplication **does not** require a common denominator.

Adding fractions: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Multiplying Fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

B. Simplify.

$$\begin{aligned}
 &\frac{1}{3} + \frac{1}{2} \cdot \frac{1}{5} \\
 &\frac{1}{3} + \frac{1}{2} \cdot \frac{1}{5} \\
 &= \frac{1}{3} + \frac{1}{10} \\
 &= \frac{10}{30} + \frac{3}{30} \\
 &= \frac{13}{30}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{5}{8} \div \frac{3}{2} \cdot \frac{14}{15} \\
 &\frac{5}{8} \div \frac{3}{2} \cdot \frac{14}{15} \\
 &= \frac{5}{8} \cdot \frac{2}{3} \cdot \frac{14}{15} \\
 &= \frac{14}{36} = \frac{7}{18}
 \end{aligned}$$

$$\begin{aligned}
 &5 \cdot \frac{2}{15} - \left(\frac{1}{2}\right)^3 \\
 &5 \cdot \frac{2}{15} - \left(\frac{1}{2}\right)^3 \leftarrow \\
 &= \frac{5}{1} \cdot \frac{2}{15} - \frac{1}{8} \\
 &= \frac{2}{3} - \frac{1}{8} \\
 &= \frac{16}{24} - \frac{3}{24} \\
 &= \frac{13}{24}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{1}{2}\right)^3 &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\
 &= \frac{1}{8}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{5}{17} \left(\frac{3}{5} - \frac{4}{35}\right) \\
 &\frac{5}{17} \left(\frac{3}{5} - \frac{4}{35}\right) \\
 &= \frac{5}{17} \left(\frac{21}{35} - \frac{4}{35}\right) \\
 &= \frac{5}{17} \left(\frac{17}{35}\right) \\
 &= \frac{1}{7}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{3}{16} \div \left(\frac{5}{12} - \frac{1}{2} + \frac{2}{3}\right) \cdot 4 \\
 &\frac{3}{16} \div \left(\frac{5}{12} - \frac{1}{2} + \frac{2}{3}\right) \cdot 4 \\
 &= \frac{3}{16} \div \left(\frac{5}{12} - \frac{6}{12} + \frac{8}{12}\right) \cdot 4 \\
 &= \frac{3}{16} \div \left(\frac{7}{12}\right) \cdot \frac{4}{1} \\
 &= \frac{3}{16} \cdot \frac{12}{7} \cdot \frac{4}{1} \\
 &= \frac{9}{7}
 \end{aligned}$$

$$\begin{aligned}
 &\left(\frac{2}{3}\right)^2 - \left(\frac{1}{2}\right)^3 \\
 &\left(\frac{2}{3}\right)^2 - \left(\frac{1}{2}\right)^3 \\
 &= \frac{4}{9} - \frac{1}{8} \\
 &= \frac{32}{72} - \frac{9}{72} \\
 &= \frac{23}{72}
 \end{aligned}$$

$$\frac{2+3 \cdot 6-4 \cdot 3}{2^2-3^2}$$

$$\frac{2+3 \cdot 6-4 \cdot 3}{2^2-3^2}$$

$$= \frac{2+18-12}{4-9}$$

$$= \frac{8}{-5} = -\frac{8}{5}$$

$$\frac{(2+7) \cdot 2-2^3}{10+9^2+3^3}$$

$$\frac{(2+7) \cdot 2-2^3}{10+9^2+3^3}$$

$$= \frac{9 \cdot 2-8}{10+81+9}$$

$$= \frac{18-8}{100} = \frac{10}{100} = \frac{1}{10}$$

$$\frac{(-1-3)^2-15}{-3 \cdot (-7+2^2)-5}$$

$$\frac{(-1-3)^2-15}{-3 \cdot (-7+2^2)-5}$$

$$= \frac{(-4)^2-15}{-3(-3)-5}$$

$$= \frac{16-15}{9-5} = \frac{1}{4}$$

We will see that some of the problems have different looking parenthesis $\{[()]\}$, treat them the same and just remember to do the **innermost parentheses first**. Some problems may involve an **absolute value**, in which case we will need to apply the innermost absolute value first as you would if it were a parenthesis.

Notation Counts!

Pay attention to proper notation and you will be less likely to make a mistake.



Apply the absolute value first then multiply here.

$$3-7|-2-3|+4^3$$

$$3-7|-2-3|+4^3$$

$$\Rightarrow = 3-7|-5|+4^3$$

$$= 3-7 \cdot 5+64$$

$$= 3-35+64$$

$$= -32+64$$

$$= 32$$

C. Simplify.

$$5-3[6-(2+7)]$$

$$5-3[6-(2+7)]$$

$$= 5-3[6-9]$$

$$= 5-3[-3]$$

$$= 5+9$$

$$= 14$$

$$1+2[(-2)^3-(-3)^2]$$

$$1+2[(-2)^3-(-3)^2]$$

$$= 1+2[-8-9]$$

$$= 1+2[-17]$$

$$= 1-34$$

$$= -33$$

$$-3[2(7-5) \div 4 \cdot (-2) + (-3)^3]$$

$$-3[2(7-5) \div 4 \cdot (-2) + (-3)^3]$$

$$= -3[2(2) \div 4(-2) - 27]$$

$$= -3[4 \div 4(-2) - 27]$$

$$= -3[1 \cdot (-2) - 27]$$

$$= -3[-29]$$

$$= 87$$

$$\frac{1}{2}[\frac{3}{4} \cdot (-4)^2 - 2]^2$$

$$\frac{1}{2}[\frac{3}{4} \cdot (-4)^2 - 2]^2$$

$$= \frac{1}{2}[\frac{3}{4} \cdot 16 - 2]^2$$

$$= \frac{1}{2}[3 \cdot 4 - 2]^2$$

$$= \frac{1}{2}[12-2]^2$$

$$= \frac{1}{2}[10]^2 = \frac{1}{2} \cdot 100 = 50$$

$$6 \cdot [(\frac{2}{3})^2 - (\frac{1}{2})^2] \div (-5)^2$$

$$6 \cdot [(\frac{2}{3})^2 - (\frac{1}{2})^2] \div (-5)^2$$

$$= 6[\frac{4}{9} - \frac{1}{4}] \div 25$$

$$= 6[\frac{16}{36} - \frac{9}{36}] \div 25$$

$$= 6[\frac{7}{36}] \div 25$$

$$= \frac{7}{6} \cdot \frac{1}{25}$$

$$= \frac{7}{150}$$

$$-7^2 - [-20 - (-3)^2] - (-10)$$

$$-7^2 - [-20 - (-3)^2] - (-10)$$

$$= -49 - [-20 - 9] + 10$$

$$= -49 - [-29] + 10$$

$$= -49 + 29 + 10$$

$$= -10$$

The first step in understanding algebra is to realize that letters are used to represent numbers. Sometimes the problem will specifically state what number the letter represents, in which case, we replace the letter with that number and evaluate.

Variable – A letter used to represent a number. Combinations of letters and numbers are called **variable expressions**.

Incorrect order of operations is the most common error when evaluating variable expressions. To avoid this problem, particularly when beginning to learn algebra, we should put a blank parenthesis wherever we see a variable, then substitute in the appropriate values.

Evaluate $b^2 - 4ac$ when $a = -1$, $b = -2$ and $c = 3$		
$b^2 - 4ac$ $\Rightarrow ()^2 - 4()()$ $(-2)^2 - 4(-1)(3) \quad \leftarrow$ $= 4 + 12$ $= 16$		

*Write a **blank parenthesis** wherever you see a variable.*

*Then “**plug-in**” the appropriate values.*

Writing the parenthesis may seem like unnecessary extra work but they will help us follow the correct order of operations and avoid many common errors.

A. Evaluate

$x + 2$ when $x = -3$
$x + 2$ when $x = -3$ $() + 2$ $(-3) + 2$ $= -1$

$2x - 3$ when $x = -2$
$2x - 3$ when $x = -2$ $2() - 3$ $2(-2) - 3$ $= -4 - 3$ $= -7$

$-3x + 7$ when $x = 5$
$-3x + 7$ when $x = 5$ $-3() + 7$ $-3(5) + 7$ $= -15 + 7$ $= -8$

$5(a - 1)$ when $a = -3$
$5(a - 1)$ when $a = -3$ $5(-3 - 1)$ $= 5(-4)$ $= -20$

$-3(2 - y)$ when $y = -2$
$-3(2 - y)$ when $y = -2$ $-3(2 - ())$ $-3(2 - (-2))$ $= -3(2 + 2)$ $= -3(4)$ $= -12$

Evaluate $-5a + 2b$ when $a = 2$ and $b = -3$
$-5a + 2b$ when $a = 2$ and $b = -3$ $-5() + 2()$ $-5(2) + 2(-3)$ $= -10 - 6$ $= -16$

When evaluating variable expressions with exponents be sure to take care with the order of operations. Remember that exponents should be evaluated before multiplication. Usually the exponents will be the first operation we perform.

Exponents – Raising a to the 5th power means $a^5 = a \cdot a \cdot a \cdot a \cdot a$

When $a = -2$ then $a^5 = (-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$

B. Evaluate

$a^2 - b^2$ when $a = 7$ and $b = -5$ $a^2 - b^2$ when $a = 7$ and $b = -5$ $(\quad)^2 - (\quad)^2$ $(7)^2 - (-5)^2$ $= 49 - 25$ $= 24$	$(x+2)(2x-3)$ when $x = 3$ $(x+2)(2x-3)$ when $x = 3$ $(3+2)(2 \cdot 3 - 3)$ $= (5)(6-3)$ $= 5 \cdot 3$ $= 15$	$2x^2 + 3x - 1$ when $x = -1$ $2x^2 + 3x - 1$ when $x = -1$ $2(-1)^2 + 3(-1) - 1$ $= 2(1) - 3 - 1$ $= 2 - 3 - 1$ $= -2$
$y^2 + 5y + 6$ when $y = 0$ $y^2 + 5y + 6$ when $y = 0$ $(0)^2 + 5(0) + 6 \leftarrow$ $= 0 + 0 + 6$ $= 6$	$3 - x^2$ when $x = -4$ $3 - x^2$ when $x = -4$ $3 - (-4)^2$ $= 3 - 16$ $= -13$	$a^3 - b^3$ when $a = 2$ and $b = -3$ $a^3 - b^3$ when $a = 2$ and $b = -3$ $(2)^3 - (-3)^3$ $= 8 - (-27)$ $= 8 + 27$ $= 35$

Zero times anything is zero.

We must substitute the given value for each occurrence of the variable. In addition, it usually is best to perform the operations within the parenthesis first rather than distribute.

C. Evaluate

$b^2 - 4ac$ when $a = 2, b = 3$ and $c = -3$ $b^2 - 4ac$ when $a = 2, b = 3, c = -3$ $(3)^2 - 4(2)(-3)$ $= 9 + 24$ $= 33$	$b^2 - 4ac$ when $a = -3, b = -2$ and $c = -5$ $b^2 - 4ac$ when $a = -3, b = -2, c = -5$ $(-2)^2 - 4(-3)(-5)$ $= 4 - 60$ $= -56$	$5(x+h)$ when $x = 5$ and $h = 0.01$ $5(x+h)$ when $x = 5$ and $h = 0.01$ $5(5 + .01)$ $= 5(5.01)$ $= 25.05$
---	--	--

$-3(x+h)+5$ when $x=5$ and $h=0.1$ $-3(x+h)+5$ when $x=5$ and $h=0.1$ $-3(5+0.1)+5$ $= -3(5.1)+5$ $= -15.3+5$ $= -10.3$	$(x+h)^2-3(x+h)-2$ when $x=-3$ and $h=0.1$ $(x+h)^2-3(x+h)-2$ when $x=-3$ and $h=0.1$ $(-3+.1)^2-3(-3+.1)-2$ $= (-2.9)^2-3(-2.9)-2$ $= 8.41+8.7-2$ $= 15.11$	$-b \div 2a$ when $a=-2$ and $b=12$ $-b \div 2a$ when $a=-2$ and $b=12$ $-(12) \div 2(-2)$ $= -12 \div 2(-2)$ $= -6(-2)$ $= 12$
--	---	--

	$\frac{(x+h)^2-2(x+h)+9}{h}$ when $x=-1$ and $h=0.01$	
Substitute the appropriate values.	$\frac{(-1+.01)^2-2(-1+.01)+9}{.01}$	
	$= \frac{(-.99)^2-2(-.99)+9}{.01}$	Work the numerator using the order of operations.
	$= \frac{.9801+1.98+9}{.01}$	
Divide last.	$= \frac{11.9601}{.01} = 1196.01$	

Typical Word Problem

Rental Truck – The daily truck rental rate is \$48.00 plus an additional \$0.45 for every mile driven. **A.** Set up an algebraic equation for the daily cost of the truck. **B.** Use it to determine the cost if someone plans to drive it 100 miles for the day.

a. Let x = the number of miles driven

then $\text{COST} = \$48 + \$0.45x$

b. Evaluate when $x = 100$ miles

$\text{COST} = \$48 + \$0.45(100)$

$= 48 + .45(100)$

$= 48 + 45 = 93$

The cost would be \$93.00 for the rental if you drive 100 miles that day.

The properties of real numbers are very important in our study of Algebra. These properties can be applied in Algebra because a variable is simply a letter that represents a real number. The distributive property is one that we apply often when simplifying algebraic expressions.

Distributive Property – Given real numbers a , b and c : $a(b + c) = ab + ac$

When multiplying an expression within parenthesis you must multiply everything inside by the number or variable you are distributing.

Simplify: $-2(3x^2 - 2x + 1)$		
Distribute the “-2” here.	$\Rightarrow -2(3x^2 - 2x + 1)$ $= -6x^2 + 4x - 2$	Notice that we have multiplied every coefficient by “-2”

A. Simplify

$5(2x - 4)$ $5(2x - 4)$ $= 10x - 20$	$(4x + 8)7$ $(4x + 8)7$ $= 28x + 56$	$-2(5 - 7x)$ $-2(5 - 7x)$ $= -10 + 14x$
--	--	---

$3(-5x^2 + 2x - 1)$ $3(-5x^2 + 2x - 1)$ $= -15x^2 + 6x - 3$	$-5(3a + 7b - 5c)$ $-5(3a + 7b - 5c)$ $= -15a - 35b + 25c$	$7(3a - 4b) + 2c$ $7(3a - 4b) + 2c$ $= 21a - 28b + 2c$
---	--	--

Multiply only the terms within the parenthesis!

$\frac{5}{2}(4x - 2)$ $\frac{5}{2}(4x - 2)$ $= 10x - 5$	$3(\frac{1}{3}x + 4)$ $3(\frac{1}{3}x + 4)$ $= x + 12$	$-\frac{2}{3}(3x^2 - 6x + 1)$ $-\frac{2}{3}(3x^2 - 6x + 1)$ $= -2x^2 + 4x - \frac{2}{3}$
---	--	--

When simplifying we will often have to combine like terms after we distribute. This step is consistent with the order of operations, multiplication before addition.

Like Terms – Constant terms or terms with the same variable factors.

$$2\text{apples} + 3\text{apples} = 5\text{apples} \quad \text{OR} \quad 2xy + 3xy = 5xy$$

B. Simplify

$$\begin{aligned} &2(-3x+2) - 4x + 7 \\ &2(-3x+2) - 4x + 7 \\ &= -6x + 4 - 4x + 7 \\ &= -10x + 11 \end{aligned}$$

$$\begin{aligned} &2(6a-7b) - 3(a-3b+c) \\ &2(6a-7b) - 3(a-3b+c) \\ &= 12a - 14b - 3a + 9b - 3c \\ &= 9a - 5b - 3c \end{aligned}$$

$$\begin{aligned} &7 - 5(3x-7) \\ &7 - 5(3x-7) \\ &= 7 - 15x + 35 \\ &= -15x + 42 \end{aligned}$$

$$\begin{aligned} &9(7z-4) - 3(9z+1) \\ &9(7z-4) - 3(9z+1) \\ &= 63z - 36 - 27z - 3 \\ &= 36z - 39 \end{aligned}$$

$$\begin{aligned} &9(-3x^2-4x+7) - 36 \\ &9(-3x^2-4x+7) - 36 \\ &= -27x^2 - 36x + 63 - 36 \\ &= -27x^2 - 36x + 27 \end{aligned}$$

$$\begin{aligned} &8y - (3y+7) \\ &8y - (3y+7) \\ &= 8y - 3y - 7 \quad \leftarrow \\ &= 5y - 7 \end{aligned}$$

Multiply each term by -1 here.

The variable parts have to be exactly the same before we can combine them. But before we combine like terms, generally, we will distribute first. When distributing negative numbers notice the operations change because you are actually multiplying by a negative number.

C. Simplify

$$\begin{aligned} &-7(2x^2-8x-4) - 13x \\ &-7(2x^2-8x-4) - 13x \\ &= -14x^2 + 56x + 28 - 13x \\ &= -14x^2 + 43x + 28 \end{aligned}$$

$$\begin{aligned} &x - 2(-5x-9) + 4 \\ &x - 2(-5x-9) + 4 \\ &= x + 10x + 18 + 4 \\ &= 11x + 22 \end{aligned}$$

$$\begin{aligned} &6(abc+4) - (7abc-5) \\ &6(abc+4) - (7abc-5) \\ &= 6abc + 24 - 7abc + 5 \\ &= -abc + 29 \end{aligned}$$

$$\begin{aligned} &4 - 2(x^2-7x+1) \\ &4 - 2(x^2-7x+1) \\ &= 4 - 2x^2 + 14x - 2 \\ &= -2x^2 + 14x + 2 \end{aligned}$$

$$\begin{aligned} &-2(a^2b-3ab^2+5) - 4a^2b+7 \\ &-2(a^2b-3ab^2+5) - 4a^2b+7 \\ &= -2a^2b + 6ab^2 - 10 - 4a^2b + 7 \\ &= -6a^2b + 6ab^2 - 3 \end{aligned}$$

$$\begin{aligned} &7 - 3(5+8) - 5 \\ &7 - 3(5+8) - 5 \\ &= 7 - 3(13) - 5 \quad \leftarrow \\ &= 7 - 39 - 5 \\ &= -37 \end{aligned}$$

No need to distribute here!

$2(3x^2 - 7x + 1) - 3(x^2 + 5x - 1)$ $\begin{aligned} & \overbrace{2(3x^2 - 7x + 1)} \quad \overbrace{-3(x^2 + 5x - 1)} \\ &= 6x^2 - 14x + 2 - 3x^2 - 15x + 3 \\ &= 3x^2 - 29x + 5 \end{aligned}$	$x^2y + 3xy^2 - (2x^2y - xy^2)$ $\begin{aligned} & \overbrace{x^2y + 3xy^2} \quad \overbrace{-(2x^2y - xy^2)} \\ &= x^2y + 3xy^2 - 2x^2y + xy^2 \\ &= -x^2y + 4xy^2 \end{aligned}$
$-a - b - c - (2a + 2b + 2c)$ $\begin{aligned} & -a - b - c - (2a + 2b + 2c) \\ &= -a - b - c - 2a - 2b - 2c \\ &= -3a - 3b - 3c \end{aligned}$	$3 - 5(ab - 3) + 2(ba - 4)$ $\begin{aligned} & 3 - 5(ab - 3) + 2(ba - 4) \\ &= 3 - 5ab + 15 + 2ba - 8 \\ &= 3 + 15 - 8 - 5ab + 2ab \\ &= 10 - 3ab \end{aligned}$

Profit is equal to revenues less cost of production. If the revenue R can be represented by $R = 2x^2 - x + 50$ and the cost C can be represented by $C = 3x^2 - 2$ where x is the number of units produced, find an equation that represents the profit.

$$\begin{aligned} \text{Profit} &= \text{Revenue} - \text{Cost} \\ &= (2x^2 - x + 50) - (3x^2 - 2) \\ &= 2x^2 - x + 50 - 3x^2 + 2 \\ &= -x^2 - x + 52 \end{aligned}$$

What is the difference between $3x - 4$ and $-2x + 5$?

$$\begin{aligned} & (3x - 4) - (-2x + 5) \\ & \overbrace{(3x - 4) - (-2x + 5)} \\ &= 3x - 4 + 2x - 5 \\ &= 5x - 9 \quad \checkmark \end{aligned}$$

Chapter 1 *Pre-Algebra Review*

Sample Exam

Please answer all the questions and show work where appropriate.

1. List three numbers that are less than -33 .
2. Simplify: $-2 + 5 - |-13|$
3. Divide: $\frac{33}{30} \div \frac{55}{25}$
4. Add: $\frac{2}{3} + \frac{3}{10}$
5. Joe's bill came to \$12.05 before a $7\frac{1}{4}\%$ tax was added. What will his total bill be when tax is added?
6. Simplify: $-9 + 2^4 - 3^2$
7. Evaluate $3x^2 - 2x + 3$ when $x = -2$.
8. Simplify: $2(3x - 4) - 3(5x + 7)$
9. Evaluate $b^2 - 4ac$ when $a = -1$, $b = 4$ and $c = -3$.
10. A home store charges \$45.00 to rent a truck with an additional charge of \$0.55 per mile the truck is driven. Build a variable expression for the rental charge.

Chapter 1 *Pre-Algebra Review*

Sample Exam Answers

1. $\{-100, -50, -34\}$ *Answers will vary*
2. -10
3. $\frac{1}{2}$
4. $\frac{29}{30}$
5. \$12.92
6. -2
7. 19
8. $-9x - 29$
9. 4
10. $45 + 0.55x$

List three numbers that are less than -33 .

$$\Rightarrow \{-100, -50, -34\}$$

Answers will vary so just choose any three numbers that are smaller than -33 .

Simplify: $-2 + 5 - |-13|$

$$\begin{aligned} & -2 + 5 - |-13| \\ & = -2 + 5 - 13 \\ & = 3 - 13 \\ & = -10 \end{aligned}$$

Divide: $\frac{33}{30} \div \frac{55}{25}$

$$\begin{aligned} & \frac{33}{30} \div \frac{55}{25} \\ & = \frac{33}{30} \cdot \frac{25}{55} \\ & = \frac{3 \cdot 11}{2 \cdot 3 \cdot 5} \cdot \frac{5 \cdot 5}{5 \cdot 11} \\ & = \frac{1}{2} \end{aligned}$$

Add: $\frac{2}{3} + \frac{3}{10}$

$$\begin{aligned} & \frac{2}{3} + \frac{3}{10} \\ & = \frac{2 \cdot 10}{3 \cdot 10} + \frac{3 \cdot 3}{10 \cdot 3} \\ & = \frac{20}{30} + \frac{9}{30} \\ & = \frac{29}{30} \end{aligned}$$

Simplify: $-9 + 2^4 - 3^2$

$$\begin{aligned} & -9 + 2^4 - 3^2 \\ & = -9 + 16 - 9 \\ & = 7 - 9 \\ & = -2 \end{aligned}$$

Evaluate $3x^2 - 2x + 3$ when $x = -2$.

$$\begin{aligned} & 3x^2 - 2x + 3 \\ & 3(-2)^2 - 2(-2) + 3 \\ & = 3 \cdot 4 + 4 + 3 \\ & = 12 + 4 + 3 \\ & = 19 \end{aligned}$$

Joe's bill came to \$12.05 before a $7\frac{1}{4}\%$ tax was added. What will his total bill be when tax is added?

The tax rate of $7\frac{1}{4}\% = 7.25\% = .0725$
So the tax amount is $\$12.05 \times .0725 = \0.87
rounded off to the nearest cent. Add this
to the bill to get the total = $\$12.05 + \$0.87 = \$12.92$.

Simplify: $2(3x - 4) - 3(5x + 7)$

$$\begin{aligned} & \overbrace{2(3x - 4)} \quad \overbrace{-3(5x + 7)} \\ & = 6x - 8 - 15x - 21 \\ & = -9x - 29 \end{aligned}$$

Evaluate $b^2 - 4ac$ when $a = -1$, $b = 4$ and $c = -3$

$$\begin{aligned} & b^2 - 4ac \\ & ()^2 - 4()() \\ & (4)^2 - 4(-1)(-3) \\ & = 16 - 12 \\ & = 4 \end{aligned}$$

A home store charges \$45.00 to rent a truck with an additional charge of \$0.55 per mile the truck is driven. Build a variable expression for the rental charge.

Let x = the number of miles driven
Then the cost = $45 + .55x$



Chapter 2 *Linear Equations and Inequalities*

Contents

This is the chapter where we really begin to learn Algebra. The techniques in this section will provide the basis for everything to come in our study of this subject. Be sure to spend some extra time with the applications and word problem section. Also, new notation is introduced in the linear inequality section.

Linear Equations and Inequalities

Solving Linear Equations – Part I ...2.1

Solving Linear Equations – Part II ...2.2

Word Problems ...2.3

Linear Inequalities ...2.4

Sample Exam

Sample Exam Solutions

One method for solving a linear equation that looks like $2x + 3 = 13$ is to simply guess and check. What value can I substitute in for x so that the result will be 13? After some thought it should be easy to see that x must equal 5 here because $2 \cdot 5 + 3 = 10 + 3 = 13$.

Linear Equation – An equation with one variable where the exponent is not greater than 1.

When solving, the idea is to isolate the variable by applying the opposite operations to both sides to retain the equality. In other words, if we add 5 to one side we must add 5 to the other side as well.



Solve: $2x - 5 = 9$

$$\begin{array}{r}
 2x - 5 = 9 \\
 +5 \quad +5 \\
 \hline
 2x = 14 \\
 \frac{2x}{2} = \frac{14}{2} \\
 x = 7
 \end{array}$$

The first opposite operation is to **add 5** to both sides.

Then to isolate the variable **divide by 2**.

⇒

Check: $2x - 5 = 9$

$$\begin{array}{l}
 2(\quad) - 5 \\
 2(7) - 5 \\
 = 14 - 5 \\
 = 9 \quad \checkmark
 \end{array}$$

Tip: We can always check to see if our solution is correct by substituting it in to the original equation to see if it works. On a test, if you have time, you should always check.

A.

Is $x = 7$ a solution to $-3x + 5 = -16$?	Is $x = 2$ a solution to $-2x - 7 = 28$?	Is $x = -3$ a solution to $\frac{1}{3}x - 4 = -5$?
$ \begin{array}{l} -3x + 5 = -16 \\ -3(7) + 5 \\ = -21 + 5 \\ = -16 \quad \text{Yes.} \end{array} $	$ \begin{array}{l} -2x - 7 = 28 \\ -2(2) - 7 \\ = -4 - 7 \\ = -11 \quad \text{No} \end{array} $	$ \begin{array}{l} \frac{1}{3}x - 4 = -5 \\ \frac{1}{3}(-3) - 4 \\ = -1 - 4 \\ = -5 \quad \text{Yes} \end{array} $

B. Solve for x

$$\begin{array}{r} 3x = -33 \\ 3x = -33 \\ \frac{3x}{3} = \frac{-33}{3} \\ x = -11 \end{array}$$

$$\begin{array}{r} 10x = 200 \\ 10x = 200 \\ \frac{10x}{10} = \frac{200}{10} \\ x = 20 \end{array}$$

$$\begin{array}{r} 27 = 9a \\ 27 = 9a \\ \frac{27}{9} = \frac{9a}{9} \\ 3 = a \end{array}$$

$$\begin{array}{r} -2x = 14 \\ -2x = 14 \\ \frac{-2x}{-2} = \frac{14}{-2} \\ x = -7 \end{array}$$

$$\begin{array}{r} -5x = -25 \\ -5x = -25 \\ \frac{-5x}{-5} = \frac{-25}{-5} \\ x = 5 \end{array}$$

$$\begin{array}{r} -x = 40 \\ -x = 40 \\ \frac{-1 \cdot x}{-1} = \frac{40}{-1} \\ x = -40 \end{array}$$

Multiply or divide by -1 here.

Multiplication is the opposite of division here.

$$\begin{array}{r} \frac{x}{5} = -10 \\ \frac{x}{5} = -10 \\ \Rightarrow (5) \frac{x}{5} = -10(5) \\ x = -50 \end{array}$$

$$\begin{array}{r} \frac{2}{3} = \frac{x}{6} \\ \frac{2}{3} = \frac{x}{6} \\ (6) \frac{2}{3} = \frac{x}{6} (6) \\ 4 = x \end{array}$$

$$\begin{array}{r} -\frac{t}{12} = \frac{1}{4} \\ -\frac{t}{12} = \frac{1}{4} \\ (-12) \left(-\frac{t}{12}\right) = (-12) \left(\frac{1}{4}\right) \\ t = -3 \end{array}$$

C. Solve for x

$$\begin{array}{r} x + 3 = 8 \\ x + 3 = 8 \\ -3 \quad -3 \\ x = 5 \end{array}$$

$$\begin{array}{r} 5 - x = -15 \\ 5 - x = -15 \\ -5 \quad -5 \\ -x = -20 \\ x = 20 \end{array}$$

$$\begin{array}{r} -4 + x = 17 \\ -4 + x = 17 \\ +4 \quad +4 \\ x = 21 \end{array}$$

$$\begin{array}{r} x + \frac{2}{3} = \frac{5}{9} \\ x + \frac{2}{3} = \frac{5}{9} \\ -\frac{2}{3} \quad -\frac{2 \cdot 3}{3 \cdot 3} \\ x = -\frac{1}{9} \end{array}$$

$$\begin{array}{r} a - \frac{1}{5} = \frac{1}{3} \\ a - \frac{1}{5} = \frac{1 \cdot 5}{3 \cdot 5} \\ +\frac{1}{5} \quad +\frac{1 \cdot 3}{5 \cdot 3} \\ a = \frac{8}{15} \end{array}$$

$$\begin{array}{r} x + 2\frac{1}{2} = 3\frac{1}{3} \\ x + 2\frac{1}{2} = 3\frac{1}{3} \\ x + \frac{5}{2} = \frac{10 \cdot 2}{3 \cdot 2} \\ -\frac{5}{2} \quad -\frac{5 \cdot 3}{2 \cdot 3} \\ x = \frac{5}{6} \end{array}$$

In Algebra, improper fractions are more natural to work with.

Maria scored 7 more points than Jose on the first mathematics exam. If Maria's total score was 95 what was Jose's total. (Set up an algebraic equation and solve it)

Let x = Jose's total score

set-up $\rightarrow 95 = x + 7$

$88 = x$ So Jose scored 88 on exam #1.

Tip: Often it is handy to *multiply both sides by the reciprocal* of the variable coefficient. This technique can save a step or two by using the property $\frac{a}{b} \cdot \frac{b}{a} = 1$.



$$-\frac{a}{7} = \frac{1}{14}$$

$$-\frac{a}{7} = \frac{1}{14}$$

$$(-7)\left(-\frac{a}{7}\right) = (-7)\left(\frac{1}{14}\right)$$

$$a = -\frac{1}{2}$$

$$\frac{3}{16}x = \frac{1}{4}$$

$$\frac{3}{16}x = \frac{1}{4}$$

$$\left(\frac{16}{3}\right)\frac{3}{16}x = \frac{1}{4}\left(\frac{16}{3}\right)$$

$$x = \frac{4}{3}$$

$$\frac{7}{3}x = -\frac{1}{2}$$

$$\frac{7}{3}x = -\frac{1}{2}$$

$$\left(\frac{3}{7}\right)\frac{7}{3}x = \left(\frac{3}{7}\right)\left(-\frac{1}{2}\right)$$

$$x = -\frac{3}{14}$$

Multiply both sides by the reciprocal..

The order of operations is reversed when solving. Do the addition and subtraction first then multiply or divide as needed.

D. Solve for x

$$2x + 5 = 15$$

$$2x + 5 = 15$$

$$\begin{array}{r} 2x + 5 = 15 \\ -5 \quad -5 \end{array}$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

$$-3a + 4 = -17$$

$$-3a + 4 = -17$$

$$\begin{array}{r} -3a + 4 = -17 \\ -4 \quad -4 \end{array}$$

$$\frac{-3a}{-3} = \frac{-21}{-3}$$

$$a = 7$$

$$\frac{1}{2}x + \frac{1}{3} = \frac{1}{3}$$

$$\frac{1}{2}x + \frac{1}{3} = \frac{1}{3}$$

$$\begin{array}{r} \frac{1}{2}x + \frac{1}{3} = \frac{1}{3} \\ -\frac{1}{3} \quad -\frac{1}{3} \end{array}$$

$$(2)\frac{1}{2}x = 0(2)$$

$$x = 0$$

$$5 - 3x = -10$$

$$5 - 3x = -10$$

$$\begin{array}{r} 5 - 3x = -10 \\ -5 \quad -5 \end{array}$$

$$\frac{-3x}{-3} = \frac{-15}{-3}$$

$$x = 5$$

$$10x + 7 = 7$$

$$10x + 7 = 7$$

$$\begin{array}{r} 10x + 7 = 7 \\ -7 \quad -7 \end{array}$$

$$\frac{10x}{10} = \frac{0}{10}$$

$$x = 0$$

$$-12 = 2x - 4$$

$$-12 = 2x - 4$$

$$\begin{array}{r} -12 = 2x - 4 \\ +4 \quad +4 \end{array}$$

$$\frac{-8}{2} = \frac{2x}{2}$$

$$-4 = x$$

$$-\frac{2}{5} + 3a = \frac{3}{5}$$

$$-\frac{2}{5} + 3a = \frac{3}{5}$$

$$\begin{array}{r} -\frac{2}{5} + 3a = \frac{3}{5} \\ +\frac{2}{5} \quad +\frac{2}{5} \end{array}$$

$$3a = \frac{5}{5}$$

$$3a = 1$$

$$a = \frac{1}{3}$$

$$\frac{1}{3}x - \frac{1}{2} = \frac{1}{5}$$

$$\frac{1}{3}x - \frac{1}{2} = \frac{1}{5}$$

$$\begin{array}{r} \frac{1}{3}x - \frac{1}{2} = \frac{1}{5} \\ +\frac{1}{2} \quad +\frac{1}{2} \end{array}$$

$$(3)\frac{1}{3}x = \frac{7}{10}(3)$$

$$x = \frac{21}{10}$$

$$5 - 5x = -20$$

$$5 - 5x = -20$$

$$\begin{array}{r} 5 - 5x = -20 \\ -5 \quad -5 \end{array}$$

$$\frac{-5x}{-5} = \frac{-25}{-5}$$

$$x = 5$$

Problems Solved! Chapter 2 Linear Equations and Inequalities
Solving Linear Equations – Part II

Often we will find that linear equations first need to be simplified before we can solve them. Typically this will involve combining like terms. Combine same side like terms first then opposite side like terms using the techniques we learned in part I.

Caution: A common error is to add or subtract a term on the same side of the equal sign. Only use the opposite operation on opposite sides of the equal sign.

Correct steps for solving: $3x + 5 - 2x = 6x - 10$	Incorrect steps for solving: $3x + 5 - 2x = 6x - 10$
Combine same side like terms first! $3x + 5 - 2x = 6x - 10$ $\Rightarrow x + 5 = 6x - 10$ $x = 6x - 15$ $-6x = -6x$ $\frac{-5x}{-5} = \frac{-15}{-5}$ $x = 3 \quad \checkmark \text{ Correct!}$	$3x + 5 - 2x = 6x - 10$ $+2x \quad +2x \quad \leftarrow$ $5x + 5 = 6x - 10$ $-5 \quad -5$ $5x = 6x - 15$ $-6x \quad -6x$ $-x = -15$ $x = 15 \quad \times \text{ Incorrect!}$ Adding to the same side will lead to incorrect results

A. Solve

$$3x - 10 = 4x - 13$$

$$3x - 10 = 4x - 13$$

$$+10 \quad +10$$

$$3x = 4x - 3$$

$$-4x \quad -4x$$

$$-x = -3$$

$$\frac{-x}{-1} = \frac{-3}{-1}$$

$$x = 3$$

$$-2a + 3 = 4a + 15$$

$$-2a + 3 = 4a + 15$$

$$-3 \quad -3$$

$$-2a = 4a + 12$$

$$-4a \quad -4a$$

$$-6a = 12$$

$$\frac{-6a}{-6} = \frac{12}{-6}$$

$$a = -2$$

$$\frac{1}{2}x - \frac{1}{3} = -\frac{1}{4}x + \frac{1}{2}$$

$$\frac{1}{2}x - \frac{1}{3} = -\frac{1}{4}x + \frac{1}{2}$$

$$+ \frac{1}{3} \quad + \frac{1}{3}$$

$$\frac{1}{2}x = -\frac{1}{4}x + \frac{5}{6}$$

$$+ \frac{1}{4}x \quad + \frac{1}{4}x \quad \leftarrow$$

$$\frac{3}{4}x = \frac{5}{6}$$

$$\left(\frac{4}{3}\right)\frac{3}{4}x = \frac{5}{6}\left(\frac{4}{3}\right)$$

$$x = \frac{10}{9}$$



Don't let fractions get in your way, the steps are the same.

$$6x + 2 - 3x = -2x - 13$$

$$6x + 2 - 3x = -2x - 13$$

$$3x + 2 = -2x - 13$$

$$+2x \quad +2x$$

$$5x = -11$$

$$x = -11/5$$

$$12 - 2x + 4 = 8 + 5x + 2 - x$$

$$12 - 2x + 4 = 8 + 5x + 2 - x$$

$$-2x + 16 = 4x + 10$$

$$-4x \quad -4x$$

$$-6x = -6$$

$$x = 1$$

$$\frac{1}{2}x + \frac{1}{3} - \frac{1}{5}x = -\frac{3}{10}x + \frac{1}{2}$$

$$\frac{1}{2}x + \frac{1}{3} - \frac{1}{5}x = -\frac{3}{10}x + \frac{1}{2}$$

$$\frac{3}{10}x + \frac{1}{3} = -\frac{3}{10}x + \frac{1}{2}$$

$$\frac{3}{10}x = -\frac{3}{10}x + \frac{1}{6}$$

$$\left(\frac{10}{6}\right)\frac{3}{10}x = \frac{1}{6}\left(\frac{10}{6}\right)$$

$$x = \frac{10}{36} = \frac{5}{18}$$

Not all equations work out to have a single solution. Some have infinitely many solutions such as $x = x$. Here any number we choose for x will work. Also, some equations have no solution such as $x + 1 = x$.

Contradiction – an equation that will always be false has no solution.

Identity – an equation that will always be true has any real number, $\mathbb{R}$, as a solution.

$$\begin{aligned}
 3x + 5 - x &= 2x + 7 \\
 3x + 5 - x &= 2x + 7 \\
 2x + 5 &= 2x + 7 \\
 -5 &\quad -5 \\
 2x &= 2x + 2 \\
 -2x &\quad -2x \\
 0 &= 2, \text{ False} \\
 \text{No Solution}
 \end{aligned}$$

$$\begin{aligned}
 -7x + 3 &= 2 - 7x + 1 \\
 -7x + 3 &= 2 - 7x + 1 \\
 -7x + 3 &= -7x + 3 \\
 -3 &\quad -3 \\
 -7x &= -7x \\
 +7x &\quad +7x \\
 0 &= 0, \text{ True} \\
 \text{All Reals, } \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \frac{7}{10}x + \frac{2}{3} &= \frac{1}{5}x + \frac{2}{3} + \frac{1}{2}x \\
 \frac{7}{10}x + \frac{2}{3} &= \frac{7}{10}x + \frac{2}{3} \\
 -\frac{7}{10}x &\quad -\frac{7}{10}x \\
 \frac{2}{3} &= \frac{2}{3}, \text{ True} \\
 \text{All Reals, } \mathbb{R}
 \end{aligned}$$

It is quite common to encounter linear equations that require us to distribute before combining like terms. Look for the parenthesis and distribute before doing anything.

B. Solve

$$\begin{aligned}
 3 - 2(x + 4) &= -3(4x - 5) \\
 3 - 2(x + 4) &= -3(4x - 5) \\
 3 - 2x - 8 &= -12x + 15 \\
 -2x - 5 &= -12x + 15 \\
 +5 &\quad +5 \\
 -2x &= -12x + 20 \\
 +12x &\quad +12x \\
 \frac{10x}{10} &= \frac{20}{10} \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 -7(x + 1) + 5(2x - 3) &= 8 \\
 -7(x + 1) + 5(2x - 3) &= 8 \\
 -7x - 7 + 10x - 15 &= 8 \\
 3x - 22 &= 8 \\
 +22 &\quad +22 \\
 3x &= 30 \\
 \frac{3x}{3} &= \frac{30}{3} \\
 x &= 10
 \end{aligned}$$

$$\begin{aligned}
 -3(2x - 3) + 2 &= 3(x + 7) \\
 -3(2x - 3) + 2 &= 3(x + 7) \\
 -6x + 9 + 2 &= 3x + 21 \\
 -6x + 11 &= 3x + 21 \\
 -11 &\quad -11 \\
 -6x &= 3x + 10 \\
 -3x &\quad -3x \\
 -9x &= 10 \\
 \frac{-9x}{-9} &= \frac{10}{-9} \\
 x &= -\frac{10}{9}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{2}(3x + 5) - \frac{3}{2} &= \frac{1}{4}(2x + 12) \\
 \frac{1}{2}(3x + 5) - \frac{3}{2} &= \frac{1}{4}(2x + 12) \\
 \frac{3}{2}x + \frac{5}{2} - \frac{3}{2} &= \frac{1}{2}x + 3 \\
 \frac{3}{2}x + \frac{2}{2} &= \frac{1}{2}x + 3 \\
 -1 &\quad -1 \\
 \frac{3}{2}x &= \frac{1}{2}x + 2 \\
 -\frac{1}{2}x &\quad -\frac{1}{2}x \\
 \frac{2}{2}x &= 2 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 10(3x + 5) - 5(4x + 2) &= 2(5x + 20) \\
 10(3x + 5) - 5(4x + 2) &= 2(5x + 20) \\
 30x + 50 - 20x - 10 &= 10x + 40 \\
 10x + 40 &= 10x + 40 \\
 -10x &\quad -10x \\
 40 &= 40, \text{ True} \\
 \text{All Reals, } \mathbb{R}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{3}(x - 2) + \frac{1}{5} &= \frac{1}{9}(3x + 3) \\
 \frac{1}{3}(x - 2) + \frac{1}{5} &= \frac{1}{9}(3x + 3) \\
 \frac{1}{3}x - \frac{2}{3} + \frac{1}{5} &= \frac{1}{3}x + \frac{1}{3} \\
 \frac{1}{3}x - \frac{7}{15} &= \frac{1}{3}x + \frac{1}{3} \\
 -\frac{7}{15} &= \frac{1}{3}, \text{ False} \\
 \text{No solution}
 \end{aligned}$$

Literal equations are difficult for many people because there will be more than one variable. Just remember that the letters are placeholders for some number so the steps for solving are the same. Isolate the variable that it asks us to solve for.

C. Solve

Solve for w: $P = 2l + 2w$ $\begin{aligned} P &= 2l + 2w \\ -2l & \quad -2l \\ \hline P - 2l &= 2w \\ \frac{P - 2l}{2} &= \frac{2w}{2} \\ \frac{P - 2l}{2} &= w \end{aligned}$	Solve for r: $C = 2\pi r$ $\begin{aligned} \frac{C}{2\pi} &= \frac{2\pi r}{2\pi} \\ \frac{C}{2\pi} &= r \end{aligned}$	Solve for t: $D = r \cdot t$ $\begin{aligned} \frac{D}{r} &= \frac{r \cdot t}{r} \\ \frac{D}{r} &= t \end{aligned}$
Solve for C: $F = \frac{9}{5}C + 32$ $\begin{aligned} F &= \frac{9}{5}C + 32 \\ -32 & \quad -32 \\ \hline F - 32 &= \frac{9}{5}C \\ (F - 32)\left(\frac{5}{9}\right) &= \frac{9}{5}C\left(\frac{5}{9}\right) \\ \frac{5}{9}F - \frac{160}{9} &= C \\ \text{OR } C &= \frac{5F - 160}{9} \end{aligned}$	Solve for a: $P = a + b + c$ $\begin{aligned} P &= a + b + c \\ -c & \quad -c \\ \hline P - c &= a + b \\ -b & \quad -b \\ \hline P - c - b &= a \end{aligned}$	Solve for h: $S = 2\pi r^2 + 2\pi rh$ $\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ -2\pi r^2 & \quad -2\pi r^2 \\ \hline S - 2\pi r^2 &= 2\pi rh \\ \frac{S - 2\pi r^2}{2\pi r} &= \frac{2\pi rh}{2\pi r} \\ \frac{S - 2\pi r^2}{2\pi r} &= h \\ \frac{S}{2\pi r} - r &= h \end{aligned}$

The internet connection at the hotel cost \$5.00 to log on and \$0.20 a minute to run. If Joe's total bill came to \$18.00 how many minutes did Joe spend on the internet? (Set up an algebraic equation and solve it)

Let x = number of minutes on the internet
 then $\text{COST} = 5.00 + .20x$
 set-up $\rightarrow 18 = 5 + .20x$

$$\begin{array}{r} 18 \\ -5 \\ \hline 13 \end{array} = \begin{array}{r} .20x \\ -.20x \\ \hline \end{array}$$

$$\frac{13}{.20} = \frac{.20x}{.20}$$

$$65 = x$$

 Joe spent 65 minutes on the internet.

In this section we will identify the major types of word problems that you will most likely encounter at this point in your study of algebra. Often students will skip the word problems, but that is not a passing strategy. Usually setting them up is the hard part but solving them after that, for the most part, is not that difficult.



Tip: Read the problem twice before starting and do not simply skip word problems!

A. Number Problems

The sum of two integers is 18. The larger number is 2 less than 3 times the smaller. Find the integers.

Let x = the smaller integer
 $3x - 2$ = the larger number
 Set-up $\rightarrow$ $x + (3x - 2) = 18$

$$\begin{array}{r} 4x - 2 = 18 \\ +2 \quad +2 \\ \hline 4x = 20 \end{array} \quad \begin{array}{l} \nearrow \\ \frac{4x}{4} = \frac{20}{4} \\ x = 5 \\ 3(5) - 2 = 15 - 2 = 13 \end{array}$$

The two integers are 5 and 13

The difference between two integers is 2. The larger number is 6 less than twice the smaller. Find the integers.

Let x = the smaller integer
 then $2x - 6$ = the larger integer
 Set-up $\rightarrow$ $(2x - 6) - x = 2$

$$\begin{array}{r} x - 6 = 2 \\ +6 \quad +6 \\ \hline x = 8 \end{array} \quad \begin{array}{l} \nearrow \text{larger} = 2x - 6 \\ = 2(8) - 6 \\ = 16 - 6 \\ = 10 \end{array}$$

The two integers are 8 and 10.

Many times we can figure out these types of word problems by guessing and checking. This is the case because the numbers are chosen to be simple so the algebraic steps will not be too tedious. You are learning to set up algebraic equations on these easier problems so that you can use these ideas to solve more difficult ones, problems that can not be guessed at.



Tip: For full credit your instructor will insist that you use Algebra to solve the word problems. Don't fight it, just identify your variables and set it up using Algebra!

One number is 3 more than another number. When two times the larger is subtracted from 3 times the smaller number, the result is 6. Find the numbers

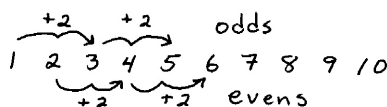
Let x = one of the numbers
 then $x+3$ = the other number
 set-up $\rightarrow$ $3x - 2(x+3) = 6$

$$\begin{array}{rcl}
 3x - 2x - 6 & = & 6 \\
 x - 6 & = & 6 \\
 +6 & +6 & \\
 \hline
 x & = & 12
 \end{array}$$

larger number $= x+3$
 $= 12+3$
 $= 15$

The two numbers are 12 and 15.

It is important to notice that both even and odd integers are separated by two units.



B. Even and Odd Integers

The sum of two consecutive odd integers is 36. Find the integers.

Let x = the first odd integer
 $x+2$ = the next odd integer
 set-up $\rightarrow$ $x + (x+2) = 36$

$$\begin{array}{rcl}
 2x + 2 & = & 36 \\
 -2 & -2 & \\
 \hline
 2x & = & 34
 \end{array}$$

$\frac{2x}{2} = \frac{34}{2}$
 $x = 17$
 $x+2 = 19$

The two odds are 17 and 19.

The sum of two consecutive even integers is 46. Find the integers

Let x = the first even integer
 $x+2$ = the next even integer
 set-up $\rightarrow$ $x + (x+2) = 46$

$$\begin{array}{rcl}
 2x + 2 & = & 46 \\
 -2 & -2 & \\
 \hline
 2x & = & 44
 \end{array}$$

$\frac{2x}{2} = \frac{44}{2}$
 $x = 22$
 $x+2 = 24$

The two evens are 22 and 24.

Notice that the set-up is the same as odds.

Caution: A common mistake is to set up consecutive odds with an x and $x + 3$. This will most likely lead to a decimal answer which certainly is not an integer.

Make sure you read the problems carefully; notice that consecutive integers are separated by 1 unit.

The sum of three consecutive integers is 24. Find the integers.

$$\begin{aligned}
 \text{Let } x &= \text{the first integer.} \\
 x+1 &= \text{the next integer.} \\
 x+2 &= \text{the third integer.} \\
 \text{Set-up } \rightarrow & \boxed{x + (x+1) + (x+2) = 24} \\
 & 3x + 3 = 24 \\
 & 3x = 21 \\
 & x = 7 \\
 & \text{The three integers are 7, 8 and 9.}
 \end{aligned}$$

You will need to ask your instructor if you will be able to use a calculator. In either case, you should be able to work with decimals by hand. When money is involved be sure to round off to two decimal places.

C. Tax and Rental Problems

If a pair of Nike shoes cost \$48.95 plus $7\frac{1}{4}\%$ tax, what will the total be at the register?

$$\begin{aligned}
 \text{Let } x &= \text{the total cost } (7\frac{1}{4}\% = 7.25\% = .0725) \\
 \text{Set-up } \rightarrow & \boxed{x = 48.95 + .0725(48.95)} \\
 & = 48.95 + 3.55 \text{ (Rounded Off)} \\
 & = 52.50 \\
 & \text{The total cost will be \$52.50.}
 \end{aligned}$$

At $8\frac{3}{4}\%$ the amount of tax on an item came to \$12.04. What was the cost of the item?

$$\begin{aligned}
 \text{Let } x &= \text{the cost of the item } (8\frac{3}{4}\% = 8.75\% = .0875) \\
 \text{set-up } \rightarrow & \boxed{12.04 = .0875(x)} \\
 & \frac{12.04}{.0875} = \frac{.0875x}{.0875} \\
 & 137.60 = x \\
 & \text{The cost of the item was \$137.60.}
 \end{aligned}$$

A local non-profit was mistakenly charged \$2,005.84 which included a $7\frac{1}{4}\%$ tax charge. If tax was not to be included how much refund is the company due?

Let x = cost of the item before tax

set-up $\rightarrow$ $2005.84 = x + .0725x$

$$2005.84 = 1.0725x$$

$$\frac{2005.84}{1.0725} = \frac{1.0725x}{1.0725}$$

$$1870.25 = x \quad (\text{Rounded OFF})$$

$$\text{So refund} = 2005.84 - 1870.25 = \$135.59$$

A taxi cab charges \$5.00 for the ride plus \$1.25 per mile. How much will a 53 mile trip cost?

Let c = the cost of the trip

set-up $\rightarrow$ $c = 5.00 + 1.25(53)$

$$= 5 + 66.25$$

$$= 71.25$$

The cost of the 53 mile trip = \$71.25

If a rental car cost Jose \$35.00 for the day plus \$0.33 per mile and his total cost was \$78.00, how many miles did he drive?

Let x = the number of miles driven

Set-up $\rightarrow$ $78 = 35 + .33x$

$$\frac{43}{.33} = \frac{.33x}{.33}$$

$$130.30 = x \quad (\text{Rounded off})$$

He drove about 130 miles.

Any Algebra text that you read will have steps or guidelines for solving word problems. These steps are all generally the same but nothing works better to improve your word problem skills than practice. You must do lots of these to get comfortable with them.

Basic Guideline for Solving Word Problems:

1. Read the problem several times and organize the given information.
2. Identify the variables by assigning a letter to the unknown quantity.
3. Set up an algebraic equation.
4. Solve the equation.
5. Finally, answer the question and make sure it makes sense.

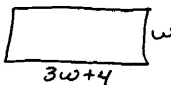
Tip: When working with geometry problems it helps to draw a picture.



D. Geometry Problems

A rectangle has a perimeter measuring 64 ft. The length is 4 ft more than 3 times the width. Find the dimensions of the rectangle.

Let w = the width of the rectangle
 $l = 3w + 4$ the length of the rectangle



Set-up $\rightarrow P = 2l + 2w$

$$64 = 2(3w + 4) + 2w$$

$$64 = 6w + 8 + 2w$$

$$64 = 8w + 8$$

$$56 = 8w$$

$$7 = w$$

$$l = 3(7) + 4 = 21 + 4 = 25$$

The rectangle measures 7 ft by 25 ft.

The perimeter of an equilateral triangle measures 63 cm. Find the length of each side.

Let s = length of each side

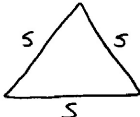
Set-up $P = s + s + s$

$$63 = 3s$$

$$\frac{63}{3} = s$$

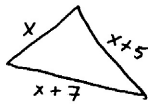
$$21 = s$$

Each side measures 21 cm.



Two sides of a triangle are 5 and 7 inches longer than the third side. If the perimeter measures 21 in, find the length of each side.

Let x = the length of the third side
 $x + 5$ = length of one side
 $x + 7$ = length of other side



Set-up $P = a + b + c$

$$21 = x + (x + 5) + (x + 7)$$

$$21 = 3x + 12$$

$$9 = 3x$$

$$x = 3$$

So the three sides measure 3 in, 8 in and 10 in

Some perimeter formulas you are expected to know: ($\pi \approx 3.14$)

Perimeter of a Rectangle: $P = 2l + 2w$

Perimeter of a Triangle: $P = a + b + c$

Perimeter of a Circle: $C = 2\pi r$

Perimeter of a Square: $P = 4s$

E. Percent Problems

A \$215,000 house requires a 20% down payment. How much will the down payment be for this home?

$$\begin{aligned} \text{Let } x &= \text{amount of Down Payment} \\ \text{set-up } \rightarrow x &= \$215,000 (.20) \\ &= 43000 \\ \text{Down payment will be } &\$43,000 \end{aligned}$$

A stock fell from \$42.00 to \$39.85 in one year. How much of a percent decrease does this represent?

$$\begin{aligned} \text{Let } x &= \text{Percent Decrease} \\ \text{set-up Amount Decrease} &= \text{what percent of the original} \\ 2.15 &= x \cdot 42 \\ \frac{2.15}{42} &= x \\ x &\approx .05 \text{ (Rounded off)} \\ \text{This was approximately a } &5\% \text{ Decrease.} \end{aligned}$$

A discount store paid \$35.50 for a dress they are selling for \$49.99. What is the store markup on this item?

$$\begin{aligned} \text{Let } x &= \text{Percent Increase} \\ \text{set-up Amount Increase} &= \text{what percent of the original} \\ 14.49 &= x \cdot 35.50 \\ \frac{14.49}{35.50} &= x \\ x &\approx .41 \text{ (Rounded off)} \\ \text{Store mark-up is approximately } &41\%. \end{aligned}$$

Whenever setting up a percent problem always use the decimal or fractional equivalent of the percent. Generally, we wish to use real numbers in Algebra, so instead of using 50 for 50% we will need to use .50 or $\frac{1}{2}$. Also, if the question asks for a percentage then do not forget to convert your answer to a percent.

Cross Multiplying –

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$.

When setting up proportions, be sure to be consistent. Units for the numerators should be the same and units for the denominators should be the same as well.

F. Proportion Problems

If 2 out of 3 dentists prefer Crest, how many prefer Crest out of 600 dentists surveyed?

Let n = the number of dentists who prefer Crest.

Set-up

$$\frac{2}{3} = \frac{n}{600}$$

Cross multiply $2 \cdot 600 = 3 \cdot n$

$$1200 = 3n$$

$$\frac{1200}{3} = n \quad \text{or} \quad n = 400$$

So 400 dentists out of 600 prefer Crest.

In Visalia 3 out of every 7 voters said yes to proposition 40. If 42,000 people voted, how many said no to proposition 40?

Let n = number of voter who said no.

If 3 out of 7 said yes then 4 out of 7 said no

Set-up $\rightarrow$

$$\frac{4}{7} = \frac{n}{42,000}$$

Cross multiply $4 \cdot 42,000 = 7n$

$$168,000 = 7n \quad \text{or} \quad n = \frac{168,000}{7}$$

24,000 voters said no.

A recipe calls for 5 tablespoons of sugar for every 8 cups of flour. How many tablespoons of sugar are required for 32 cups of flour?

Let n = number of tablespoons of sugar

Set-up

$$\frac{5}{8} = \frac{n}{32}$$

$$5 \cdot 32 = 8n$$

$$\frac{160}{8} = \frac{8n}{8} \quad \text{or} \quad n = 20$$

We will need 20 tablespoons of sugar.

G. Mixture Problems

Sally has 12 coins consisting of quarter and dimes. The value adds to \$2.25, how many of each coin does she have?

Let x = the number of quarters

$12 - x$ = the number of dimes

$$\text{Set-up} \rightarrow .25(x) + .10(12 - x) = 2.25$$

$$.25x + 1.2 - .10x = 2.25$$

$$.15x + 1.2 = 2.25$$

$$\begin{array}{r} -1.2 \quad -1.2 \end{array}$$

$$\frac{.15x}{.15} = \frac{1.05}{.15}$$

$$x = 7 \quad \left(\begin{array}{l} \text{\# of} \\ \text{so } \wedge \text{ Dimes} = 12 - 7 = 5 \end{array} \right)$$

Sally has 7 quarters and 5 dimes.

A 50% alcohol solution is mixed with a 10% alcohol solution to create 8 oz of a 32% alcohol solution. How much of each is needed?

Let x = amount of 50% alcohol solution

$8 - x$ = amount of 10% alcohol solution

$$\text{Set-up} \rightarrow .50(x) + .10(8 - x) = .32(8)$$

$$.50x + .8 - .10x = 2.56$$

$$.40x + .8 = 2.56$$

$$\begin{array}{r} -.8 \quad -.8 \end{array}$$

$$\frac{.40x}{.40} = \frac{1.76}{.40}$$

$$x = 4.4 \text{ oz}$$

We need 4.4 oz of 50% solution and

$8 - 4.4 = 3.6 \text{ oz}$ of 10% solution.

Mary invested her total savings of \$3,400 in two accounts. Her mutual fund account earned 8% last year and her CD earned 5%. If her total interest for the year was \$245, how much was in each account?

Let x = Amount invested in mutual funds at 8%

$3400 - x$ = Amount invested in the CD at 5%

$$\text{set-up} \rightarrow .08(x) + .05(3400 - x) = 245$$

$$.08x + 170 - .05x = 245$$

$$.03x + 170 = 245$$

$$.03x = 75$$

$$x = 75 / .03$$

$$x = 2500 \text{ and } 3400 - 2500 = 900$$

Mary invested \$2,500 in mutual funds and \$900 in the CD.

H. Distance Problems

The 375 mile drive to Las Vegas took 5 hours. What was the average speed?

Let r = Average speed to Las Vegas

Set-up $\rightarrow D = r \cdot t$

$$\boxed{375 = r \cdot 5}$$

$$375/5 = r \quad \text{or} \quad r = 75$$

The average rate was 75 mph.

Two trains leave the station at the same time traveling in opposite directions. One travels at 70 mph and the other travels at 60 mph. How long does it take for them to be 390 miles apart?

Let t = time it takes to be 390 miles apart

Set-up $\rightarrow$ Distance train 1 travels + Distance train 2 travels = 390

$$\boxed{70 \cdot t + 60t = 390}$$

$$\frac{130t}{130} = \frac{390}{130}$$

$$t = 3 \text{ hrs}$$

	D	r	t
train 1	D_1	70	t
train 2	D_2	60	t
total	390		

It will take 3 hrs to separate 390 miles.

Joe and Bill are traveling across the country. Joe leaves one hour earlier than Bill at a rate of 60 mph. If Bill leaves at a rate of 70 mph, how long will it take him to catch up?

Let t = the time it takes bill to catch up to Joe.

Set-up $\rightarrow$ when Bill catches up
the distances will be equal

$$\boxed{60(t+1) = 70t}$$

$$\begin{array}{r} 60t + 60 = 70t \\ -60t \quad -60t \end{array}$$

$$\frac{60}{10} = \frac{10t}{10}$$

$$6 = t$$

	D	r	t
Joe	D	60	$t+1$
Bill	D	70	t

Bill will catch up
in 6 hours.

For the word problems early in Algebra, we generally want to set up our equations with one variable. Remember that we are in the chapter dealing with linear equations. Later in our study we will learn how to deal with multiple variable systems. For now, try to avoid using a second variable when setting up your equations.

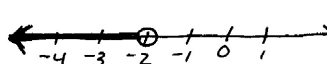
Problems Solved! Chapter 2 Linear Equations and Inequalities

Linear Inequalities

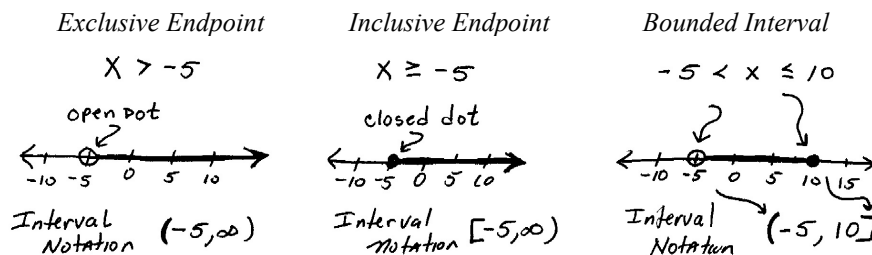
All the steps that we have learned for solving linear equations are the same for solving linear inequalities except one. We may add or subtract any real number to both sides of an inequality and we may multiply or divide both sides by any positive real number. The only new rule comes from multiplying or dividing by a negative number.

Dividing by 5 here leads to a true statement.	$\Rightarrow$	$\begin{array}{l} 10 > 5 \\ \hline \frac{10}{5} > \frac{5}{5} \\ 2 > 1 \end{array}$	$\mid$	$\begin{array}{l} 10 > 5 \\ \hline \frac{10}{-5} < \frac{5}{-5} \\ -2 < -1 \end{array}$	$\Leftarrow$	To obtain a true statement after we divide by -5 here we must reverse the inequality.
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So whenever we **divide or multiply by a negative number we must reverse the inequality**. It is easy to forget to do this so take special care to watch out for negative coefficients.

Solve: $-2x + 5 > 9$	
	$\begin{array}{l} -2x + 5 > 9 \\ -5 \quad -5 \\ \hline -2x > 4 \end{array}$
Here we divide by a negative number so we must reverse the inequality.	$\Rightarrow \begin{array}{l} -2x > 4 \\ \hline \frac{-2x}{-2} < \frac{4}{-2} \\ x < -2 \end{array}$
	
OR $(-\infty, -2)$ in interval notation.	
Check, say $x = -5$ $-2x + 5 > 9$ $-2(-5) + 5$ $= 10 + 5$ $= 15 > 9 \checkmark$	

Notice that we obtain infinitely many solutions for these linear inequalities. Because of this we have to present our solution set in some way other than a big list. The two most common ways to express solutions to an inequality are by **graphing them on a number line** and **interval notation**. (Notation: ∞ reads infinity and $-\infty$ reads negative infinity.)



Tip: Always use a round parenthesis and open dots for inequalities without the equal and always use square parenthesis and closed dots for inequalities with the equal



A. Solve and Graph the solution set.

$$3x + 7 < 16$$

$$\begin{array}{r} 3x + 7 < 16 \\ -7 \quad -7 \end{array}$$

$$\frac{3x}{3} < \frac{9}{3}$$

$$x < 3$$

$$\text{I.N. } (-\infty, 3)$$

$$5x - 4 \geq 6$$

$$\begin{array}{r} 5x - 4 \geq 6 \\ +4 \quad +4 \end{array}$$

$$\frac{5x}{5} \geq \frac{10}{5}$$

$$x \geq 2$$

$$\text{I.N. } [2, \infty)$$

$$12x + 7 > -17$$

$$\begin{array}{r} 12x + 7 > -17 \\ -7 \quad -7 \end{array}$$

$$\frac{12x}{12} > \frac{-24}{12}$$

$$x > -2$$

$$\text{I.N. } (-2, \infty)$$

$$-2x + 5 < -7$$

$$\begin{array}{r} -2x + 5 < -7 \\ -5 \quad -5 \end{array}$$

$$-2x < -12$$

$$\frac{-2x}{-2} > \frac{-12}{-2}$$

$$x > 6$$

$$\text{I.N. } (6, \infty)$$

$$-5x + 10 > 30$$

$$\begin{array}{r} -5x + 10 > 30 \\ -10 \quad -10 \end{array}$$

$$-5x > 20$$

$$\frac{-5x}{-5} < \frac{20}{-5}$$

$$x < -4$$

$$\text{I.N. } (-\infty, -4)$$

$$-7x + 1 \leq 29$$

$$\begin{array}{r} -7x + 1 \leq 29 \\ -1 \quad -1 \end{array}$$

$$-7x \leq 28$$

$$\frac{-7x}{-7} \geq \frac{28}{-7}$$

$$x \geq -4$$

$$\text{I.N. } [-4, \infty)$$

$$\frac{1}{2}x - \frac{1}{3} > \frac{2}{3}$$

$$\begin{array}{r} \frac{1}{2}x - \frac{1}{3} > \frac{2}{3} \\ +\frac{1}{3} \quad +\frac{1}{3} \end{array}$$

$$\frac{1}{2}x > \frac{3}{3}$$

$$(2)\frac{1}{2}x > 1(2)$$

$$x > 2$$

$$\text{I.N. } (2, \infty)$$

$$\frac{5}{3}x + \frac{1}{2} \leq \frac{1}{3}$$

$$\begin{array}{r} \frac{5}{3}x + \frac{1}{2} \leq \frac{1}{3} \\ -\frac{1}{2} \quad -\frac{1}{2} \end{array}$$

$$\frac{5}{3}x \leq -\frac{1}{6}$$

$$\left(\frac{3}{5}\right)\frac{5}{3}x \leq -\frac{1}{6}\left(\frac{3}{5}\right)$$

$$x \leq -\frac{1}{10}$$

$$\text{I.N. } (-\infty, -\frac{1}{10}]$$

$$-\frac{3}{4}x - \frac{1}{2} \geq \frac{5}{2}$$

$$\begin{array}{r} -\frac{3}{4}x - \frac{1}{2} \geq \frac{5}{2} \\ +\frac{1}{2} \quad +\frac{1}{2} \end{array}$$

$$-\frac{3}{4}x \geq \frac{6}{2}$$

$$\left(-\frac{4}{3}\right)\frac{3}{4}x \leq \frac{3}{1}\left(-\frac{4}{3}\right)$$

$$x \leq -4$$

$$\text{I.N. } (-\infty, -4]$$

Compound inequalities can be split up or solved in one step like these below.

B. Solve and Graph the solution set.

$$-15 < 3x - 6 \leq 6$$

$$\begin{array}{r} -15 < 3x - 6 \leq 6 \\ +6 \quad +6 \quad +6 \end{array}$$

$$-9 < \frac{3x}{3} \leq \frac{12}{3}$$

$$-3 < x \leq 4$$

$$\text{I.N. } (-3, 4]$$

$$-3 \leq 2x + 5 < 17$$

$$\begin{array}{r} -3 \leq 2x + 5 < 17 \\ -5 \quad -5 \quad -5 \end{array}$$

$$-8 \leq \frac{2x}{2} < \frac{12}{2}$$

$$-4 \leq x < 6$$

$$\text{I.N. } [-4, 6)$$

$$-1 \leq \frac{1}{2}x - 5 \leq 1$$

$$\begin{array}{r} -1 \leq \frac{1}{2}x - 5 \leq 1 \\ +5 \quad +5 \quad +5 \end{array}$$

$$4 \leq \frac{1}{2}x \leq 6$$

$$(2)4 \leq (2)\frac{1}{2}x \leq (2)6$$

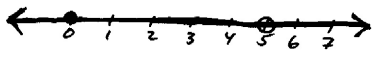
$$8 \leq x \leq 12$$

$$\text{I.N. } [8, 12]$$

$-3 \leq 3(x-1) \leq 3$ $-3 \leq 3(x-1) \leq 3$ $\begin{array}{r} -3 \leq 3x-3 \leq 3 \\ +3 \quad +3 \quad +3 \end{array}$ $0 \leq 3x \leq 6$ $\frac{0}{3} \leq \frac{3x}{3} \leq \frac{6}{3}$ $0 \leq x \leq 2$ I.N. $[0, 2]$	$4 < -2(x+3) < 6$ $4 < -2x-6 < 6$ $\begin{array}{r} +6 \quad +6 \quad +6 \end{array}$ $10 < -2x < 12$ $\frac{10}{-2} > \frac{-2x}{-2} > \frac{12}{-2}$ $-5 > x > -6$ OR $-6 < x < -5$ I.N. $(-6, -5)$	$-4 \leq -\frac{1}{3}(3x+12) < 4$ $-4 \leq -x-4 < 4$ $\begin{array}{r} +4 \quad +4 \quad +4 \end{array}$ $0 \leq -x < 8$ $(-1) \cdot 0 \geq (-1) \cdot -x > (-1) \cdot 8$ $0 \geq x > -8$ OR $-8 < x \leq 0$ I.N. $(-8, 0]$
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C. Solve and Graph the solution set.

$3(x-2)+5 > 2(3x+5)+1$ $3x-6+5 > 6x+10+1$ $3x-1 > 6x+11$ $\begin{array}{r} +1 \quad +1 \end{array}$ $3x > 6x+12$ $\begin{array}{r} -6x \quad -6x \end{array}$ $-3x > 12$ $\frac{-3x}{-3} < \frac{12}{-3}$ $x < -4$ I.N. $(-\infty, -4)$	$-4(3x-1)+2x \leq 2(4x-1)-3$ $-12x+4+2x \leq 8x-2-3$ $-10x+4 \leq 8x-5$ $\begin{array}{r} -4 \quad -4 \end{array}$ $-10x \leq 8x-9$ $\begin{array}{r} -8x \quad -8x \end{array}$ $-18x \leq -9$ $\frac{-18x}{-18} \geq \frac{-9}{-18}$ $x \geq \frac{1}{2}$ I.N. $[\frac{1}{2}, \infty)$
$3x-5 < 4 \quad \text{or} \quad x-6 > 1$ $\begin{array}{r} +5 \quad +5 \end{array} \quad \begin{array}{r} +6 \quad +6 \end{array}$ $3x < 9$ $x < 3 \quad \text{OR} \quad x > 7$ Interval Notation $(-\infty, 3) \cup (7, \infty)$	$-3x+10 < -20 \quad \text{or} \quad 7-x > 8$ $\begin{array}{r} -10 \quad -10 \end{array} \quad \begin{array}{r} -7 \quad -7 \end{array}$ $-3x < -30 \quad \text{OR} \quad -x > 1$ $\frac{-3x}{-3} > \frac{-30}{-3} \quad \frac{-x}{-1} < \frac{1}{-1}$ $x > 10 \quad \text{OR} \quad x < -1$ I.N. $(-\infty, -1) \cup (10, \infty)$

$2x + 3 < 13$ or $4x - 7 \geq -7$	$13 < 8x + 5 < 5$
$\begin{array}{rcl} 2x + 3 < 13 & \text{or} & 4x - 7 \geq -7 \\ -3 & & +7 \quad +7 \\ \hline 2x < 10 & & 4x \geq 0 \\ \frac{2x}{2} < \frac{10}{2} & & \frac{4x}{4} \geq \frac{0}{4} \\ x < 5 & \text{or} & x \geq 0 \end{array}$  Any Real, $\mathbb{R} = (-\infty, \infty)$	$\begin{array}{rcl} 13 < 8x + 5 < 5 \\ -5 & & -5 \quad -5 \\ \hline \frac{8}{8} < \frac{8x}{8} < \frac{0}{8} \\ 1 < x < 0 \end{array}$ No Solution ← There is no number that is greater than 1 at the same time less than 0.

Clint wishes to earn a B which is at least 80 but not more than 90. What range must he score on the fourth exam if the first three were 65, 75 and 90?

Let x = his score on the 4th exam

Then for a "B" his average must be between 80 and 90,

or $80 \leq \text{AVG} < 90$

set-up $\rightarrow \boxed{80 \leq \frac{65+75+90+x}{4} < 90}$

$$(4) \cdot 80 \leq (4) \left(\frac{230+x}{4} \right) < (4) \cdot 90$$

$$\begin{array}{rcl} 320 & \leq & 230 + x < 360 \\ -230 & & -230 \quad -230 \end{array}$$

$$90 \leq x < 130$$

He must score at least a 90 on exam #4.

Bill earns \$12.00 plus \$0.25 for every person he gets to register to vote. How many people must he register to earn at least \$50.00 for the day?

Let x = # of people Bill signs up to vote

Bill wishes his salary $\geq \$50.00$

so then $\boxed{12.00 + .25x \geq 50.00}$

-12

-12

$$.25x \geq 38$$

$$x \geq 38/.25$$

$$x \geq 152$$

He must get at least 152 people to register.

Chapter 2 *Linear Equations and Inequalities*

Sample Exam

Please answer all the questions and show work where appropriate.

1. Is $x = -2$ a solution to the linear equation $3x + 5 = -1$?
2. Solve for x :
 - a. $7x - 5 = 30$
 - b. $2 - 4x = -6$
3. Solve: $-2(3x - 3) = 4x - 9$
4. Solve: $2x - \frac{3}{15} = \frac{1}{5}x + \frac{2}{3}$
5. Find three consecutive odd integers whose sum is 51. (*Set-up an algebraic equation then solve it.*)
6. The perimeter of a rectangle is 110ft. Find the dimensions if the length is 5ft less than twice the width.
7. Solve and graph your solutions on a number line:
 - a. $3x + 7 \geq 22$
 - b. $-3 < 2x + 3 < 11$
8. Solve and express your answer in interval notation:
 $-3(x + 1) \leq \frac{1}{2}(2x + 10) - 7$
9. If a student scores 60, 65 and 71 on the first three exams what must he score on the fourth exam to earn at least a 70 average?
10. How long will it take Joe to drive 510 miles at an average speed of 60mph?

Chapter 2 *Linear Equations and Inequalities*

Sample Exam Answers

1. Yes
2. a. $x = 5$
b. $x = 2$

3. $x = \frac{3}{2}$

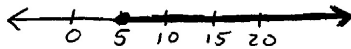
4. $x = \frac{13}{27}$

5. 15, 17 and 19

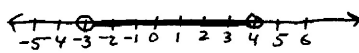
6. Width = 20 ft

Length = 35 ft

7. a. $x \geq 5$



- b. $-3 < x < 4$



8. $x \geq -\frac{1}{4}$

Interval notation $[-\frac{1}{4}, \infty)$

9. At least 84

10. 8.5 hours

Is $x = -2$ a solution to the linear equation $3x + 5 = -1$?

$$\begin{aligned} 3x + 5 &= -1 \\ 3(-2) + 5 & \\ = -6 + 5 & \\ = -1 & \quad \text{Yes} \end{aligned}$$

Solve for x :
 $7x - 5 = 30$

$$\begin{aligned} 7x - 5 &= 30 \\ +5 &+5 \\ \hline 7x &= 35 \\ \hline \frac{7x}{7} &= \frac{35}{7} \\ x &= 5 \end{aligned}$$

Solve for x :
 $2 - 4x = -6$

$$\begin{aligned} 2 - 4x &= -6 \\ -2 &-2 \\ \hline -4x &= -8 \\ \hline \frac{-4x}{-4} &= \frac{-8}{-4} \\ x &= 2 \end{aligned}$$

$$-2(3x - 3) = 4x - 9$$

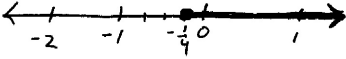
$$\begin{aligned} -2(3x - 3) &= 4x - 9 \\ -6x + 6 &= 4x - 9 \\ -6x &= 4x - 15 \\ -4x &-4x \\ \hline -10x &= -15 \\ \hline \frac{-10x}{-10} &= \frac{-15}{-10} \\ x &= \frac{3}{2} \end{aligned}$$

$$2x - \frac{3}{15} = \frac{1}{5}x + \frac{2}{3}$$

$$\begin{aligned} 2x - \frac{3}{15} &= \frac{1}{5}x + \frac{2}{3} \\ +\frac{3}{15} &+\frac{3}{15} \\ \hline 2x &= \frac{1}{5}x + \frac{13}{15} \\ -\frac{1}{5}x &-\frac{1}{5}x \\ \hline \frac{9}{5}x &= \frac{13}{15} \\ x &= \frac{13}{15} \left(\frac{5}{9} \right) \\ x &= \frac{13}{27} \end{aligned}$$

$$-3(x+1) \leq \frac{1}{2}(2x+10) - 7$$

$$\begin{aligned} -3(x+1) &\leq \frac{1}{2}(2x+10) - 7 \\ -3x - 3 &\leq x + 5 - 7 \\ -x &-x \\ \hline -4x - 3 &\leq -2 \\ +3 &+3 \\ \hline -4x &\leq 1 \\ \hline \frac{-4x}{-4} &\leq \frac{1}{-4} \\ x &\geq -\frac{1}{4} \end{aligned}$$



Interval Not. $[-\frac{1}{4}, \infty)$

Find three consecutive odd integers whose sum is 51. (Set-up an algebraic equation then solve it.)

Let x = the first odd integer
 $x+2$ = the next odd integer
 $x+4$ = the third odd integer

$$\text{Set-up} \rightarrow x + x + 2 + x + 4 = 51$$

$$3x + 6 = 51$$

$$\frac{3x}{3} = \frac{45}{3}$$

$$x = 15$$

The three consecutive odd integers are 15, 17 and 19.

The perimeter of a rectangle is 110ft. Find the dimensions if the length is 5ft less than twice the width.

Let w = the width of the rectangle

$2w-5$ = the length of the rectangle

Perimeter $P = 2l + 2w$

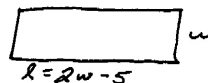
$$\text{Set-up} \rightarrow 110 = 2(2w-5) + 2w$$

$$110 = 4w - 10 + 2w$$

$$110 = 6w - 10$$

$$\frac{120}{6} = \frac{6w}{6}$$

$$20 = w$$



$$\text{Ans: } w = 20 \text{ ft } \checkmark$$

$$\text{length } l = 2(20) - 5$$

$$= 40 - 5$$

$$= 35 \text{ ft } \checkmark$$

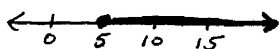
Solve and Graph the solution set:

$$3x + 7 \geq 22$$

$$\begin{array}{r} 3x + 7 \geq 22 \\ -7 \quad -7 \end{array}$$

$$\frac{3x}{3} \geq \frac{15}{3}$$

$$x \geq 5$$



$$[5, \infty)$$

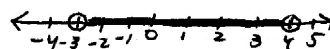
Solve and Graph the solution set:

$$-3 < 2x + 3 < 11$$

$$\begin{array}{r} -3 < 2x + 3 < 11 \\ -3 \quad -3 \quad -3 \end{array}$$

$$\frac{-6}{2} < \frac{2x}{2} < \frac{8}{2}$$

$$-3 < x < 4$$



$$(-3, 4)$$

If a student scores 60, 65 and 71 on the first three exams what must he score on the fourth exam to earn at least a 70 average?

Let x = score on 4th exam

$$\frac{60 + 65 + 71 + x}{4} \geq 70$$

$$\star: \frac{196 + x}{4} \geq 70 \cdot 4$$

$$\begin{array}{r} 196 + x \geq 280 \\ -196 \quad -196 \end{array}$$

$$x \geq 84$$

At least 84 on the 4th exam.

How long will it take Joe to drive 510 miles at an average speed of 60mph?

$$D = r \cdot t$$

$$\frac{510}{60} = \frac{60 \cdot t}{60}$$

$$8.5 = t$$

or $8\frac{1}{2}$ hours
to travel 510 miles.

Graphing is a barrier in Algebra for many students. In this chapter we will learn three methods for graphing lines. Each method has strengths and weaknesses. Learning how to graph just takes practice. Be patient, take your time in this chapter and before long you will learn to enjoy graphing lines.

Graphing Lines

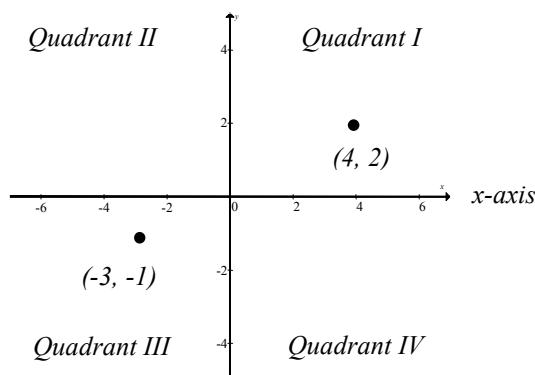
*3.1... Method 1 – Plotting Points**3.2... Method 2 – Using x- and y-intercepts**3.3... Slope**3.4... Method 3 – Slope Intercept Form**3.5... Parallel and Perpendicular Lines**3.6... Function Notation**3.7... Linear Inequalities**Sample Exam**Sample Exam Solutions*

Problems Solved! Chapter 3 *Graphing Linear Equations*
Method 1 – Plotting Points

Graphing lines can be done in a number of ways. This first section describes a method, **Plotting Points**, that always works and can be used for many other types of equations. The drawback is that it is tedious and sometimes time consuming.

Cartesian Coordinate System – Sometimes referred to as the rectangular coordinate system, consists of two perpendicular real number lines intersecting at zero.

Positions on this grid system can be identified in ordered pairs, (x, y) . The center of the system, $(0, 0)$ is called the *origin*.



The horizontal real number line is usually called the x -axis and is used for the *independent variable*. The vertical real number line is called the y -axis and is used for the *dependent variable*.

Notice that in a linear equation with two variables, $y = 3x - 2$, the y -value depends on what the x -value is. Since x is independent here, choose any real number, say $x = 4$, and you can find the corresponding y -value by evaluating $y = 3(4) - 2 = 12 - 2 = 10$. So the ordered pair $(4, 10)$ is a point on the line.

Graph $y = -2x + 6$ by plotting 5 points.

x	y
-1	8
0	6
1	4
2	2
3	0

Step 1: Choose any five x -values.

Step 2: Find the corresponding y -values.

Step 3: Plot the points and connect the dots.

When choosing x -values it is wise to pick negative and positive numbers as well as zero. Try to find the points where the line crosses the x and y axes. These special points are called the x - and y -intercepts.

A. Is the given point a solution?

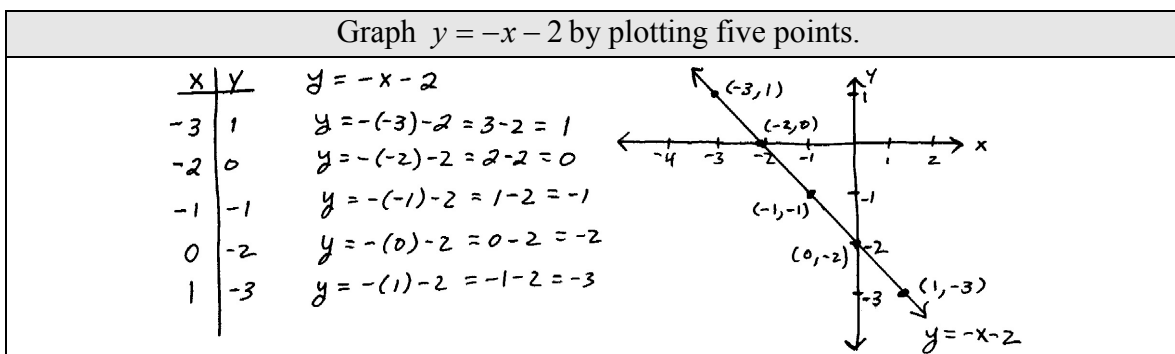
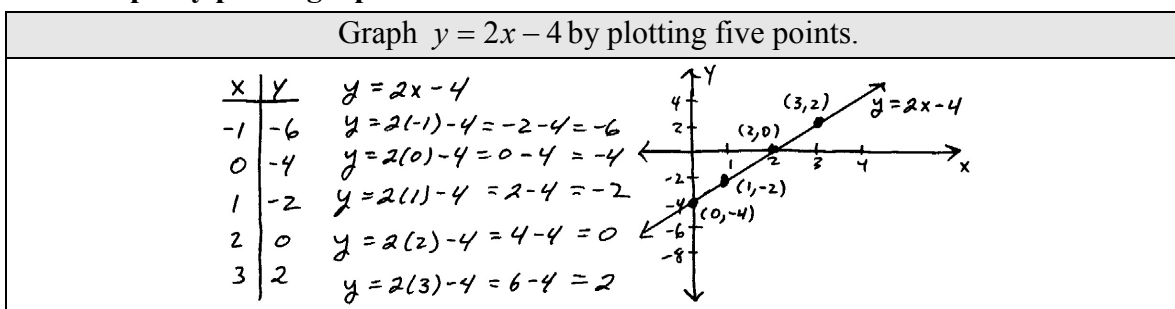
Is $(-5, 2)$ a solution for $2x + 3y = -4$?	Is $(1, -2)$ a solution for $-4x - 2y = 1$?	Is $(-3, -1)$ a solution for $-x + 3y = 0$?
$\begin{array}{l l} (x, y) & 2x + 3y = -4 \\ \hline (-5, 2) & 2(-5) + 3(2) \stackrel{?}{=} -4 \\ & -10 + 6 \\ & = -4 \quad \checkmark \text{ Yes.} \end{array}$	$\begin{array}{l l} (x, y) & -4x - 2y = 1 \\ \hline (1, -2) & -4(1) - 2(-2) \stackrel{?}{=} 1 \\ & -4 + 4 \\ & = 0 \neq 1 \text{ No} \end{array}$	$\begin{array}{l l} (x, y) & -x + 3y = 0 \\ \hline (-3, -1) & -(-3) + 3(-1) \stackrel{?}{=} 0 \\ & 3 - 3 = 0 \quad \checkmark \text{ Yes} \end{array}$

Remember that we are less likely to make a mistake if we insert a parenthesis where we see a variable and then substitute in the appropriate values.

B. Find the corresponding value.

Find y when $x = -5$ and $-2x + y = 3$	Find x when $y = 0$ and $3x - 5y = 15$	Find y when $x = -3$ and $x - y = -12$
$\begin{array}{l} -2x + y = 3 \\ -2(-5) + y = 3 \\ 10 + y = 3 \\ -10 \quad -10 \\ \hline y = -7 \end{array}$	$\begin{array}{l} 3x - 5y = 15 \\ 3x - 5(0) = 15 \\ 3x = 15 \\ \frac{3x}{3} = \frac{15}{3} \\ x = 5 \end{array}$	$\begin{array}{l} x - y = -12 \\ -3 - y = -12 \\ +3 \quad +3 \\ \hline -y = -9 \\ y = 9 \end{array}$

C. Graph by plotting 5 points.

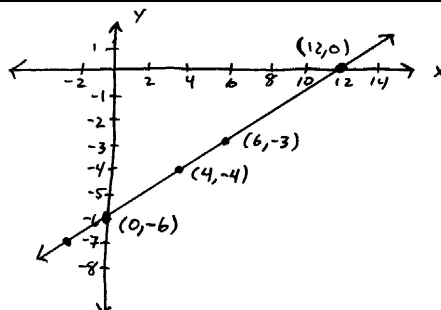


Tip: Choosing a scale when creating a blank coordinate system will take some thought. Keep in mind that the scale on the x-axis need not be the same as the scale on the y-axis.



Graph $y = \frac{1}{2}x - 6$ by plotting five points.

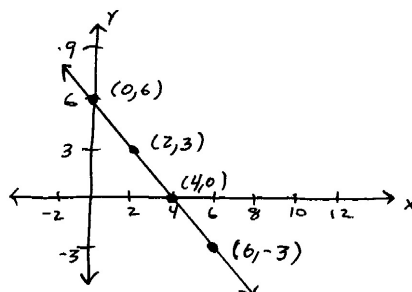
x	y	$y = \frac{1}{2}(x) - 6$
-2	-7	$y = \frac{1}{2}(-2) - 6 = -1 - 6 = -7$
0	-6	$y = \frac{1}{2}(0) - 6 = 0 - 6 = -6$
4	-4	$y = \frac{1}{2}(4) - 6 = 2 - 6 = -4$
6	-3	$y = \frac{1}{2}(6) - 6 = 3 - 6 = -3$
12	0	$y = \frac{1}{2}(12) - 6 = 6 - 6 = 0$



Tip: When the coefficient of x is a fraction, choose x -values to be multiples of the denominator so that you might avoid unnecessarily tedious calculations.

Graph $y = -\frac{3}{2}x + 6$ by plotting five points.

x	y	$y = -\frac{3}{2}x + 6$
-2	9	$y = -\frac{3}{2}(-2) + 6 = 3 + 6 = 9$
0	6	$y = -\frac{3}{2}(0) + 6 = 0 + 6 = 6$
2	3	$y = -\frac{3}{2}(2) + 6 = -3 + 6 = 3$
4	0	$y = -\frac{3}{2}(4) + 6 = -6 + 6 = 0$
6	-3	$y = -\frac{3}{2}(6) + 6 = -9 + 6 = -3$



Graph $2x - 3y = 6$ by plotting five points.

First solve for y :

$$2x - 3y = 6$$

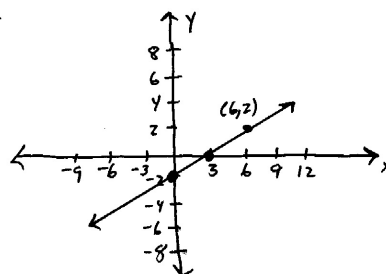
$$\Rightarrow \frac{-3y}{-3} = \frac{-2x+6}{-3}$$

$$y = \frac{-2}{-3}x + \frac{6}{-3}$$

$$y = \frac{2}{3}x - 2$$

$$y = \frac{2}{3}x - 2$$

x	y
-3	-4
0	-2
3	0
6	2
9	4



Divide both terms by -3.

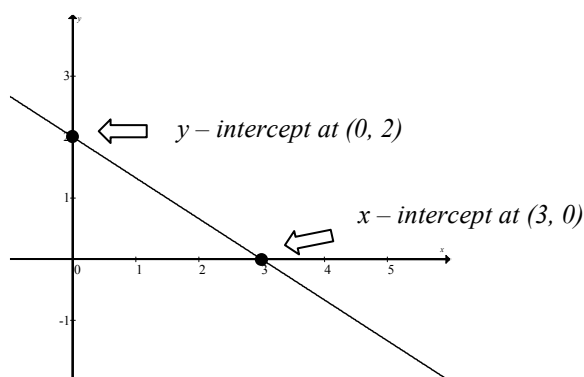
When dividing a binomial by a number you must **divide both terms** by that number. For example, treat the -3 as a common denominator as in the previous problem,

$$\frac{-2x+6}{-3} = \frac{-2x}{-3} + \frac{6}{-3} = \frac{2}{3}x - 2. \text{ A common error is to just divide the 6 by -3.}$$

You might be familiar with the basic fact that **two points determine a line**. This fact leads to a nice and easy way to graph lines using the two points called x and y-intercepts.

x-intercept – The point at which a line crosses the x-axis has the form $(x, 0)$.

y-intercept – The point at which a line crosses the y-axis has the form $(0, y)$.



All x-intercepts, if they exist, must have a corresponding y-value of zero. All y-intercepts must have a corresponding x-value of zero. This might sound confusing but just remember the following steps to algebraically find intercepts.

Graph $3x - 5y = 15$ using the intercepts.

To find the y – intercepts plug $x = 0$.

$$3(0) - 5y = 15$$

$$-5y = 15$$

$$y = -3$$

y-intercept is $(0, -3)$

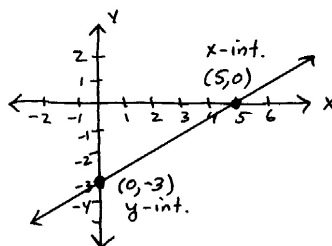
To find the x – intercepts plug $y = 0$.

$$3x - 5(0) = 15$$

$$3x = 15$$

$$x = 5$$

x-intercept is $(5, 0)$



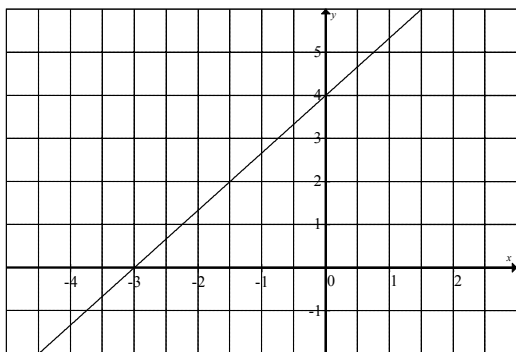
Plot the points and connect the dots.

This is a nice and easy method for determining the two points you need for graphing a line. In fact, we will use this exact technique for finding intercepts when we study the graphs of all the conic sections later in the course. Be careful not to say that $y = -3$ is the

y-intercept because the intercepts, actually, are ordered pairs or points on the graph so you should take care to say $(0, -3)$ is the y-intercept.

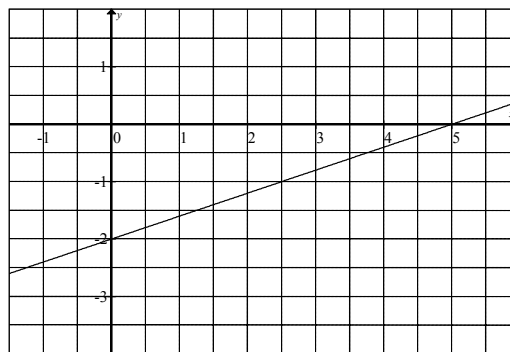
A. Use the given graph to answer the question.

Find the x and y – intercepts.



Just read the graph
x-intercept $(-3, 0)$
y-intercept $(0, 4)$

Find the x and y – intercepts.



Just read the graph
x-intercept $(5, 0)$
y-intercept $(0, -2)$

Be sure to pay attention to the scale. Notice that two ticks span one unit in the second example above. Misreading the scale is the most common error in this type of problem.

B. Find the intercepts then graph.

Graph $-4x + 3y = 12$ using the intercepts.

y-int. plug $x=0$

$$-4(0) + 3y = 12$$

$$3y = 12$$

$$y = 4$$

y-intercept is $(0, 4)$

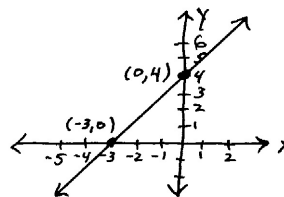
x-int. plug $y=0$

$$-4x + 3(0) = 12$$

$$-4x = 12$$

$$x = -3$$

x-intercept is $(-3, 0)$



Graph $-4x + 2y = -6$ using the intercepts.

y-int plug $x=0$

$$-4(0) + 2y = -6$$

$$2y = -6$$

$$y = -3$$

y-intercept $(0, -3)$

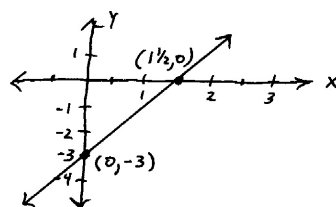
x-int. plug $y=0$

$$-4x + 2(0) = -6$$

$$-4x = -6$$

$$x = \frac{6}{4} = \frac{3}{2}$$

x-intercept $(\frac{3}{2}, 0)$



Graph $y = -5x + 15$ using the intercepts.y-int. plug $x=0$

$$y = -5(0) + 15$$

$$y = 15$$

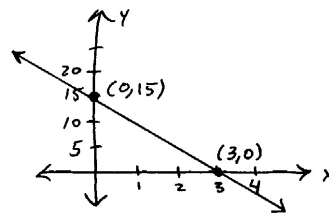
y-intercept $(0, 15)$ x-int. plug $y=0$

$$0 = -5x + 15$$

$$-15 = -5x$$

$$-15 = -5x$$

$$3 = x$$

x-intercept $(3, 0)$ Graph $y = -\frac{3}{4}x + 9$ using the intercepts.y-int. plug $x=0$

$$y = -\frac{3}{4}(0) + 9$$

$$y = 9$$

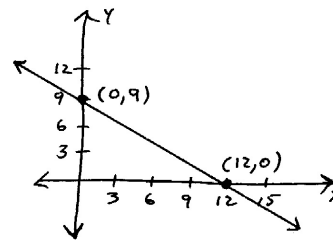
y-intercept $(0, 9)$ x-int. plug $y=0$

$$0 = -\frac{3}{4}x + 9$$

$$-9 = -\frac{3}{4}x$$

$$-9(-\frac{4}{3}) = -\frac{3}{4}x(-\frac{4}{3})$$

$$12 = x$$

x-intercept $(12, 0)$ 

This brings us to one of the most popular questions in linear graphing. **Do all lines have x and y-intercepts?** The answer is NO. Horizontal lines, of the form $y = b$, do not necessarily have x-intercepts. Vertical lines, of the form $x = a$, do not necessarily have y-intercepts.

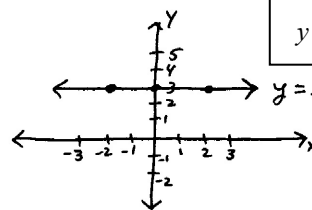
Graph $y = 3$ x | y $y = 3$

$$-2 \quad 3$$

$$0 \quad 3$$

$$2 \quad 3$$

x is free to be any real number but $y = 3$ always.



Horizontal line
 $y = 3$ has no x-intercept

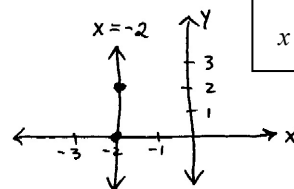
Graph $x = -2$ x | y $x = -2$

$$-2 \quad -2$$

$$-2 \quad 0$$

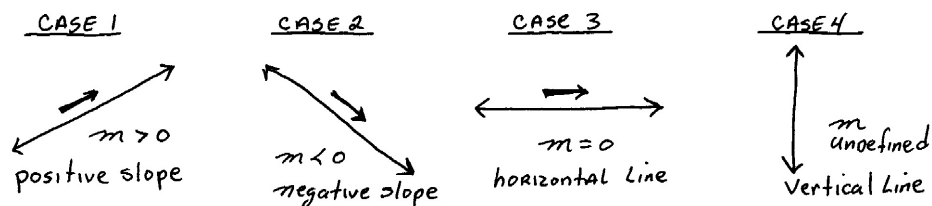
$$-2 \quad 2$$

y is free to be any real number but $x = -2$ always.



Vertical line
 $x = -2$ has no y-intercept

Before we can understand our third method for graphing a line we must learn the concept of slope. Slope is a way we measure the steepness of a line that will aid us in graphing. When thinking about the steepness of a line, consider reading them from left to right, there will be four cases.

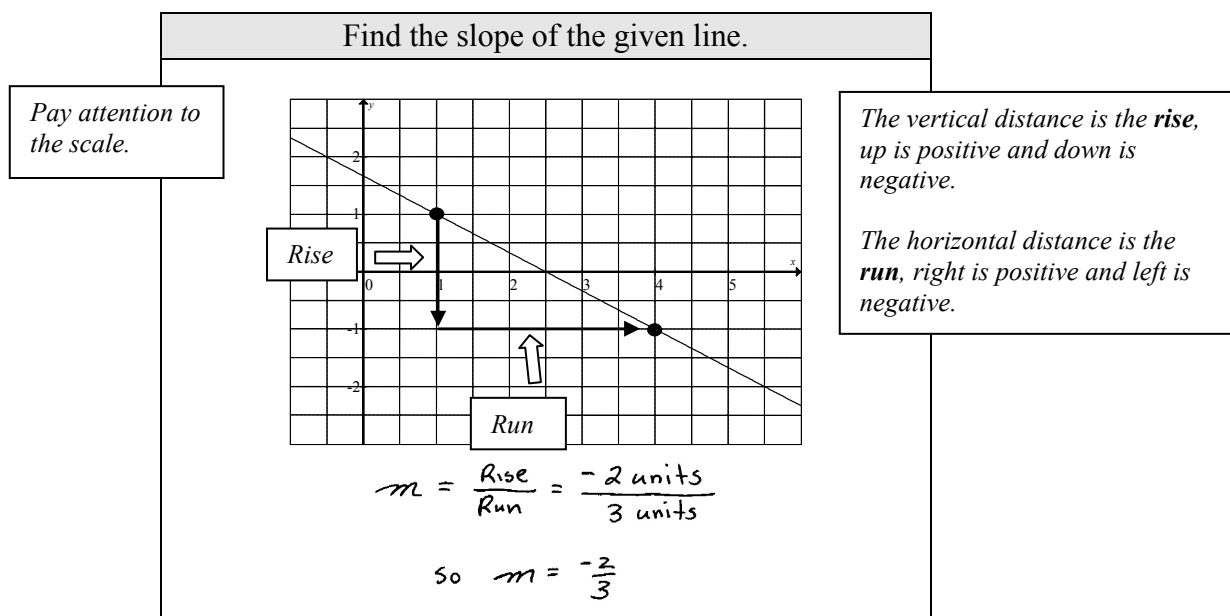


Notice that the phrase “no slope” is not listed. This phrase is ambiguous so do not use it, the slope is zero if the line is horizontal and the slope is undefined if the line is vertical.

Slope – denoted m , measures the steepness of a line:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

One way to determine the slope is to count the number of units that it takes to rise and run from point to point. This is what you are asked to do if the graph is given.



When dealing with a negative fraction, remember that $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$. It is correct to use any one of these ratios for the slope. You will get the same line. Try it.

If we are given two points, (x_1, y_1) and (x_2, y_2) , we can calculate the slope algebraically using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, difference in the y-value divided by the difference in the x-values.

Find the slope of the line passing through
(1, 1) and (4, -1)

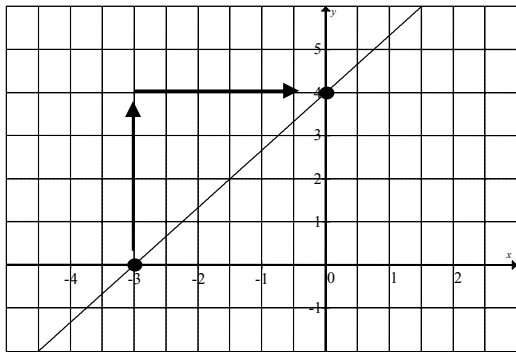
	(x_1, y_1)	(x_2, y_2)
Given	(1, 1)	and (4, -1)
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{4 - 1} = \frac{-2}{3}$	
	$m = -\frac{2}{3} \checkmark$	

Notice that we get the same answer as in the previous example.

Write a blank formula first then substitute into it.

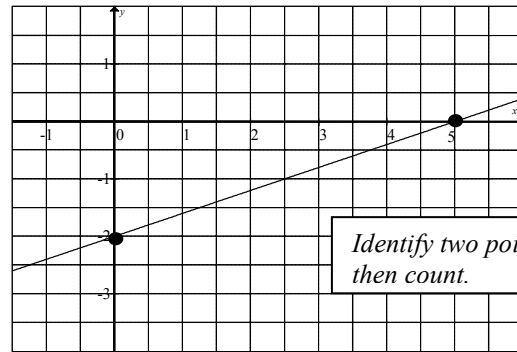
A. Find the slope of the given line.

Find the slope of the line.



$$m = \frac{\text{Rise}}{\text{Run}} = \frac{4}{3}$$

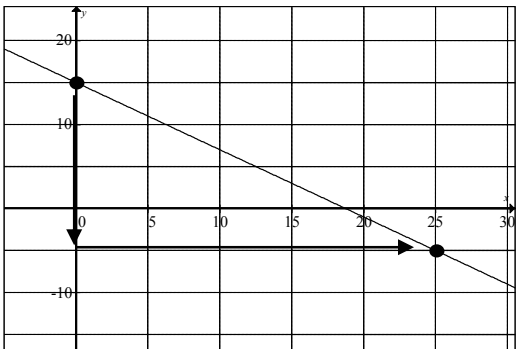
Find the slope of the line.



Identify two points then count.

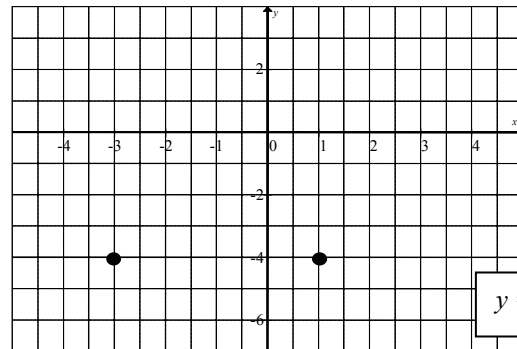
$$m = \frac{\text{Rise}}{\text{Run}} = \frac{2}{5}$$

Find the slope of the line.



$$m = \frac{\text{Rise}}{\text{Run}} = \frac{-20}{25} = \frac{-4}{5}$$

Find the slope of the line.



$$y = -4$$

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{0}{4} = 0$$

Horizontal Line

B. Find the slope of the line passing through

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{5 - (-2)} \\ &= \frac{-5 - 3}{5 + 2} \\ &= \frac{-8}{7} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - (-1)}{3 - (-1)} \\ &= \frac{-7 + 1}{3 + 1} \\ &= \frac{-6}{4} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-8)}{-14 - 10} \\ &= \frac{4 + 8}{-14 - 10} \\ &= \frac{12}{-24} = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 5}{0 - (-6)} \\ &= \frac{11 - 5}{0 + 6} \\ &= \frac{6}{6} = 1 \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-2)}{-3 - 7} \\ &= \frac{-2 + 2}{-3 - 7} \\ &= \frac{0}{-10} = 0 \\ &\text{(horizontal line)} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-4)}{6 - 6} \\ &= \frac{-5 + 4}{6 - 6} \\ &= \frac{-1}{0} \text{ undefined} \\ &\text{(vertical line)} \end{aligned}$$

Tip: You should always write in the formula before you plug into it so that whoever reads your work, including yourself, will understand what you are doing.



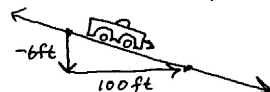
We might run into other books that refer to slope as change in y , or Δy , divided by the change in x , or Δx .

C. Word Problems

While driving on the Grapevine, Joe encountered a sign warning of a 6% downgrade in the road. What does this say about the steepness of the road?

A 6% downgrade warns of a steep incline or slope.

$$m = -6\% = \frac{-6}{100} = \frac{\text{Rise}}{\text{Run}}$$



So for every 100ft forward the road drops 6ft.

The average tuition at a public four-year college was \$2977 in 1995 and \$3489 in 1998. Find the rate at which tuition was increasing.

Treat time as the x -values because it is independent
Then cost is the y -value because it is dependent
on the year.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\$3489 - \$2977}{1998 - 1995} = \frac{\$512}{3\text{yrs}}$$

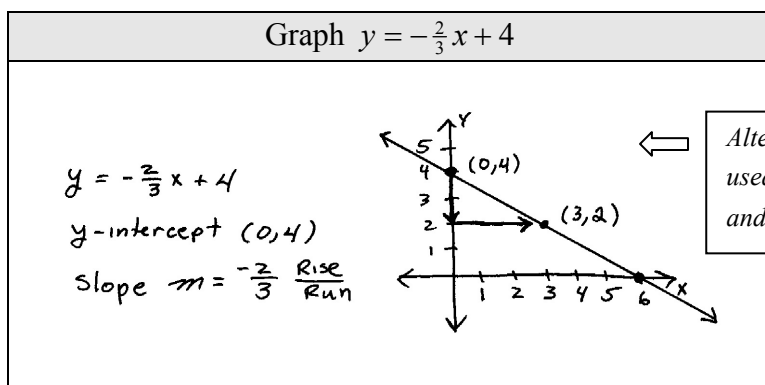
$$\text{or } m \approx \frac{\$170.67}{1\text{yr}} \text{ after we divide}$$

So the cost increased \$170.67 per year.

Our final method for graphing lines will, in most cases, be the method of choice. All lines are completely determined by their y-intercept and slope. Once a linear equation is in **Slope Intercept Form** graphing it becomes easy.

Slope Intercept Form: $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept.

Tip: Always start with the y-intercept and mark off the slope from there. If we continue marking off the slope we can find many other points on the line.



Alternatively, we could have used the slope $-\frac{2}{3}$, up two units and left 3 units.

Notice that if you continue marking off the slope in this particular example you will get the x-intercept, $(6, 0)$. This is very special and does not always happen.

Slope Intercept Form also allows us to easily find the equation of the line given the graph. If you are given the slope and any point on the line you can find its equation by following the steps below.

Find the equation of the line passing through $(2, 3)$ with slope $m = \frac{1}{2}$.

- Start with $y = mx + b$
- ① m is given: $y = \frac{1}{2}x + b$
 - ② to find b use $(2, 3)$:

$$3 = \frac{1}{2}(2) + b$$

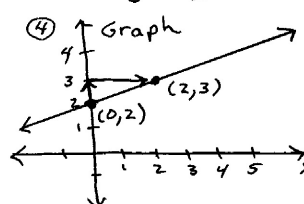
$$3 = 1 + b$$

$$2 = b$$

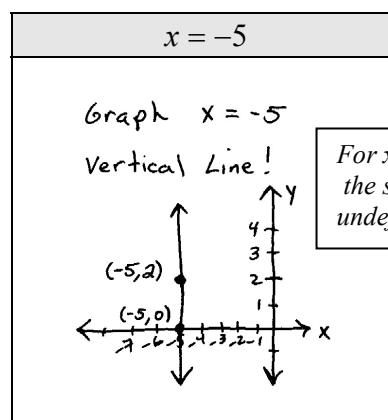
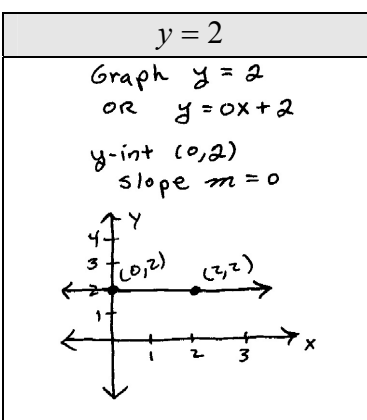
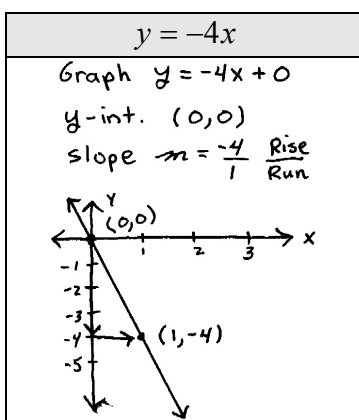
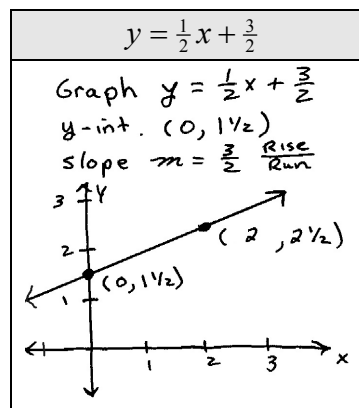
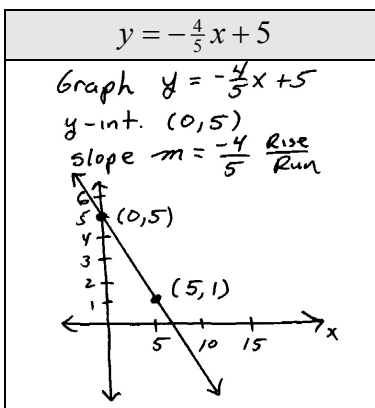
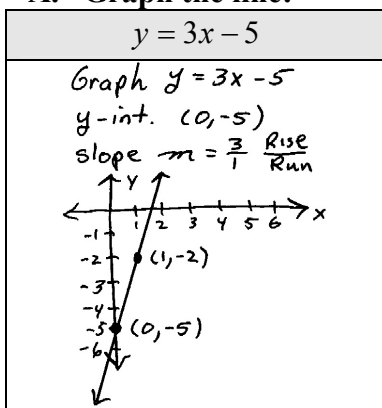
Use the given point to find b .

③ Put it all together:

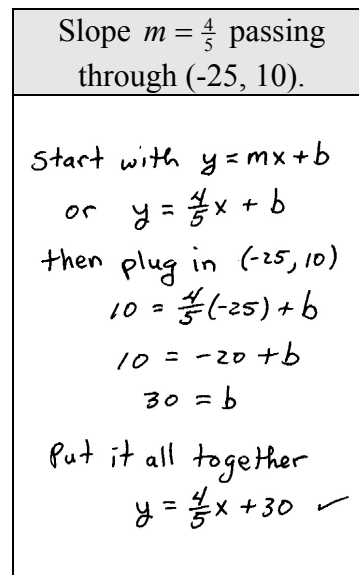
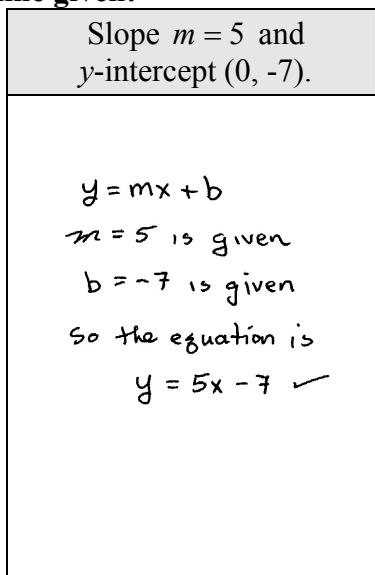
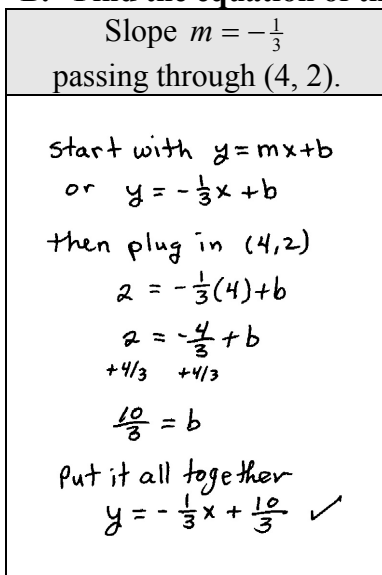
$$y = \frac{1}{2}x + 2$$



This equation is the answer to the question.

A. Graph the line:

Since two points determine a line it is nice to label at least two points on all our graphs, even if it does not ask for us to do so.

B. Find the equation of the line given:

Slope $m = 0$ passing through $(-8, 3)$. start with $y = mx + b$ or $y = 0x + b$ Plug in $(-8, 3)$ $3 = 0(-8) + b$ $3 = b$ Put it together $y = 0x + 3$ or $y = 3$ ✓	Slope $m = \frac{5}{7}$ passing through $(0, 0)$. start with $y = mx + b$ or $y = \frac{5}{7}x + b$ Plug in $(0, 0)$ $0 = \frac{5}{7}(0) + b$ $0 = b$ Put it all together $y = \frac{5}{7}x + 0$ or $y = \frac{5}{7}x$ ✓	Slope m undefined passing through $(-5, 6)$. Slope m undefined means we have a Vertical Line ($x = a$) So here we say $x = -5$ ✓
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C. Graph the line.

$3x - 4y = 12$ $\begin{array}{r} 3x - 4y = 12 \\ -3x \quad -3x \\ \hline -4y = -3x + 12 \\ \frac{-4y}{-4} = \frac{-3x + 12}{-4} \\ y = \frac{3}{4}x - 3 \end{array}$ y-int $(0, -3)$ $m = \frac{3}{4}$ Rise Run	$-2x + 5y = 20$ $\begin{array}{r} -2x + 5y = 20 \\ +2x \quad +2x \\ \hline 5y = 2x + 20 \\ \frac{5y}{5} = \frac{2x + 20}{5} \\ y = \frac{2}{5}x + 4 \end{array}$ y-int. $(0, 4)$ $m = \frac{2}{5}$ Rise Run
$x - y = 5$ $\begin{array}{r} x - y = 5 \\ -x \quad -x \\ \hline -y = -x + 5 \\ \frac{-y}{-1} = \frac{-x + 5}{-1} \\ y = x - 5 \end{array}$ y-int. $(0, -5)$ $m = \frac{1}{1}$ Rise Run	$x + \frac{1}{2}y = 3$ $\begin{array}{r} x + \frac{1}{2}y = 3 \\ -x \quad -x \\ \hline \frac{1}{2}y = -x + 3 \\ 2(\frac{1}{2}y) = 2(-x + 3) \\ y = -2x + 6 \end{array}$ y-int $(0, 6)$ $m = \frac{-2}{1}$ Rise Run

Given that the cell count, y , of a certain bacteria is given by $y = 500x + 30,000$ where x is the time in minutes, what does the 30,000 represent in the linear equation?

If we let $x = 0$ minutes then $y = 500(0) + 30,000$
 or $y = 30,000$ cells when we start.
 So the 30,000 is the initial amount of cells.

Many of the problems that we will encounter in this chapter involve parallel or perpendicular lines. To study this we must focus on the slopes of the lines.

Two non-vertical lines with slopes m_1 and m_2 are:

Parallel – if the lines have the same slope, $m_1 = m_2$.

Perpendicular – if the slopes are negative reciprocals, $m_2 = -\frac{1}{m_1}$

or equivalently, if $m_1 \cdot m_2 = -1$.

There is notation that corresponds to these ideas, for example, $m_{\perp}$ reads “the perpendicular slope.” To find the slope of the perpendicular line simply reciprocate the number and change the sign.

Find the corresponding perpendicular slope.

If $m = \frac{2}{3}$ then $m_{\perp} = -\frac{3}{2}$

If $m = 2$ then $m_{\perp} = -\frac{1}{2}$

If $m = -\frac{1}{3}$ then $m_{\perp} = 3$

If $m = -7$ then $m_{\perp} = \frac{1}{7}$

If $m = 1$ then $m_{\perp} = -\frac{1}{1} = -1$

If $m = 0$ then $m_{\perp} = \frac{1}{0}$ undefined
(Vertical Line)

Any integer can be thought of as a fraction by dividing by 1, for example $m = 9 = \frac{9}{1}$.

Of course, just use the same slope if you are asked to find the slope of the parallel line.

Find the equation of the line perpendicular to $y = -\frac{1}{4}x + 2$ passing through the point $(-1, -5)$.

Step 1: Find m .

① Given $m = -\frac{1}{4}$
So $m_{\perp} = 4$

③ Put it all together

$y = mx + b$

$y = 4x - 1$ ✓

Step 3: put it all together using $y = mx + b$

Step 2 Find b .

② $y = 4x + b$
Plug in $(-1, -5)$

$-5 = 4(-1) + b$

$-5 = -4 + b$

$+4 \quad +4$

$-1 = b$

This three step process always works. We could solve this problem using point – slope form as well.

A. Are the lines parallel, perpendicular or neither?

$$\begin{cases} y = -\frac{1}{3}x + 7 \\ y = 3x + 2 \end{cases}$$

Given $m_1 = -\frac{1}{3}$
and $m_2 = 3$

Perpendicular, since
they are negative
reciprocals, or
 $(-\frac{1}{3}) \cdot 3 = -1$.

$$\begin{cases} 5x - y = 6 \\ 10x - 2y = 1 \end{cases}$$

Line 1: $5x - y = 6$ Line 2: $10x - 2y = 1$
 $\quad \quad \quad -5x \quad \quad \quad -10x$

$$\frac{-y}{-1} = \frac{-5x+6}{-1}$$

$$y = 5x - 6$$

So $\boxed{m_1 = 5}$

$$\frac{-2y}{-2} = \frac{-10x+1}{-2}$$

$$y = 5x - \frac{1}{2}$$

$\boxed{m_2 = 5}$

compare $\leftarrow$
Since the slopes are the same: parallel

$$\begin{cases} 3x - 2y = 12 \\ -2x + 3y = 3 \end{cases}$$

Line 1: $3x - 2y = 12$ Line 2: $-2x + 3y = 3$
 $\quad \quad \quad -3x \quad \quad \quad +2x$

$$\frac{-2y}{-2} = \frac{-3x+12}{-2}$$

$$y = \frac{3}{2}x - 6$$

So $\boxed{m_1 = \frac{3}{2}}$

$$\frac{3y}{3} = \frac{2x+3}{3}$$

$$y = \frac{2}{3}x + 1$$

So $\boxed{m_2 = \frac{2}{3}}$

Since the slopes are neither the same
nor negative reciprocals: Neither

$$\begin{cases} x = 6 \\ y = -1 \end{cases}$$

$x = 6$ is a vertical line
with undefined slope

$y = -1$ is a horizontal
line with slope $m = 0$

Perpendicular ✓

B. Are the lines parallel, perpendicular or neither?

$$-2x + y = -3 \text{ and the line passing through } (6, 5) \text{ and } (2, 3).$$

Line 1: $-2x + y = -3$ Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $\quad \quad \quad +2x \quad \quad \quad +2x$

$$y = 2x - 3$$

So $\boxed{m_1 = 2}$

$$= \frac{5 - 3}{2 - 6} = \frac{2}{-4}$$

So $\boxed{m_2 = -\frac{1}{2}}$

Neither parallel nor perpendicular

$$y = 2 \text{ and the line passing through } (-3, 2) \text{ and } (5, 2).$$

Line 1: $y = 2 \Rightarrow \boxed{m_1 = 0}$
(horizontal line)

Line 2: $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-3)}$

$$= \frac{0}{5+3} = \frac{0}{8} = 0$$

$\boxed{m_2 = 0}$

Parallel. ✓

$7x + 14y = 1$ and the line passing through $(-6, -4)$ and $(-5, -2)$.	$x = 4$ and the line passing through $(-1, 4)$ and $(-1, 3)$.
Line 1: $7x + 14y = 1$ $-7x$ $-7x$ Line 2: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{14y}{14} = \frac{-7x + 1}{14}$ $= \frac{-2 - (-4)}{-5 - (-6)} = \frac{-2 + 4}{-5 + 6}$ $y = \frac{-7}{14}x + \frac{1}{14}$ $= \frac{2}{1} = 2$ $y = -\frac{1}{2}x + \frac{1}{14}$ so $m_1 = -\frac{1}{2}$ $\longleftrightarrow$ compare $m_2 = 2$ perpendicular $\checkmark$	Line 1: $x = 4 \Rightarrow m_1$ undefined (Vertical line) Line 2: $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{-1 - (-1)}$ $= \frac{-1}{-1 + 1} = \frac{-1}{0}$ (vertical) so m_2 undefined Parallel $\checkmark$

C. Find the slope of the line perpendicular.

$-x - 3y = 6$	$4x - y = 1$	$x = -3$
$-x - 3y = 6$ $+x$ $+x$ $-3y = x + 6$ $\frac{-3y}{-3} = \frac{x+6}{-3}$ $y = -\frac{1}{3}x - 2$ so $m = -\frac{1}{3}$ and $m_{\perp} = 3 \checkmark$	$4x - y = 1$ $-4x$ $-4x$ $-y = -4x + 1$ $\frac{-y}{-1} = \frac{-4x+1}{-1}$ $y = 4x - 1$ so $m = 4$ and $m_{\perp} = -\frac{1}{4} \checkmark$	$x = -3$ (Vertical Line) m undefined and $m_{\perp} = 0 \checkmark$ (horizontal Line)

D. Find the equation of the line:

Parallel to $y = \frac{1}{2}x + 2$ passing through $(6, -1)$	Perpendicular to $-3x + 2y = -1$ passing through $(-4, 0)$	Perpendicular to $6x + 3y = 1$ passing through $(8, -2)$
① Given $m = \frac{1}{2}$ so $m_{\perp} = \frac{1}{2}$ ② $y = \frac{1}{2}x + b$ Plug in pt. $(6, -1)$ $-1 = \frac{1}{2}(6) + b$ $-1 = 3 + b$ $-4 = b$ ③ $y = \frac{1}{2}x - 4 \checkmark$	① $-3x + 2y = -1$ $+3x$ $+3x$ $2y = 3x - 1$ $y = \frac{3}{2}x - \frac{1}{2}$ so $m = \frac{3}{2} \Rightarrow m_{\perp} = -\frac{2}{3}$ ② $y = -\frac{2}{3}x + b$ Plug in pt. $(-4, 0)$ $0 = -\frac{2}{3}(-4) + b$ $0 = \frac{8}{3} + b$ $-\frac{8}{3} = b$ ③ $y = -\frac{2}{3}x - \frac{8}{3} \checkmark$	① $6x + 3y = 1$ $-6x$ $-6x$ $3y = -6x + 1$ $y = -2x + \frac{1}{3}$ ② If $m = -2$ then $m_{\perp} = \frac{1}{2}$ Plug in $(8, -2)$ $y = \frac{1}{2}x + b$ $-2 = \frac{1}{2}(8) + b$ $-2 = 4 + b$ $-6 = b$ ③ $y = \frac{1}{2}x - 6 \checkmark$

Point-Slope Form

We have been finding the equation of a line using slope-intercept form.. It turns out this is not the only way to do it. We can also find lines using point-slope form.

Slope-Intercept form of a line: $y = mx + b$

Point-Slope form of a line: $y - y_o = m(x - x_o)$ where (x_o, y_o) is the given point.

To illustrate this new method we will rework our first example using point-slope form.

Find the equation of the line perpendicular to $y = -\frac{1}{4}x + 2$ passing through the point $(-1, -5)$.		
Step 1: Determine what point and slope to use.	$y = -\frac{1}{4}x + 2$ <u>Slope:</u> $m_{\perp} = 4$ <u>Point:</u> $(-1, -5)$	
	$y - y_o = m(x - x_o)$ $y - (-5) = 4(x - (-1))$ $y + 5 = 4(x + 1)$ $y + 5 = 4x + 4$ $y = 4x - 1$ ✓	Step 2: Substitute the appropriate values into point-slope form
	Step 3: Solve for y.	

Tip: We know two points determine a line. We can also say that one point and a slope can determine a line as well.



E. Find the equation of the line:

Parallel to $y = \frac{1}{2}x + 2$ passing through $(6, -1)$	Perpendicular to $-3x + 2y = -1$ passing through $(-4, 0)$	Perpendicular to $6x + 3y = 1$ passing through $(8, -2)$
$y = \frac{1}{2}x + 2$ <u>Slope</u> $m_{\parallel} = \frac{1}{2}$ <u>Point</u> $(6, -1)$ $y - y_o = m(x - x_o)$ $y - (-1) = \frac{1}{2}(x - 6)$ $y + 1 = \frac{1}{2}x - 3$ $y = \frac{1}{2}x - 4$ ✓	$-3x + 2y = -1$ $2y = 3x - 1$ $y = \frac{3}{2}x - \frac{1}{2}$ <u>Slope</u> $m_{\perp} = -\frac{2}{3}$ <u>Point</u> $(-4, 0)$ $y - y_o = m(x - x_o)$ $y - 0 = -\frac{2}{3}(x - (-4))$ $y = -\frac{2}{3}(x + 4)$ $y = -\frac{2}{3}x - \frac{8}{3}$ ✓	$6x + 3y = 1$ $3y = -6x + 1$ $y = -2x + \frac{1}{3}$ <u>Slope</u> $m_{\perp} = \frac{1}{2}$ <u>Point</u> $(8, -2)$ $y - y_o = m(x - x_o)$ $y - (-2) = \frac{1}{2}(x - 8)$ $y + 2 = \frac{1}{2}x - 4$ $y = \frac{1}{2}x - 6$ ✓

You may have noticed that we have been evaluating or “plugging in” lots of numbers to find the corresponding y -values. We have been doing this to find points on the graph (x, y) . This process can be streamlined using function notation:

$$y = f(x)$$

We will read this new notation above, “ y is a function of x .” So, at this point, we can think of our lines as functions:

$$y = mx + b \quad \Longleftarrow \quad \begin{array}{l} \text{Same line, just} \\ \text{different notation!} \end{array}$$

$$f(x) = mx + b$$

Function – A rule that uniquely assigns one output to every one input.

The **domain** of a function is the set of inputs, usually the x -values. The **range** of a function is the set of outputs, usually the y -values.

Given the linear function $f(x) = 2x - 4$ evaluate $f(-3)$.	
$f(-3) = -10$ says that when $x = -3$ the corresponding $y = -10$	$ \begin{aligned} f(x) &= 2x - 4 \\ \downarrow & \quad \downarrow \\ f(-3) &= 2(-3) - 4 \\ &= -6 - 4 \\ &= -10 \end{aligned} $

Think of $f(-3)$ as saying “plug -3 in wherever you see an x .”

Given the linear function $f(x) = 2x - 4$ find x when $f(x) = 6$.	
$ \begin{aligned} f(x) &= 2x - 4 \\ \downarrow & \\ 6 &= 2x - 4 \\ 10 &= 2x \\ 5 &= x \quad \checkmark \end{aligned} $	Here the y -value was given and you were asked to solve for x .

A. Evaluate the given function.

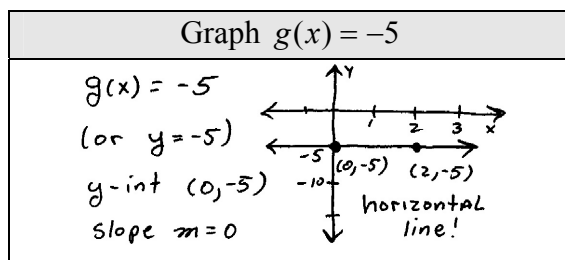
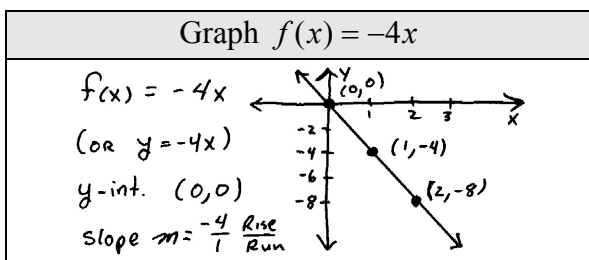
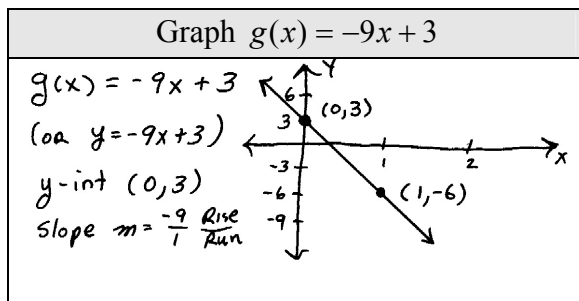
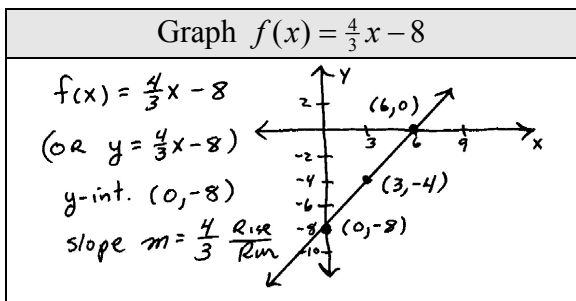
$f(x) = 2x - 7$ find $f(-5)$ $f(x) = 2x - 7$ $f(-5) = 2(-5) - 7$ $= -10 - 7$ $= -17$	$f(x) = -3x - 5$ find $f(2)$ $f(x) = -3x - 5$ $f(2) = -3(2) - 5$ $= -6 - 5$ $= -11$	$f(x) = -x$ find $f(-10)$ $f(x) = -x$ $f(-10) = -(-10)$ $= 10$
$g(x) = -x - 1$ find $g(-9)$ $g(x) = -x - 1$ $g(-9) = -(-9) - 1$ $= 9 - 1$ $= 8$	$g(x) = 4x - 1$ find $g(a)$ $g(x) = 4x - 1$ $g(a) = 4(a) - 1$ $= 4a - 1$	$g(x) = -2x + 3$ find $g(a+h)$ $g(x) = -2x + 3$ $g(a+h) = -2(a+h) + 3$ $= -2a - 2h + 3$
$h(x) = x - 8 $ find $h(-1)$ $h(x) = x - 8 $ $h(-1) = -1 - 8 $ $= -9 $ $= 9$	$h(x) = 3x - 7 $ find $h(0)$ $h(x) = 3x - 7 $ $h(0) = 3 \cdot 0 - 7 $ $= 0 - 7 $ $= -7 = 7$	$h(x) = -x + 6 $ find $h(10)$ $h(x) = -x + 6 $ $h(10) = -10 + 6 $ $= -4 $ $= 4$

Work all operations within the absolute values first then apply the absolute value last.

B. Evaluate the given function.

$f(x) = 2x - 7$ find x when $f(x) = -8$ $f(x) = 2x - 7$ $-8 = 2x - 7$ $-\frac{1}{2} = \frac{2x}{2}$ $-\frac{1}{2} = x$	$f(x) = -2x - 5$ find x when $f(x) = 5$ $f(x) = -2x - 5$ $5 = -2x - 5$ $10 = -2x$ $-5 = x$	$f(x) = -x - 3$ find x when $f(x) = -7$ $f(x) = -x - 3$ $-7 = -x - 3$ $-4 = -x$ $4 = x$
$g(x) = -x - 1$ find x when $g(x) = 14$ $g(x) = -x - 1$ $14 = -x - 1$ $15 = -x$ $-15 = x$	$g(x) = -3x$ find x when $g(x) = 12$ $g(x) = -3x$ $12 = -3x$ $-4 = x$	$g(x) = 5x + 2$ find x when $g(x) = 12$ $g(x) = 5x + 2$ $12 = 5x + 2$ $10 = 5x$ $2 = x$

Do not let the function notation discourage you, it takes some practice. The main idea is to think of $y = f(x)$, sometimes x is given and sometimes y or $f(x)$ is given.

C. Graph the given linear function.**D. Typical Word Problem**

Bill has a popular software company which sells copies of its program for \$149. If the initial start up cost for the company was \$10,000 and it costs \$12 to produce each copy:

- Find a cost function $C(x)$ that models this business.
- Find a revenue function $R(x)$ that models this business.
- Find the profit function $P(x)$ using your functions above.
- Find the profit when 1000 programs are produced and sold.
- Find the number of programs that must be sold to break even.

Let x = the number of copies produced and sold.

⇒ (A) Cost: $C(x) = 10,000 + 12x$ (B) Revenue: $R(x) = 149x$

(C) Profit: $P(x) = R(x) - C(x)$ (Revenue less cost)
 $= 149x - (10,000 + 12x)$
 $= 149x - 10,000 - 12x$

or $P(x) = -10,000 + 137x$

(D) find $P(1000) = -10,000 + 137(1000)$ ($x = 1000$ here)
 $= -10,000 + 137,000$
 $= 127,000$

Profit for 1000 copies sold is \$127,000

(E) Break even ⇒ $P(x) = 0$ (or profit is zero.)

$0 = -10,000 + 137x$

$10,000 = 137x$

$72.99 \approx x$

must sell 73 copies to break even.

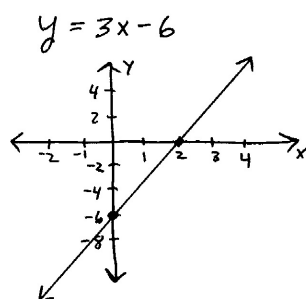
Revenue Function – include all proceeds of the sales of the produced item.

Profit Function – simply the revenue less the cost of production

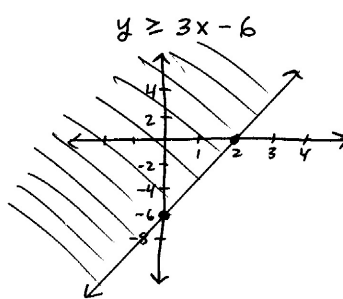
Cost Function – include all fixed and variable costs of production.

When we graph an equation like $y = 3x - 6$ we know that it will be a line. The graph of a linear inequality, on the other hand, gives us a region.

Graph of the line



Graph of the linear inequality



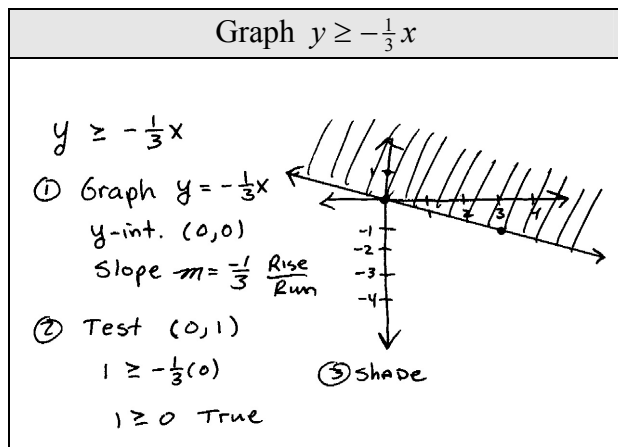
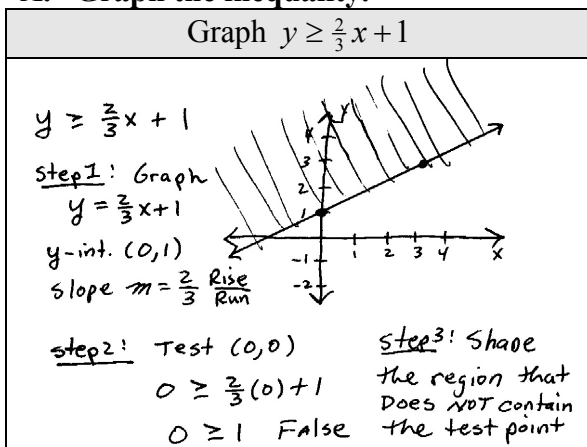
Not only do the points on the line satisfy this linear inequality but so does any point in the region that we have shaded. So from the above graph $(0, 0)$ and $(-2, 4)$ should solve the inequality.

$$\begin{aligned} (0, 0) \quad y &\geq 3x - 6 \\ 0 &\geq 3(0) - 6 \\ 0 &\geq -6 \quad \text{True} \end{aligned}$$

$$\begin{aligned} (-2, 4) \quad y &\geq 3x - 6 \\ 4 &\geq 3(-2) - 6 \\ 4 &\geq -6 - 6 \\ 4 &\geq -12 \quad \text{True} \end{aligned}$$

In practice, you need only to choose one test point not on the line. This will determine which side of the line to shade when graphing a linear inequality. Usually the origin is easiest point to test as long as it is not a point on the line.

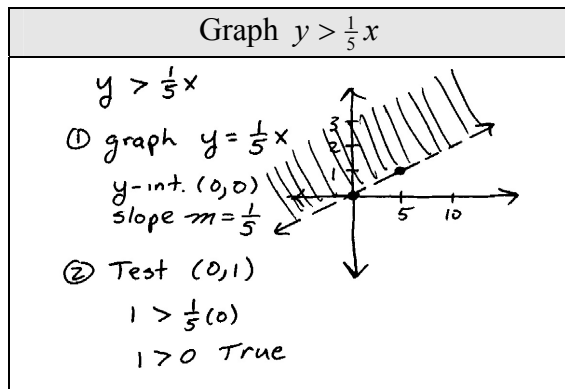
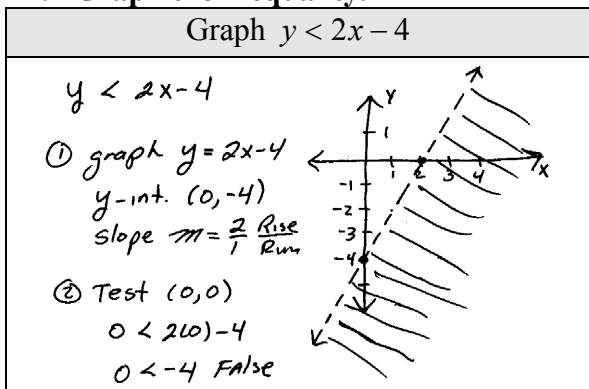
A. Graph the inequality.



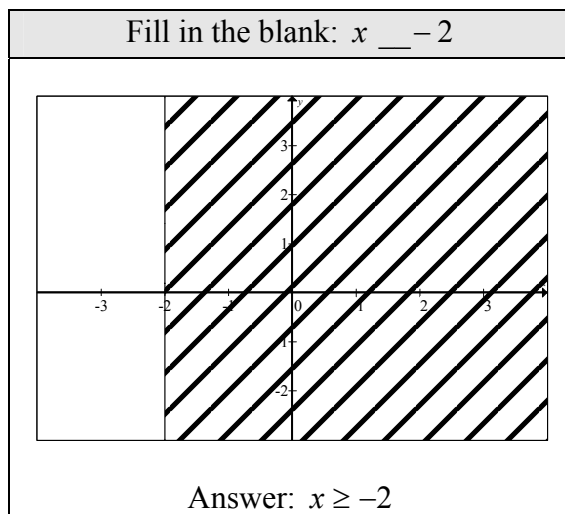
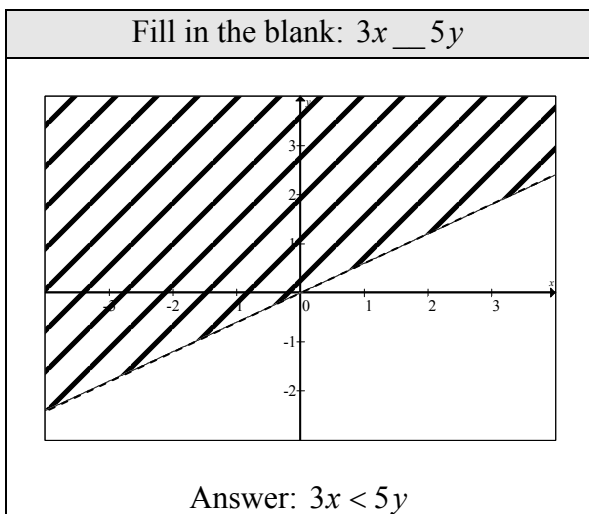
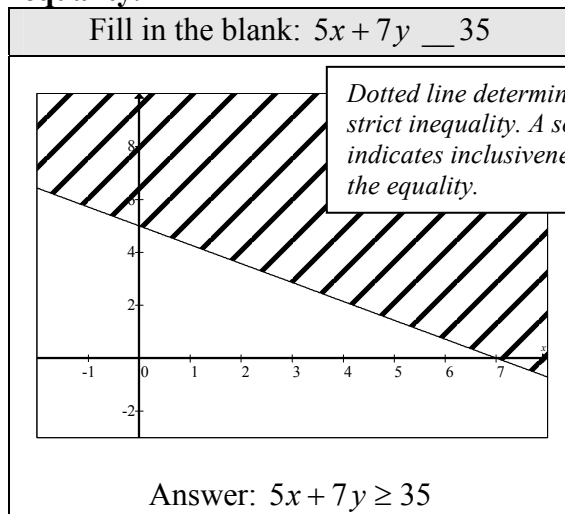
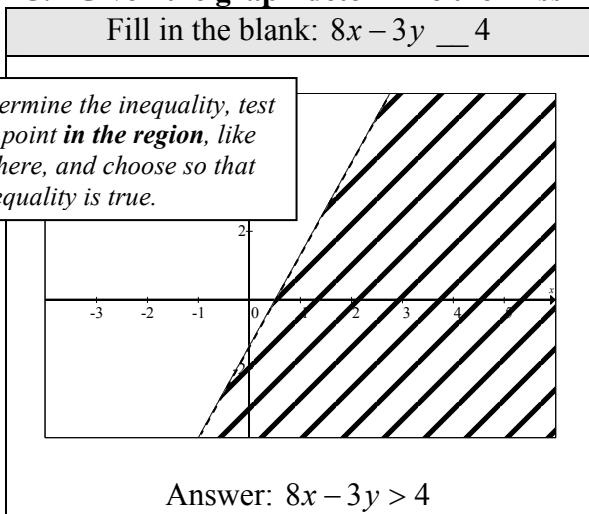
If the test point yields a true inequality shade the side of the line it is on. If the test point yields a false inequality shade the opposite side.

When trying to graph strict inequalities, inequalities without the equal, the points on the line will not satisfy the inequality so we will use a dotted line to indicate this. Otherwise, the steps are the same.

B. Graph the inequality.



C. Given the graph determine the missing inequality.



Chapter 3 *_Graphing Linear Equations*

Sample Exam

Please answer all the questions and show work where appropriate.

1. Find the slope of the line perpendicular to the line passing through $(-3, -2)$ and $(5, 1)$.
2. Find the slope of the line passing through the given points:
 - a. $(-3, 2)$ and $(5, 2)$
 - b. $(-2, 3)$ and $(-2, -3)$
3. Given the linear equation $3x - 5y = 30$:
 - a. Express it in slope-intercept form and graph it.
 - b. Label the x - and y -intercepts on the graph.
4. Find the x - and y -intercepts and use them to graph $2x + 3y = 6$.
5. Graph the linear function $f(x) = -\frac{2}{3}x + 8$ and calculate $f(6)$.
6. Are the given two lines parallel, perpendicular or neither?
$$\left. \begin{array}{l} -10x + 2y = 4 \\ -x + 5y = -10 \end{array} \right\}$$
7. Graph the following two lines on the same set of axes and indicate where they cross:
$$\left. \begin{array}{l} y = 2x + 1 \\ y = -x + 4 \end{array} \right\}$$
8. Find the equation of the line that is passing through $(8, -2)$ and parallel to the line passing through $(-3, -2)$ and $(5, 1)$.
9. Find the equation of the line that is passing through $(2, -1)$ and perpendicular to $y = \frac{3}{5}x + 2$.
10. Find the equation of the line passing through $(3, 5)$ and $(3, -4)$. Graph the line and label any intercepts.

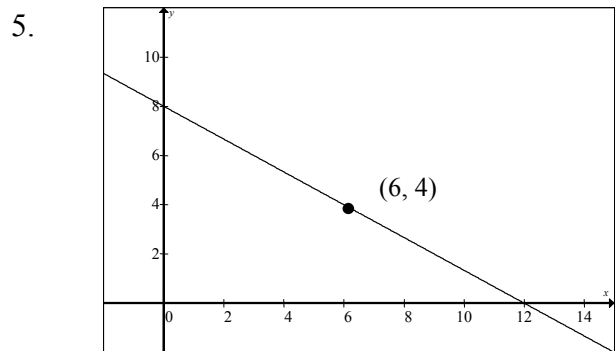
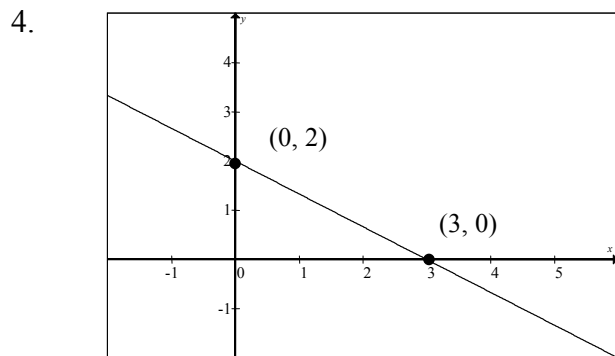
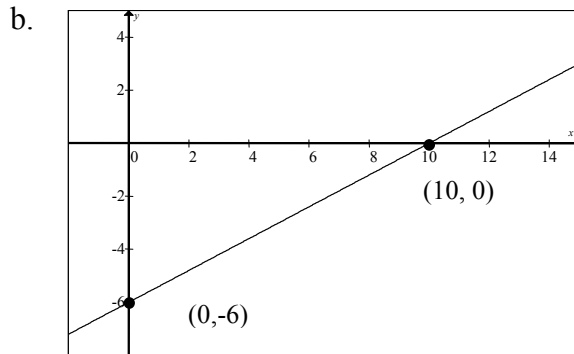
Chapter 3 *Graphing Linear Equations*

Sample Exam Answers

1. $m_{\perp} = -\frac{8}{3}$

2. a. $m = 0$
b. m undefined

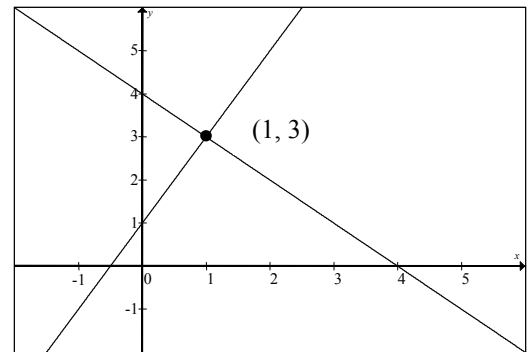
3. a. $y = \frac{3}{5}x - 6$



$f(6) = 4$

6. Neither

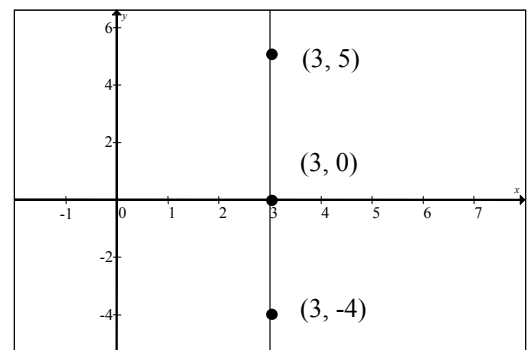
7. $(1, 3)$



8. $y = \frac{3}{8}x - 5$

9. $y = -\frac{5}{3}x + \frac{7}{3}$

10. $x = 3$



Find the slope of the line perpendicular to the line passing through $(-3, -2)$ and $(5, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{5 - (-3)}$$

$$= \frac{1+2}{5+3} = \frac{3}{8}$$

so if $m = \frac{3}{8}$
then $m_{\perp} = -\frac{8}{3}$

Calculate the slope m :
 $(-3, 2)$ and $(5, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 2}{5 - (-3)}$$

$$= \frac{0}{8} = 0$$

$$m = 0$$

Calculate the slope m :
 $(-2, 3)$ and $(-2, -3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 3}{-2 - (-2)}$$

$$= \frac{-6}{-2 + 2} = \frac{-6}{0}$$

m undefined

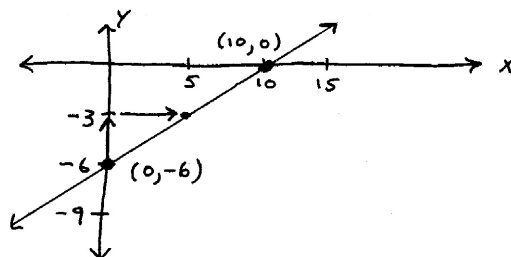
Given the linear equation $3x - 5y = 30$: Express it in slope-intercept form and graph it and label the x - and y -intercepts on the graph.

$$3x - 5y = 30$$

$$\begin{array}{r} -3x \\ \hline -5y = -3x + 30 \\ -5 \quad -5 \end{array}$$

$$y = \frac{3}{5}x - 6$$

slope $m = \frac{3}{5}$ $\frac{\text{Rise}}{\text{Run}}$



Find the x - and y -intercepts and use them to graph $2x + 3y = 6$.

$$2x + 3y = 6$$

x -intercepts plug $y=0$:

$$2x + 3 \cdot 0 = 6$$

$$2x = 6$$

$$x = 3$$

$$x\text{-int: } (3, 0)$$

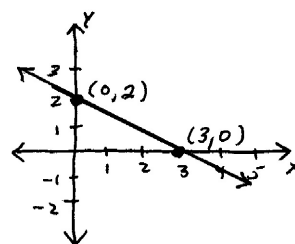
y -intercepts plug $x=0$:

$$2 \cdot 0 + 3y = 6$$

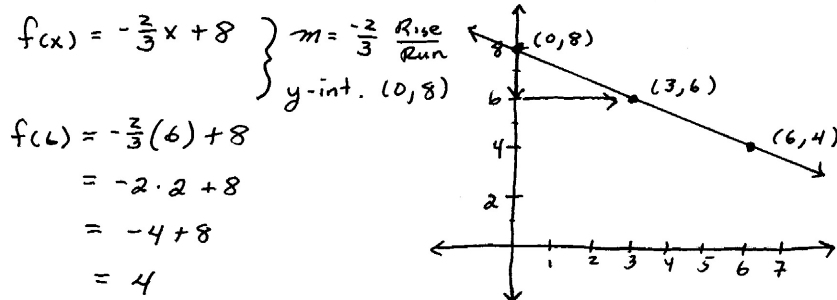
$$3y = 6$$

$$y = 2$$

$$y\text{-int } (0, 2)$$



Graph the linear function $f(x) = -\frac{2}{3}x + 8$ and calculate $f(6)$.



Are the given two lines parallel, perpendicular or neither?

$$\begin{cases} -10x + 2y = 4 \\ -x + 5y = -10 \end{cases}$$

Line 1: $-10x + 2y = 4$
 $+10x$ $+10x$

$$\frac{2y}{2} = \frac{10x + 4}{2}$$

$$y = 5x + 2$$

$$m_1 = 5$$

Neither parallel nor perpendicular.

Line 2: $-x + 5y = -10$
 $+x$ $+x$

$$\frac{5y}{5} = \frac{x - 10}{5}$$

$$y = \frac{1}{5}x - 2$$

$$m_2 = \frac{1}{5}$$

Graph the following two lines on the same set of axes and indicate where they cross:

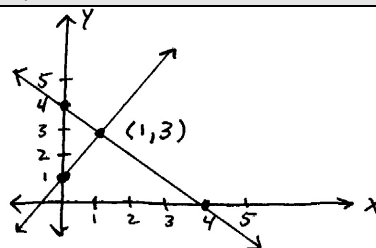
$$\begin{cases} y = 2x + 1 \\ y = -x + 4 \end{cases}$$

Line 1: $y = 2x + 1$

$\hookrightarrow m = \frac{2}{1}$ y-int (0, 1)

Line 2: $y = -x + 4$

$\hookrightarrow m = -\frac{1}{1}$ y-int (0, 4)



Find the equation of the line that is passing through (8, -2) and parallel to the line passing through (-3, -2) and (5, 1).

step 1: Find m

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{5 - (-3)}$$

$$= \frac{1 + 2}{5 + 3} = \frac{3}{8}$$

$$m = \frac{3}{8} \checkmark$$

step 2: Find b

$$y = mx + b$$

$$-2 = \left(-\frac{3}{8}\right)(8) + b$$

$$-2 = 3 + b$$

$$-5 = b \checkmark$$

step 3: Put it together

$$y = mx + b$$

$$\boxed{y = \frac{3}{8}x - 5}$$

Find the equation of the line that is passing through $(2, -1)$ and perpendicular to $y = \frac{3}{5}x + 2$.

$$y = \frac{3}{5}x + 2$$

Given $m = \frac{3}{5}$

so $m_{\perp} = -\frac{5}{3}$

To find b :

$$y = -\frac{5}{3}x + b$$

$$-1 = -\frac{5}{3}(2) + b$$

$$-1 = -\frac{10}{3} + b$$

$$+\frac{10}{3} \quad +\frac{10}{3}$$

$$\frac{7}{3} = b$$

Put it together

$$y = mx + b$$

$$y = -\frac{5}{3}x + \frac{7}{3}$$

Find the equation of the line passing through $(3, 5)$ and $(3, -4)$. Graph the line and label any intercepts.

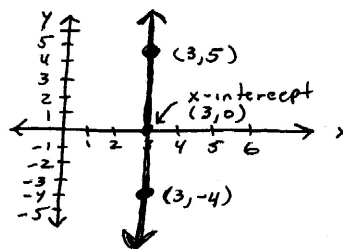
$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4 - 5}{3 - 3} = \frac{-9}{0}$$

m undefined

$\Rightarrow$ Vertical Line!

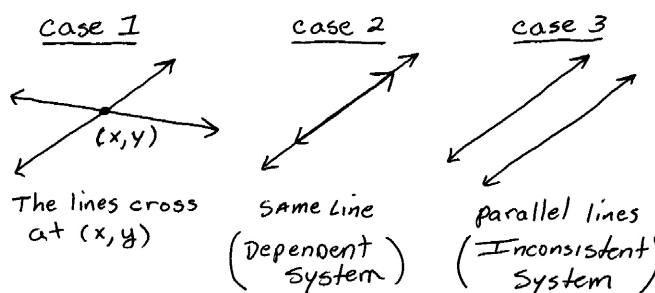
Ans: $x = 3$



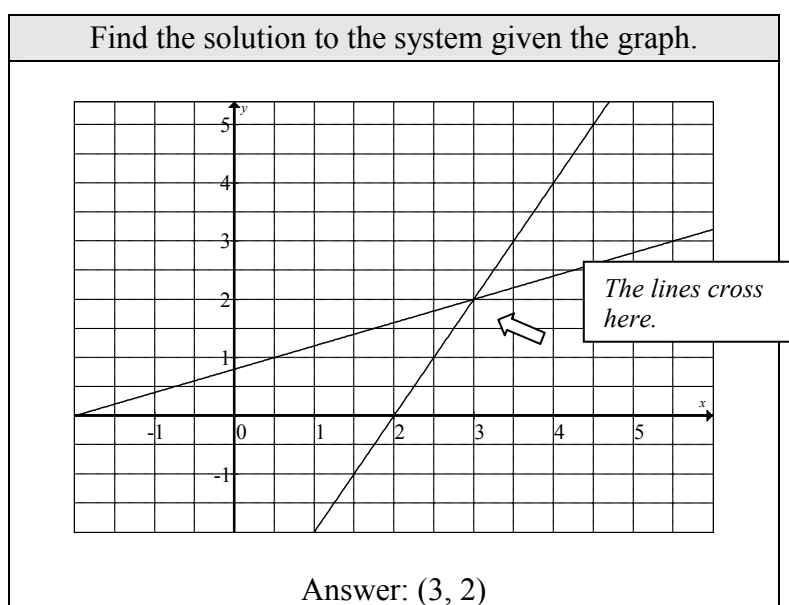
We will learn three methods for solving systems of two equations and two unknowns. The basic idea is to find the common point or the point where the two lines cross. Once we learn the techniques for solving linear systems we can set up our word problems with two variables. This chapter is a nice review of everything we have studied up to this point.

Solving Linear Systems*Method 1 – Solve by Graphing ...4.1**Method 2 – Solve by Substitution ...4.2**Method 3 – Solve by Elimination ...4.3**Word Problems ...4.4**Systems of Linear Inequalities ...4.5**Sample Exam**Sample Exam Solutions****Cumulative Review****Sample Midterm Exam**Sample Midterm Exam Answers*

The linear systems in this section will consist of two equations and two unknowns. Given two lines we are asked to find out if they have simultaneous solutions. In other words, where do the two lines cross? This question brings up three cases.



Most of the time the system will have a common point, (x, y) . The point where they cross is the solution to the system.

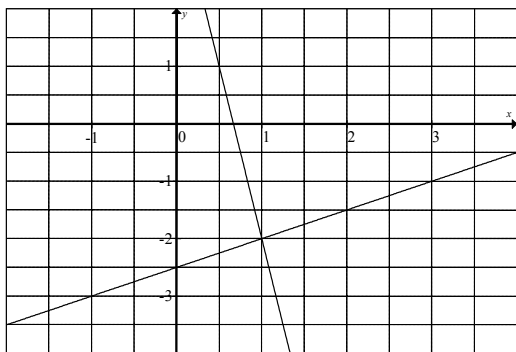


But not all systems have one ordered pair solution, some have no common points and others have infinitely many. Imagine if you were asked to solve the system consisting of two parallel lines, where do they cross? In that case, there is no simultaneous solution and the system of two parallel lines is inconsistent.

In the case where the system consists of two lines that happen to be the same line there are infinitely many common points. This system is dependent and solutions can be presented in the form (x, y) where $y = mx + b$ or $(x, mx + b)$.

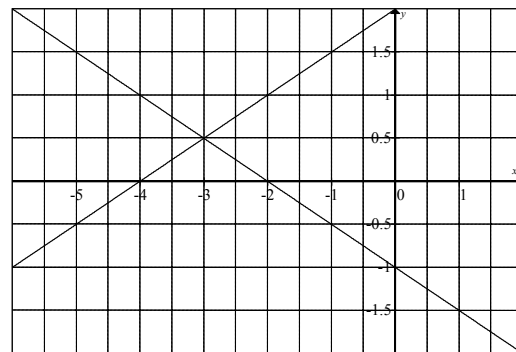
A. Solve the systems given the graph.

Find the solution to the system.

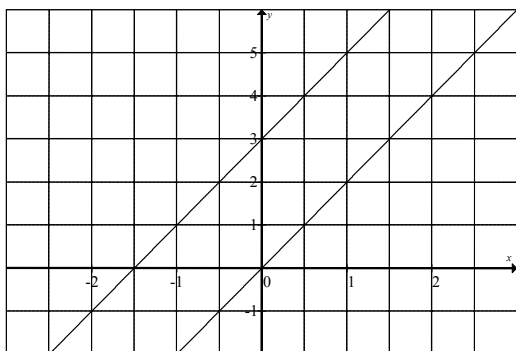


Answer: (1, -2)

Find the solution to the system.

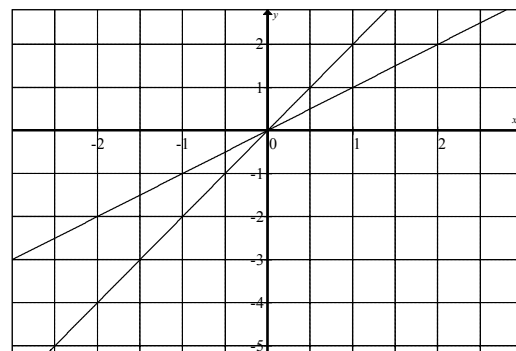
Answer: $(-3, \frac{1}{2})$

Find the solution to the system.



Answer: No Solution

Find the solution to the system.



Answer: (0, 0)

B. Is the ordered pair a solution to the system?Is $(-2, 4)$ a solution to $\begin{cases} x - y = -6 \\ -2x + 3y = 16 \end{cases}$?

<u>line 1</u>	<u>line 2</u>
$x - y = -6$	$-2x + 3y = 16$
$(-2) - (4)$	$-2(-2) + 3(4)$
$= -6 \checkmark$	$= 4 + 12 = 16 \checkmark$
Yes, it solves both equations	

Is $(1, 0)$ a solution to $\begin{cases} x - y = 1 \\ -4x + 3y = 5 \end{cases}$?

<u>line 1</u>	<u>line 2</u>
$x - y = 1$	$-4x + 3y = 5$
$(1) - (0)$	$-4(1) + 3(0)$
$= 1 \checkmark$	$= -4 \times$
No, it must solve both!	

The next example shows how to find the common point, the point where the two lines cross, if we are given the system in equation form.

Step 1: Put the lines in slope-intercept form.

Line 1

$$\begin{aligned} 2x - 3y &= 0 \\ -3y &= -2x \\ \boxed{y = \frac{2}{3}x} \end{aligned}$$

Line 2

$$\begin{aligned} -4x + 2y &= -8 \\ 2y &= 4x - 8 \\ \boxed{y = 2x - 4} \end{aligned}$$

Solve the system: $\begin{cases} 2x - 3y = 0 \\ -4x + 2y = -8 \end{cases}$

Step 2: Graph the lines and use it to find the common point.

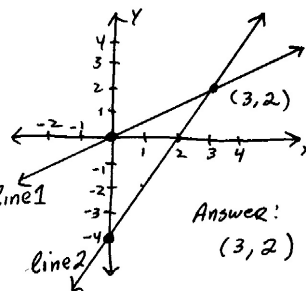
Original System

$$\begin{cases} 2x - 3y = 0 \\ -4x + 2y = -8 \end{cases}$$

$\Rightarrow$

Equivalent System

$$\begin{cases} y = \frac{2}{3}x \\ y = 2x - 4 \end{cases}$$



Step 3: Check.

Check:

$$\begin{aligned} 2x - 3y &= 0 \\ -4x + 2y &= -8 \end{aligned} \quad \begin{aligned} 2(3) - 3(2) &= 0 \\ -4(3) + 2(2) &= -8 \end{aligned} \quad \checkmark$$

Sometimes graph paper helps when graphing these. Be sure to use a straight edge when graphing so that your results will be more accurate.

C. Solve the systems.

Solve the system using the graphing method:

$$\begin{cases} x - y = -4 \\ 2x + y = 1 \end{cases}$$

$$\begin{cases} x - y = -4 \\ 2x + y = 1 \end{cases}$$

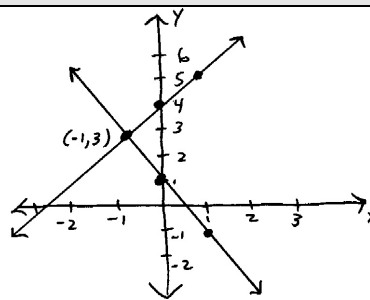
$$\begin{cases} y = x + 4 \\ y = -2x + 1 \end{cases}$$

Line 1

$$\begin{aligned} x - y &= -4 \\ -x & \quad -x \\ \hline -y &= -x - 4 \\ \frac{-y}{-1} &= \frac{-x - 4}{-1} \\ \boxed{y} &= x + 4 \end{aligned}$$

Line 2

$$\begin{aligned} 2x + y &= 1 \\ -2x & \quad -2x \\ \hline y &= -2x + 1 \\ \boxed{y} &= -2x + 1 \end{aligned}$$



Answer: $(-1, 3)$

Solve the system using the graphing method:

$$\begin{cases} x + y = -7 \\ 4x + 2y = -18 \end{cases}$$

$$\begin{cases} x + y = -7 \\ 4x + 2y = -18 \end{cases}$$

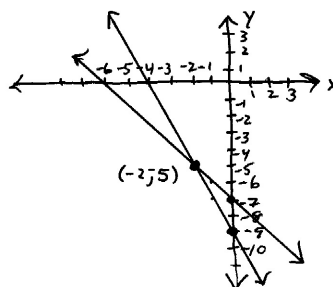
$$\begin{cases} y = -x - 7 \\ y = -2x - 9 \end{cases}$$

Line 1

$$\begin{aligned} x + y &= -7 \\ -x & \quad -x \\ \hline y &= -x - 7 \\ \boxed{y} &= -x - 7 \end{aligned}$$

Line 2

$$\begin{aligned} 4x + 2y &= -18 \\ -4x & \quad -4x \\ \hline 2y &= -4x - 18 \\ \frac{2y}{2} &= \frac{-4x - 18}{2} \\ \boxed{y} &= -2x - 9 \end{aligned}$$



Answer: $(-2, 5)$

Tip: Take extra care with the scale when graphing to find the intersection. Make sure all your tick marks are equal in size; this will make your graphs more accurate and easier to read. Check to see that your answer works for BOTH lines.



Solve the system using the graphing method:

$$\begin{cases} -2x + 5y = -15 \\ -4x + 10y = 10 \end{cases}$$

$$\begin{cases} -2x + 5y = -15 \\ -4x + 10y = 10 \end{cases}$$

line 1

$$\begin{array}{r} -2x + 5y = -15 \\ +2x \quad +2x \\ \hline 5y = 2x - 15 \\ y = \frac{2}{5}x - 3 \end{array}$$

$$\begin{cases} y = \frac{2}{5}x - 3 \\ y = \frac{2}{5}x + 1 \end{cases}$$

line 2

$$\begin{array}{r} -4x + 10y = 10 \\ +4x \quad +4x \\ \hline 10y = 4x + 10 \\ y = \frac{2}{5}x + 1 \end{array}$$

same slope.
Parallel Lines!
Answer: No Solution

Solve the system using the graphing method:

$$\begin{cases} -3x + y = -2 \\ -6x + 2y = -4 \end{cases}$$

$$\begin{cases} -3x + y = -2 \\ -6x + 2y = -4 \end{cases}$$

line 1

$$\begin{array}{r} -3x + y = -2 \\ +3x \quad +3x \\ \hline y = 3x - 2 \end{array}$$

$$\begin{cases} y = 3x - 2 \\ y = 3x - 2 \end{cases}$$

line 2

$$\begin{array}{r} -6x + 2y = -4 \\ +6x \quad +6x \\ \hline 2y = 6x - 4 \\ \frac{2y}{2} = \frac{6x - 4}{2} \\ y = 3x - 2 \end{array}$$

same line
Dependent
(x, mx + b)
Ans: (x, 3x - 2)

Dependent systems seem to give beginning Algebra students the most trouble. Remember that we are looking for points where the two lines cross. If the lines are the same, well then they will cross at infinitely many points. Because of this we have to use special notation to indicate an infinite set. Notice that we have already put the line in y-intercept form, $y = mx + b$, so it is not a big leap to write the final answer as $(x, mx + b)$. You might have seen different notation in other texts such as $\{(x, y) \mid y = mx + b\}$.

The **substitution method** for solving systems is completely algebraic. There is no need to graph the lines unless you are asked to. This method is fairly straight forward and always works, the steps are as follows:

Solve $\begin{cases} 2x + y = 7 \\ 3x - 2y = -7 \end{cases}$ by substitution.	
$\begin{aligned} 2x + y &= 7 \\ \Rightarrow 3x - 2y &= -7 \end{aligned} \Rightarrow y = -2x + 7$	Step 1: Solve for one of the variables.
$\text{Substitute: } 3x - 2(-2x + 7) = -7$ $3x + 4x - 14 = -7$ $7x = 7$ $x = 1$	Step 2: Substitute into the other equation.
$\text{Back substitute: } y = -2(1) + 7$ $= -2 + 7$ $= 5$	Step 3: Solve for the remaining variable.
Step 4: Substitute back to find the value for the other variable. Answer: $(1, 5)$	

Tip: It does not matter if you choose to solve for x or y first but make sure you do not substitute into the same equation in step 2.



Remember that we are trying to find the simultaneous solutions or the points where the two lines cross. Next we will see what happens when the system is dependent, in other words, when the system consists of two lines that are the same.

Solve $\begin{cases} -5x + y = -1 \\ 10x - 2y = 2 \end{cases}$ by substitution.	
$\begin{aligned} -5x + y &= -1 \\ 10x - 2y &= 2 \end{aligned} \Rightarrow y = 5x - 1$	
$\text{subst: } 10x - 2(5x - 1) = 2$ $10x - 10x + 2 = 2$ $2 = 2 \text{ True, Same Line}$	
Dependent System: $(x, 5x - 1)$	
Any true statement, including $0=0$, indicates a dependent system.	

The next system consists of two parallel lines which has no simultaneous solution.

Solve $\begin{cases} x-3y=-3 \\ 2x-6y=12 \end{cases}$ by substitution.
$\begin{aligned} &\begin{cases} x-3y=-3 \\ 2x-6y=12 \end{cases} \Rightarrow x=3y-3 \\ &\text{Subst: } 2(3y-3)-6y=12 \\ &\quad 6y-6-6y=12 \\ &\quad -6=12 \text{ False, } \leftarrow \\ &\quad \text{Parallel Lines} \\ &\text{Inconsistent System: No Solution} \end{aligned}$

Any false statement indicates an inconsistent system, or no solution.

A. Solve the systems using the substitution method.

Solve $\begin{cases} y=\frac{2}{3}x-4 \\ 3x-6y=5 \end{cases}$ by substitution.	Solve $\begin{cases} 2x-y=12 \\ x-y=3 \end{cases}$ by substitution.
$\begin{aligned} &\begin{cases} y=\frac{2}{3}x-4 \\ 3x-6y=5 \end{cases} \Rightarrow y=\frac{2}{3}x-4 \\ &\text{Subst: } 3x-6(\frac{2}{3}x-4)=5 \\ &\quad 3x-4x+24=5 \\ &\quad -x+24=5 \\ &\quad \quad -24 \quad -24 \\ &\quad -x=-19 \\ &\quad x=19 \\ &\text{Back subst: } y=\frac{2}{3}(19)-4 \\ &\quad =\frac{38}{3}-\frac{4 \cdot 3}{1 \cdot 3} \\ &\quad =\frac{26}{3} \\ &\text{Answer } (19, 8\frac{2}{3}) \end{aligned}$	$\begin{aligned} &\begin{cases} 2x-y=12 \\ x-y=3 \end{cases} \Rightarrow x=y+3 \\ &\text{Subst: } 2(y+3)-y=12 \\ &\quad 2y+6-y=12 \\ &\quad y+6=12 \\ &\quad \quad -6 \quad -6 \\ &\quad y=6 \\ &\text{Back subst: } x=6+3 \\ &\quad x=9 \\ &\text{Answer: } (9, 6) \end{aligned}$

Solve $\begin{cases} x=-3 \\ 3x-2y=-9 \end{cases}$ by substitution.	Solve $\begin{cases} y=-\frac{1}{2} \\ 5x+4y=12 \end{cases}$ by substitution.
$\begin{aligned} &\begin{cases} x=-3 \\ 3x-2y=-9 \end{cases} \Rightarrow x=-3 \\ &\text{Subst: } 3(-3)-2y=-9 \\ &\quad -9-2y=-9 \\ &\quad \quad -2y=0 \\ &\quad y=0 \\ &\text{Answer: } (-3, 0) \end{aligned}$	$\begin{aligned} &\begin{cases} y=-\frac{1}{2} \\ 5x+4y=12 \end{cases} \Rightarrow y=-\frac{1}{2} \\ &\text{Subst: } 5x+4(-\frac{1}{2})=12 \\ &\quad 5x-2=12 \\ &\quad 5x=14 \Rightarrow x=\frac{14}{5} \\ &\text{Answer } (2\frac{4}{5}, -\frac{1}{2}) \end{aligned}$

Solve $\begin{cases} 2x - 5y = 3 \\ 4x - 10y = 6 \end{cases}$ by substitution. $\begin{cases} 2x - 5y = 3 \\ 4x - 10y = 6 \end{cases} \Rightarrow x = \frac{5}{2}y + \frac{3}{2}$ Subst: $4(\frac{5}{2}y + \frac{3}{2}) - 10y = 6$ $10y + 6 - 10y = 6$ $6 = 6 \text{ True,}$ same line Dependent System: $(x, \frac{2}{5}x - \frac{3}{5})$	Solve $\begin{cases} -7x + 3y = 3 \\ 14x - 6y = -16 \end{cases}$ by substitution. $\begin{cases} -7x + 3y = 3 \\ 14x - 6y = -16 \end{cases} \Rightarrow y = \frac{7}{3}x + 1$ Subst: $14x - 6(\frac{7}{3}x + 1) = -16$ $14x - 14x - 6 = -16$ $-6 = -16 \text{ False}$ Inconsistent System: No Solution
Solve $\begin{cases} \frac{1}{2}x - \frac{1}{2}y = \frac{3}{2} \\ \frac{2}{3}x - y = \frac{4}{3} \end{cases}$ by substitution. $\begin{cases} \frac{1}{2}x - \frac{1}{2}y = \frac{3}{2} \\ \frac{2}{3}x - y = \frac{4}{3} \end{cases} \Rightarrow y = \frac{2}{3}x - \frac{4}{3}$ Subst: $\frac{1}{2}x - \frac{1}{2}(\frac{2}{3}x - \frac{4}{3}) = \frac{3}{2}$ $\frac{1}{2}x - \frac{1}{3}x + \frac{2}{3} = \frac{3}{2}$ $\frac{1}{6}x = \frac{5}{6}$ $x = 5$ Back subst: $y = \frac{2}{3}(5) - \frac{4}{3}$ $= \frac{10}{3} - \frac{4}{3}$ $= \frac{6}{3} = 2$ Answer $(5, 2)$	Solve $\begin{cases} 5x - 4y = 1 \\ 12x - 11y = 1 \end{cases}$ by substitution. $\begin{cases} 5x - 4y = 1 \\ 12x - 11y = 1 \end{cases} \Rightarrow x = \frac{4}{5}y + \frac{1}{5}$ Subst: $12(\frac{4}{5}y + \frac{1}{5}) - 11y = 1$ $\frac{48}{5}y + \frac{12}{5} - 11y = 1$ $-\frac{7}{5}y = -\frac{7}{5}$ $y = 1$ Back subst: $x = \frac{4}{5}(1) + \frac{1}{5}$ $= \frac{4}{5} + \frac{1}{5}$ $= \frac{5}{5} = 1$ Answer: $(1, 1)$

B. Typical word problem.

When Joe walked away from the craps table he had 45 chips. He had a combination of \$5 and \$25 chips that added to a total of \$625. How many of each did he have?

Let x = the number of \$5 chips and y = the number of \$25 chips

Set-up $\Rightarrow \begin{cases} x + y = 45 \\ 5x + 25y = 625 \end{cases} \begin{matrix} \leftarrow \text{Total \# of chips} (y = -x + 45) \\ \leftarrow \text{Total \$ value} \end{matrix}$

Subst: $\begin{aligned} 5x + 25(-x + 45) &= 625 \\ 5x - 25x + 1125 &= 625 \\ -20x &= -500 \\ x &= 25 \end{aligned}$

Back subst: $\begin{aligned} 25 + y &= 45 \\ y &= 20 \end{aligned}$

Answer:
He had 25 - \$5 chips and 20 - \$25 chips.

Problems Solved! Chapter 4 Solving Linear Systems
Method 3 – Solve by Elimination

When solving linear systems the **elimination method**, sometimes called the addition method, will be the method of choice. This technique is completely algebraic and quick once you get the hang of it. The idea is to eliminate one variable by adding equations.

Solve the system: $\begin{cases} x + y = -7 \\ 4x + 2y = -18 \end{cases}$

Multiply Line 1 by -2 to match Line 2

Original System	Equivalent System
$\Rightarrow \begin{cases} x + y = -7 \\ 4x + 2y = -18 \end{cases}$	$\begin{cases} -2x - 2y = 14 \\ 4x + 2y = -18 \end{cases}$
	$\oplus \quad \begin{cases} -2x - 2y = 14 \\ 4x + 2y = -18 \end{cases}$
	$2x = -4 \quad \leftarrow$
	$x = -2$

Step 1: Multiply by what you need to eliminate one of the variables.

Step 2: Add the equations together. This should eliminate one of the variables.

Step 3: Back substitute into any one of the previous equations.

$\Rightarrow$ Back Substitute: $\begin{matrix} -2 + y = -7 \\ +2 \quad +2 \end{matrix}$

$y = -5$

The solution to the system is $(-2, -5)$

Tip: If you multiply an equation by a number please remember to distribute!



How did I know to multiply by -2 in the previous example? This takes some practice, just look at the variable coefficients and try to match them with one positive and one negative. Sometimes you will have to multiply both equations as in the next example.

Solve the system: $\begin{cases} 4x + 2y = 4 \\ -5x - 3y = -7 \end{cases}$

Original System	Equivalent System
$\Rightarrow \begin{cases} 4x + 2y = 4 \\ -5x - 3y = -7 \end{cases}$	$\begin{cases} 12x + 6y = 12 \\ -10x - 6y = -14 \end{cases}$
$\begin{matrix} \times(3) \\ \times(2) \end{matrix}$	$\oplus \quad \begin{cases} 12x + 6y = 12 \\ -10x - 6y = -14 \end{cases}$
	$2x = -2 \quad \leftarrow$
	$x = -1$

Notice that the variable y is ready to be eliminated, just add the equations together.

$\Rightarrow$ BACK subst: $4(-1) + 2y = 4$

$-4 + 2y = 4$

$2y = 8$

$y = 4$

Ans: $(-1, 4)$

To eliminate the variable y we must multiply line 1 by 3 and line 2 by 2.

You will always have to back substitute to find the ordered pair solution.

A. Solve the systems using the elimination method.

Solve the system: $\begin{cases} x - y = -6 \\ -2x + 3y = 16 \end{cases}$

$$\begin{array}{rcl} \begin{cases} x - y = -6 \\ -2x + 3y = 16 \end{cases} & \begin{matrix} \times (2) \\ \oplus \end{matrix} & \begin{cases} 2x - 2y = -12 \\ -2x + 3y = 16 \end{cases} \\ & & \hline & & y = 4 \end{array}$$

Back subst: $x - 4 = -6$
 $x = -2$

Answer: $(-2, 4)$

Solve the system: $\begin{cases} x - y = 1 \\ -4x + 3y = 5 \end{cases}$

$$\begin{array}{rcl} \begin{cases} x - y = 1 \\ -4x + 3y = 5 \end{cases} & \begin{matrix} \times (4) \\ \oplus \end{matrix} & \begin{cases} 4x - 4y = 4 \\ -4x + 3y = 5 \end{cases} \\ & & \hline & & -y = 9 \\ & & y = -9 \end{array}$$

Back Subst.
 $x - (-9) = 1$
 $x + 9 = 1$
 $x = -8$

Ans: $(-8, -9)$

Solve the system: $\begin{cases} 5x - 3y = -1 \\ 3x + 2y = 7 \end{cases}$

$$\begin{array}{rcl} \begin{cases} 5x - 3y = -1 \\ 3x + 2y = 7 \end{cases} & \begin{matrix} \times (2) \\ \times (3) \\ \oplus \end{matrix} & \begin{cases} 10x - 6y = -2 \\ 9x + 6y = 21 \end{cases} \\ & & \hline & & 19x = 19 \\ & & x = 1 \end{array}$$

Back subst.
 $3(1) + 2y = 7$
 $2y = 4$
 $y = 2$

Ans: $(1, 2)$

Solve the system: $\begin{cases} 3x - 4y = -5 \\ 5x + 8y = -1 \end{cases}$

$$\begin{array}{rcl} \begin{cases} 3x - 4y = -5 \\ 5x + 8y = -1 \end{cases} & \begin{matrix} \times (2) \\ \oplus \end{matrix} & \begin{cases} 6x - 8y = -10 \\ 5x + 8y = -1 \end{cases} \\ & & \hline & & 11x = -11 \\ & & x = -1 \end{array}$$

Back Subst.
 $3(-1) - 4y = -5$
 $-3 - 4y = -5$
 $-4y = -2$
 $y = \frac{-2}{-4} = \frac{1}{2}$

Ans: $(-1, \frac{1}{2})$

Solve the system: $\begin{cases} 3x - y = 7 \\ 6x - 2y = 14 \end{cases}$

$$\begin{array}{rcl} \begin{cases} 3x - y = 7 \\ 6x - 2y = 14 \end{cases} & \begin{matrix} \times (-2) \\ \oplus \end{matrix} & \begin{cases} -6x + 2y = -14 \\ 6x - 2y = 14 \end{cases} \\ & & \hline & & 0 = 0 \end{array}$$

Solve for y:
 $\begin{array}{r} 3x - y = 7 \\ -3x \quad -3x \\ \hline -y = -3x + 7 \\ y = 3x - 7 \end{array}$

Dependent
 $(x, 3x - 7)$

Solve the system: $\begin{cases} -x + 3y = 9 \\ 2x - 6y = 12 \end{cases}$

$$\begin{array}{rcl} \begin{cases} -x + 3y = 9 \\ 2x - 6y = 12 \end{cases} & \begin{matrix} \times (2) \\ \oplus \end{matrix} & \begin{cases} -2x + 6y = 18 \\ 2x - 6y = 12 \end{cases} \\ & & \hline & & 0 = 30 \end{array}$$

False,
 No Solution

We will run into systems that are not lined up in standard form. In this case, we need to first rearrange the equation before we can use the elimination steps.

Original System
out of order

$$\begin{cases} y = 7x - 9 \\ 2x - 3y = 5 \end{cases}$$

Equivalent System
in standard form

$$\begin{cases} -7x + y = -9 \\ 2x - 3y = 5 \end{cases}$$

Clearing Fractions

If we are given an equation with fractional coefficients, we can clear them out by multiplying both sides by a common multiple of the denominator. This is a handy technique which we will use often in our study of Algebra. Do not abuse this, as it only works on equations and not expressions.

$$\frac{1}{2}x - \frac{1}{3}y = \frac{1}{5}$$

Distribute then simplify.

$\Rightarrow (30)\frac{1}{2}x - (30)\frac{1}{3}y = 30(\frac{1}{5})$
 $15x - 10y = 6$

The LCM of the denominators is 30.

$\Leftarrow$ No more fractions, now that is nice!

$$\frac{1}{2}x - \frac{1}{3}y = \frac{1}{5} \xleftrightarrow{\text{equivalent}} 15x - 10y = 6$$

The above linear equations are equivalent, which would you rather work with? This technique, on the street, is sometimes referred to as a “fraction buster.”



B. Solve the systems using the elimination method.

Solve the system:

$$\begin{cases} \frac{1}{3}x - \frac{2}{3}y = 3 \\ \frac{1}{3}x - \frac{1}{2}y = \frac{8}{3} \end{cases}$$

$$\begin{aligned} \frac{1}{3}x - \frac{2}{3}y &= 3 & \times (3) & \quad x - 2y = 9 & \times (2) \\ \frac{1}{3}x - \frac{1}{2}y &= \frac{8}{3} & \times (6) & \quad 2x - 3y = 16 & \end{aligned}$$

$$\begin{aligned} -2x + 4y &= -18 \\ \oplus 2x - 3y &= 16 \\ \hline y &= -2 \end{aligned}$$

BACK subst. $x - 2y = 9$

$$\begin{aligned} x - 2(-2) &= 9 \\ x + 4 &= 9 \\ x &= 5 \end{aligned}$$

Answer: (5, -2)

Solve the system:

$$\begin{cases} -\frac{1}{10}x + \frac{1}{2}y = \frac{4}{5} \\ \frac{1}{7}x + \frac{1}{3}y = -\frac{2}{21} \end{cases}$$

$$\begin{aligned} -\frac{1}{10}x + \frac{1}{2}y &= \frac{4}{5} & \times (10) & \quad -x + 5y = 8 & \times (3) \\ \frac{1}{7}x + \frac{1}{3}y &= -\frac{2}{21} & \times (21) & \quad 3x + 7y = -2 & \end{aligned}$$

$$\begin{aligned} -3x + 15y &= 24 \\ \oplus 3x + 7y &= -2 \\ \hline 22y &= 22 \\ y &= 1 \end{aligned}$$

BACK subst. $-x + 5y = 8$

$$\begin{aligned} -x + 5(1) &= 8 \\ -x + 5 &= 8 \\ -x &= 3 \\ x &= -3 \end{aligned}$$

Answer: (-3, 1)

Solve the system:

$$\begin{cases} y = -\frac{5}{3}x + \frac{1}{2} \\ \frac{1}{3}x + \frac{1}{5}y = \frac{1}{10} \end{cases}$$

$$\begin{aligned} y &= -\frac{5}{3}x + \frac{1}{2} & \times (6) & \quad 6y = -10x + 3 \\ \frac{1}{3}x + \frac{1}{5}y &= \frac{1}{10} & \times (30) & \quad 10x + 6y = 3 \end{aligned}$$

$$\begin{aligned} 10x + 6y &= 3 & \times (-1) & \quad -10x - 6y = -3 \\ \oplus 10x + 6y &= 3 & \times (1) & \quad 10x + 6y = 3 \\ \hline 0 &= 0 \text{ True} \end{aligned}$$

Dependent System $(x, -\frac{5}{3}x + \frac{1}{2})$

Solve the system:

$$\begin{cases} -\frac{1}{7}x + y = -\frac{2}{3} \\ -\frac{1}{14}x + \frac{1}{2}y = \frac{1}{3} \end{cases}$$

$$\begin{aligned} -\frac{1}{7}x + y &= -\frac{2}{3} & \times (21) & \quad -3x + 21y = -14 & \times (-1) \\ -\frac{1}{14}x + \frac{1}{2}y &= \frac{1}{3} & \times (42) & \quad -3x + 21y = 14 \end{aligned}$$

$$\begin{aligned} 3x - 21y &= 14 \\ \oplus -3x + 21y &= 14 \\ \hline 0 &= 28 \text{ False: No Solution} \end{aligned}$$

Now that we have techniques for solving systems we can set up our word problems with two variables. If we use two variables we will need two equations. With this in mind, look for two relationships when reading the questions.

A. Number Problems

Last season two running backs on the Steelers football team rushed for a combined total of 1550 yards. One rushed 4 times as many yards as the other. How many yards were rushed by each one?

Let x = the number of yards rushed by one of them
 y = the number of yards rushed by the other.

$$\text{Set-up} \rightarrow \begin{cases} x + y = 1550 \\ y = 4x \end{cases} \quad \text{Back! Substitute:}$$

$$\begin{array}{rcl} \text{Substitution:} & x + 4x = 1550 & 310 + y = 1550 \\ & 5x = 1550 & -310 \quad -310 \\ & x = 310 & y = 1240 \end{array}$$

One rushed for 310 yds and the other 1240 yds.

The set-up determines the method we will choose to solve the system. Since the y variable was isolated the easiest method to choose was the substitution method. Although, it does not matter which method we choose the answer will be the same.

A particular Algebra text has a total of 1382 pages which is broken up into two parts. The second part of the book has 64 more pages than the first part. How many pages are in each part of the book?

Let x = the number of pages in part one.

y = the number of pages in part two.

$$\text{Set-up} \rightarrow \begin{cases} x + y = 1382 \\ y = x + 64 \end{cases} \quad \text{Back Subst.}$$

$$\begin{array}{rcl} \text{Substitution:} & x + (x + 64) = 1382 & 659 + y = 1382 \\ & 2x + 64 = 1382 & y = 723 \\ & -64 \quad -64 & \\ & 2x = 1318 & \\ & \frac{2x}{2} = \frac{1318}{2} & \\ & x = 659 & \end{array}$$

659 pages in part one and 723 pages in part two.

B. Mixture Problems

Dennis mowed his next door neighbor's lawn for a handful of dimes and nickels, 80 coins in all. Upon completing the job he counted out the coins and it came to \$6.60. How many of each coin did he earn?

Let x = number of dimes
 y = number of nickels

Set-up $\rightarrow$
$$\begin{cases} x + y = 80 \\ .10x + .05y = 6.60 \end{cases} \quad \left. \begin{array}{l} \\ \end{array} \right\} \times (-.05)$$

$$\begin{array}{r} -.05x - .05y = -.4 \\ \oplus .10x + .05y = 6.60 \\ \hline .05x = 2.60 \\ x = 52 \end{array}$$

BACK substitute:

$$\begin{array}{r} 52 + y = 80 \\ y = 28 \end{array}$$

ANS: 52 dimes and 28 nickels.

On Monday Joe bought 10 cups of coffee and 5 doughnuts for his office at the cost of \$16.50. It turns out that the doughnuts were more popular than the coffee. On Tuesday he bought 5 cups of coffee and 10 doughnuts for a total of \$14.25. How much was each cup of coffee?

Let x = cost of each cup of coffee.
 y = cost of each doughnut.

Set-up $\rightarrow$
$$\begin{cases} 10x + 5y = 16.50 \\ 5x + 10y = 14.25 \end{cases} \quad \left. \begin{array}{l} \\ \end{array} \right\} \times (-2)$$

$$\begin{array}{r} -20x - 10y = -33 \\ \oplus 5x + 10y = 14.25 \\ \hline -15x = -18.75 \\ x = 1.25 \end{array}$$

ANS: The cost of each cup of coffee was \$1.25

A bartender wishes to mix an 8 ounce drink with a 20% alcohol content. He has two liquors, one with a 50% alcohol content and another with 10%. How much of each liquor does he need to mix together?

Let x = amount of the 50% alcohol liquor.
 y = amount of the 10% alcohol liquor.

Set-up $\rightarrow$
$$\begin{cases} x + y = 8 \\ .50x + .10y = .20(8) \end{cases} \quad \left. \begin{array}{l} \leftarrow \text{total Amount} \\ \leftarrow \text{total Alcohol content} \end{array} \right\} \quad \leftarrow$$

$$\begin{array}{r} -.10x - .10y = -.8 \\ \oplus .50x + .10y = 1.6 \\ \hline .40x = .8 \\ x = 2 \end{array}$$

BACK substitute:

$$\begin{array}{r} 2 + y = 8 \\ y = 6 \end{array}$$

A common error is to put = .20 here. Total alcohol content is to be 20% of 8 ounces.

He needs 2 ounces of the 50% liquor and 6 ounces of the 10% liquor.

C. Geometry Problems

Two angles are supplementary. The larger angle is 48 degrees more than 10 times the smaller angle. Find the measure of each angle.

Let x = the measure of the smaller angle.

y = the measure of the larger angle.

$$\text{Set-up} \rightarrow \begin{cases} x + y = 180 \\ y = 10x + 48 \end{cases}$$



Supplementary angles add to 180 degrees.

Substitution: $x + (10x + 48) = 180$ Back Substitute:

$$11x + 48 = 180$$

$$12 + y = 180$$

$$11x = 132$$

$$y = 168$$

$$x = 12$$

The two angles are 12° and 168° .

Two angles are complementary. The larger angle is 3 degrees less than twice the measure of the smaller angle. Find the measure of each angle.

Let x = the measure of the smaller angle

y = the measure of the larger angle.

$$\text{Set-up} \rightarrow \begin{cases} x + y = 90 \\ y = 2x - 3 \end{cases}$$



Complementary angles add to 90 degrees.

Substitution: $x + (2x - 3) = 90$ Back Substitution:

$$3x - 3 = 90$$

$$31 + y = 90$$

$$3x = 93$$

$$y = 59$$

$$x = 31$$

The two angles are 31° and 59° .

The perimeter of a rectangular garden is 62 feet. The length is 1 foot more than twice the width. Find the dimension of the garden.

Let w = width of the garden in feet.

l = length of the garden in feet.

$$\text{Set-up} \rightarrow \begin{cases} l = 2w + 1 \\ 2l + 2w = 62 \end{cases}$$

Back substitution:

$$l = 2(10) + 1$$

$$= 20 + 1$$

$$= 21$$

$$\text{Substitution} \quad 2(2w + 1) + 2w = 62$$

$$4w + 2 + 2w = 62$$

$$6w + 2 = 62$$

$$6w = 60$$

$$w = 10$$

Ans: The garden measures
10ft by 21ft.

D. Interest Problems

Sally's \$1800 savings is in two accounts. Her total interest for the year was \$93 from one account earning 6% annual interest and another earning 3%. How much does she have in each account?

Let x = the amount invested at 3%

y = the amount invested at 6%

Set-up $\rightarrow x + y = 1800$ } $\times (-.03)$ Back Substitute:

$$.03x + .06y = 93$$

$$x + 1300 = 1800$$

$$x = 500$$

$$-.03x - .03y = -54$$

$$\textcircled{+} \quad .03x + .06y = 93$$

Ans: \$500 invested at 3%

\$1300 invested at 6%.

$$.03y = 39$$

$$y = 1300$$

When setting up these word problems look for totals. The above example is very typical, notice that one on the equations consists of the total amount invested, $x + y = 1800$. The other equation represents the total amount of interest for the year, $.03x + .06y = 93$. Two linear equations allow you to solve for the variables.

Also notice that it is wise to identify your variables every time. This focuses your efforts and aids us in finding the solution. It also tells us what our answers mean at the end.

Millicent has \$10,000 invested in two accounts. For the year she earned \$535 more in interest from her 7% Mutual Fund account than she did from her 4% CD. How much does she have in each account?

Let x = amount invested in mutual funds at 7%

y = amount invested in the CD at 4%.

Set-up $\rightarrow x + y = 10,000$ }

$$.07x = .04y + 535$$

$$x + y = 10,000$$
 } $\times (.04)$ $.04x + .04y = 400$ }

$$.07x - .04y = 535$$
 } $\textcircled{+} \quad .07x - .04y = 535$ }

$$.11x = 935$$

$$x = 8,500$$

She invested \$8,500 in mutual funds
and \$1,500 in the CD.

Always check to make sure your answer makes sense in terms of the word problem. If you come up with an answer of, say $x = 20,000$ in the problem above you know this is unreasonable since the total amount is 10,000. At that point you should first go back and check your set-up then check your algebra steps from there.

The idea behind **distance problems**, sometimes called uniform motion problems, is to organize the given data. First identify the variables then try to fill in the chart with the appropriate values. Sometimes your set up can come from columns in the chart and other times the set up will come from the rows. Remember $D = r \cdot t$.

E. Uniform Motion Problems

An executive traveled 1930 miles by car and plane. He drove to the airport at an average speed of 60 mph and the plane averaged 350 mph. The total trip took 8 hours. How long did it take to get to the airport?

Let x = time spent traveling by car.

y = time spent traveling by air.

Set-up $\rightarrow x + y = 8$

$$60x + 350y = 1930$$

$$-60x - 60y = -480$$

$$+ 60x + 350y = 1930$$

$$290y = 1450$$

$$y = 5$$

	D	r	t
CAR	60x	60	x
plane	350y	350	y
	1930		8

NOTE: $D = r \cdot t$

Ans: It took 3 hrs to drive to the airport.

A boat traveled 24 miles downstream in 2 hours. The return trip took twice as long. What is the speed of the boat in still water?

Let x = speed of the boat in still water

y = speed of the current.

Set-up $\rightarrow 24 = (x+y) \cdot 2$

$$24 = (x-y) \cdot 4$$

$$24 = 2x + 2y \quad \times(2) \quad 48 = 4x + 4y$$

$$24 = 4x - 4y \quad \oplus \quad 24 = 4x - 4y$$

$$72 = 8x$$

$$9 = x$$

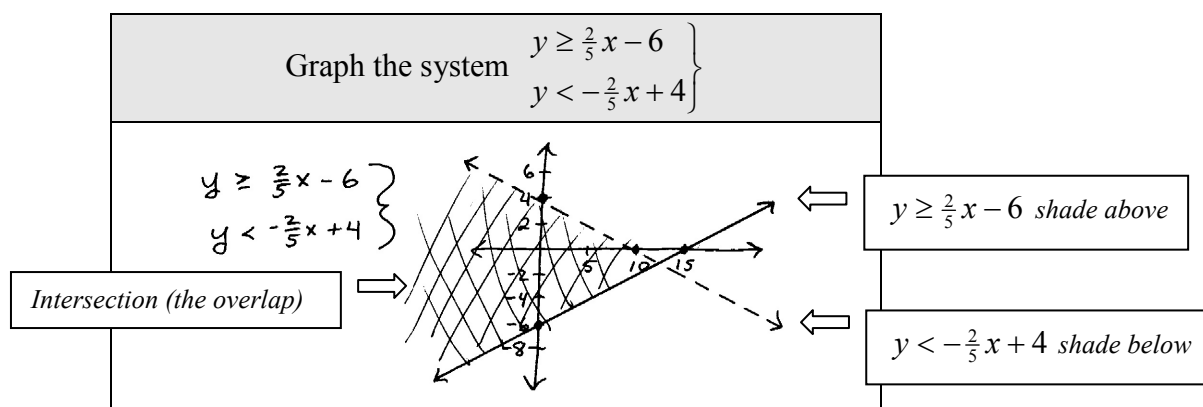
	D	r	t
Down-Stream	24	$x+y$	2
UP-Stream	24	$x-y$	4

($D = r \cdot t$)

Ans: SPEED of the boat is 9 mph.

Word problems take practice. Be sure to do all of the assigned word problems and review them often. Do not plan on skipping them on the exams. That is not a winning strategy. Usually, once we set our word problems up correctly, the algebra is easier than the other problems.

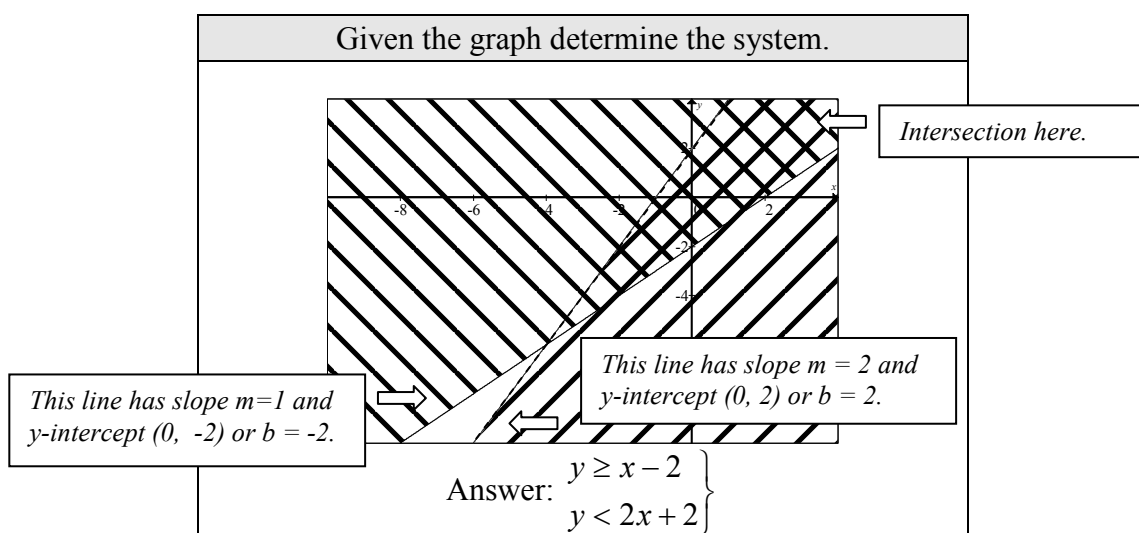
The systems of linear inequalities that we will be solving consist of two linear inequalities and two variables. To solve these we will graph them both and determine where they intersect. Any point in the overlap of the graphs will be a common point.

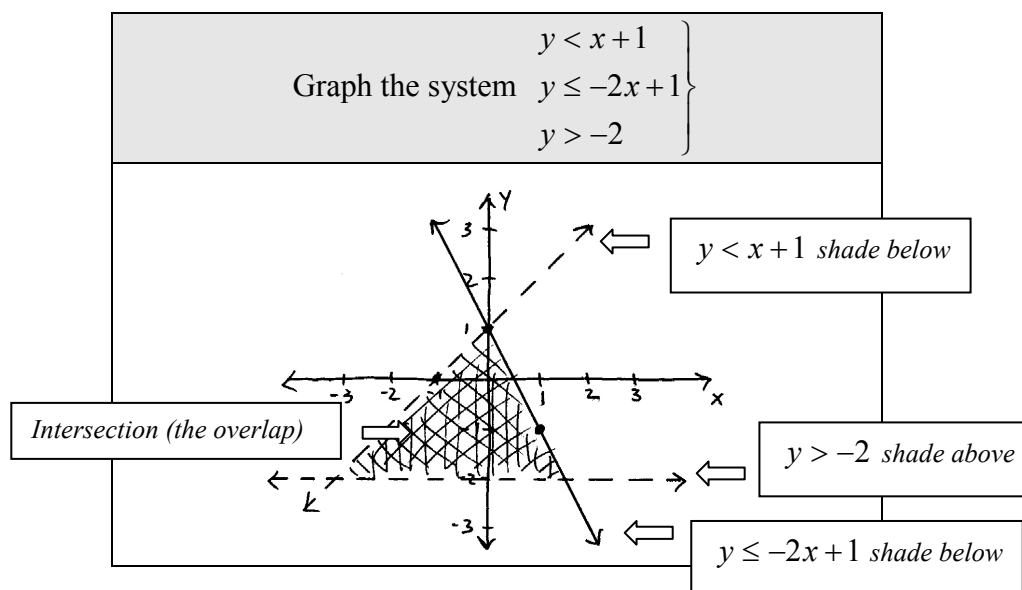
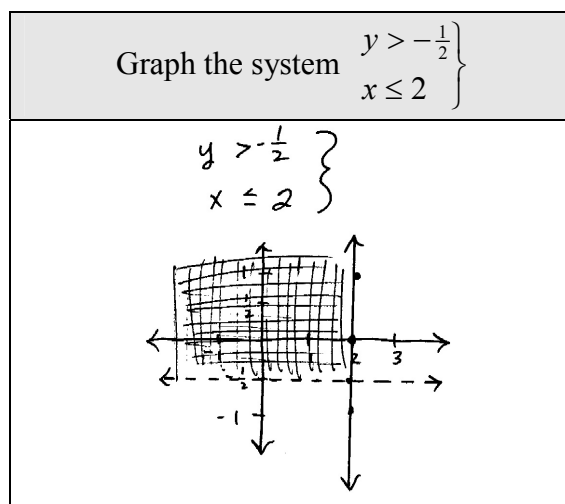
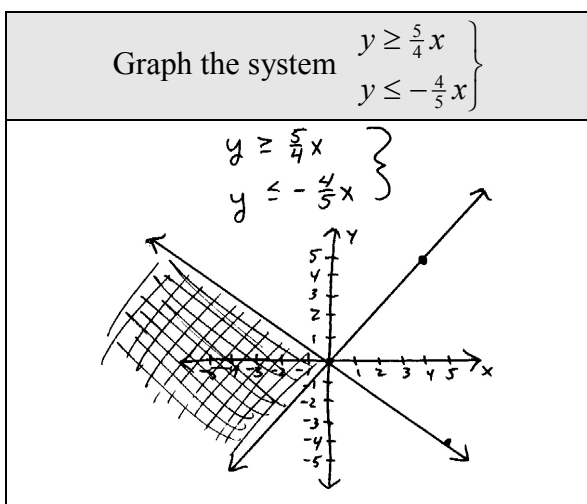
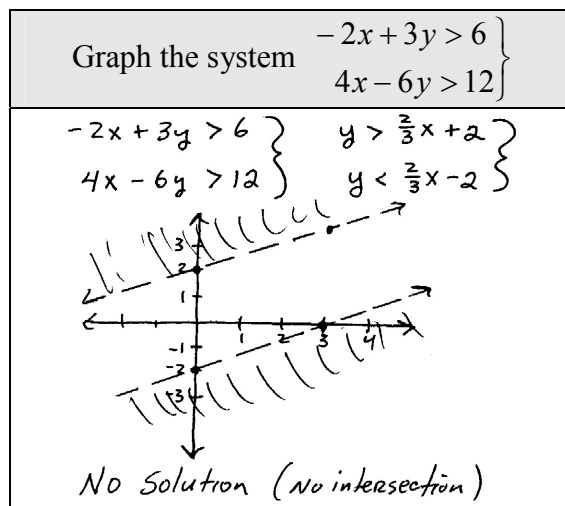
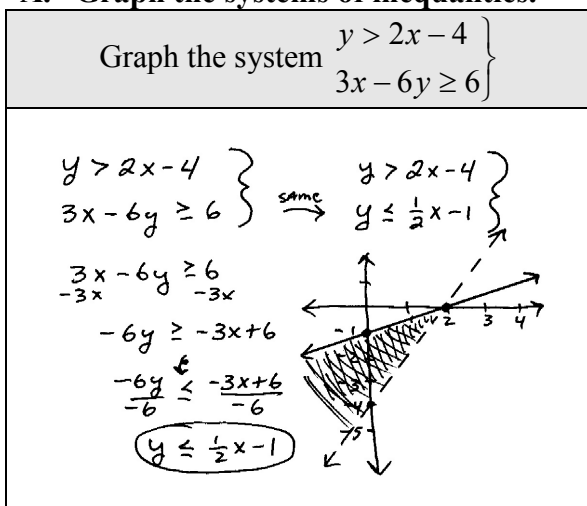


The above solution suggests that $(-5, 3)$ is a common point because it is shaded. Check it and others to see if it works for both inequalities.

$(-5, 3)$	$y \geq \frac{2}{5}x - 6$	$(-5, 3)$	$y < -\frac{2}{5}x + 4$
	$3 \geq \frac{2}{5}(-5) - 6$		$3 < -\frac{2}{5}(-5) + 4$
	$3 \geq -2 - 6$		$3 < 2 + 4$
	$3 \geq -8$ True		$3 < 6$ True

These graphs can sometimes get messy so do your best to think about the solution before actually shading. Use a pencil and a good eraser when working on these problems.



A. Graph the systems of inequalities.

Chapter 4_Solving Linear Systems

Sample Exam

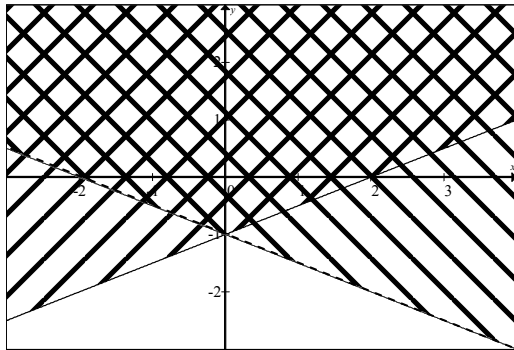
Please answer all the questions and show work where appropriate.

1. Solve the system using the graphing method:
$$\begin{cases} 2x + 3y = 12 \\ -4x + 3y = -6 \end{cases}$$
2. Solve the system using the substitution method:
$$\begin{cases} x + y = 7 \\ 18x + 2y = -2 \end{cases}$$
3. Solve the system using the addition method:
$$\begin{cases} 5x - 2y = -4 \\ 3x - 6y = 12 \end{cases}$$
4. Solve the following linear systems using any method:
 - a.
$$\begin{cases} 10x - 14y = 2 \\ 7y = 5x - 1 \end{cases}$$
 - b.
$$\begin{cases} x + 3y = 3 \\ 2x + 6y = -6 \end{cases}$$
5. Where do $x = 3$ and $y = -2$ cross?
6. Marsha has \$4,500 saved up in two different accounts. She has a savings account earning 2% in annual interest and a CD earning 5% annual interest. How much does she have in each account if her total interest for the year was \$198?
7. The length of a rectangular garden is 3ft less than twice the width. If the perimeter measures 54ft then what are the dimensions of the garden?
8. Al can row his boat the 30 miles downstream to the general store in 3 hours. The return trip, against the current, will take him 5 hours. How fast is the river current?
9. Two angles are supplementary and one angle is 45° less than 4 times the other. Find the two angles.
10. Graph all solutions to the system of inequalities:
$$\begin{cases} 2y \geq x - 2 \\ -2y < x + 2 \end{cases}$$

Chapter 4 *Solving Linear Systems*

Sample Exam Answers

1. (3, 2)
2. (-1, 8)
3. (-2, -3)
4. a. $(x, \frac{5}{7}x - \frac{1}{7})$
b. No Solution
5. (3, -2)
6. \$900 in savings and
\$3,600 in the CD
7. Width = 10ft
Length = 17ft
8. Speed of current = 2mph
9. 45° and 135°
- 10.

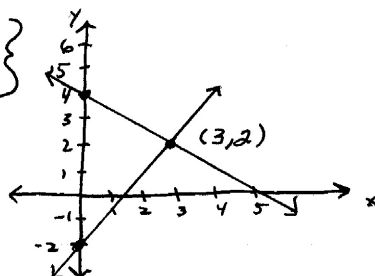


Solve the system using the graphing method:

$$\begin{cases} 2x + 3y = 12 \\ -4x + 3y = -6 \end{cases}$$

$$\begin{cases} 2x + 3y = 12 \\ -4x + 3y = -6 \end{cases} \Rightarrow \begin{cases} y = -\frac{2}{3}x + 4 \\ y = \frac{4}{3}x - 2 \end{cases}$$

They cross at
Solution: $(3, 2)$



Solve the system using the substitution method:

$$\begin{cases} x + y = 7 \\ 18x + 2y = -2 \end{cases}$$

$$\begin{cases} x + y = 7 \\ 18x + 2y = -2 \end{cases} \Rightarrow y = -x + 7$$

Substitution $18x + 2(-x + 7) = -2$

$$18x - 2x + 14 = -2$$

$$16x + 14 = -2$$

$$16x = -16$$

$$x = -1$$

So solution $(-1, 8)$ ✓

Back substitute

$$y = -x + 7$$

$$y = -(-1) + 7$$

$$= 1 + 7$$

$$y = 8$$

Solve the system using the addition method:

$$\begin{cases} 5x - 2y = -4 \\ 3x - 6y = 12 \end{cases}$$

$$\begin{cases} 5x - 2y = -4 \\ 3x - 6y = 12 \end{cases} \xrightarrow{(x-3)} \begin{cases} -15x + 6y = 12 \\ \oplus 3x - 6y = 12 \end{cases}$$

$$\begin{array}{r} -15x + 6y = 12 \\ \oplus 3x - 6y = 12 \\ \hline -12x = 24 \end{array}$$

$$x = -2$$

Solution: $(-2, -3)$ ✓

Back substitute

$$5(-2) - 2y = -4$$

$$-10 - 2y = -4$$

$$-2y = 6$$

$$y = -3$$

Solve the following linear system using any method:

$$\begin{cases} 10x - 14y = 2 \\ 7y = 5x - 1 \end{cases}$$

$$\begin{aligned} 10x - 14y &= 2 \\ 7y &= 5x - 1 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ (x2) \end{array}$$

$$\begin{aligned} 10x - 14y &= 2 \\ -5x + 7y &= -1 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (x2)$$

$$\begin{aligned} 10x - 14y &= 2 \\ \oplus -10x + 14y &= -2 \end{aligned}$$

$$0 = 0, \text{ True}$$

same line

Ans: $(x, \frac{5}{7}x - \frac{1}{7})$

Solve the following linear system using any method:

$$\begin{cases} x + 3y = 3 \\ 2x + 6y = -6 \end{cases}$$

$$\begin{aligned} x + 3y &= 3 \\ 2x + 6y &= -6 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \\ \\ (x2) \end{array}$$

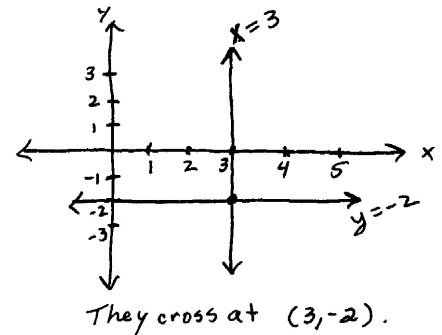
$$\begin{aligned} x + 3y &= 3 \\ \oplus 2x + 6y &= -6 \end{aligned}$$

$$0 = -12 \text{ False}$$

Parallel lines

No solution ✓

Where do $x = 3$ and $y = -2$ cross?



Marsha has \$4,500 saved up in two different accounts. She has a savings account earning 2% in annual interest and a CD earning 5% annual interest. How much does she have in each account if her total interest for the year was \$198?

Let x = amount in savings at 2%

y = amount in CD at 5%

Set-up $\rightarrow \begin{cases} x + y = 4500 \\ 0.02x + 0.05y = 198 \end{cases} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} x(-2) \\ x(100) \end{array}$

$$\begin{aligned} -2x - 2y &= -9000 \\ \oplus 2x + 5y &= 19800 \end{aligned}$$

$$3y = 10800$$

$$y = 3600$$

Back substitute

$$\begin{aligned} x + 3600 &= 4500 \\ -3600 &-3600 \end{aligned}$$

$$x = 900$$

Ans: \$900 in savings and \$3,600 in the CD.

The length of a rectangular garden is 3ft less than twice the width. If the perimeter measures 54ft then what are the dimensions of the garden?

Let w = the measure of the width
 $l = 2w - 3$ the length and $P = 2l + 2w$

Set-up $\left. \begin{array}{l} l = 2w - 3 \\ 54 = 2l + 2w \end{array} \right\}$ substitution method.

↓

$$54 = 2(2w - 3) + 2w$$

Back substitute

$$54 = 4w - 6 + 2w$$

$$54 = 6w - 6$$

$$\begin{array}{r} +6 \\ 60 = 6w \\ 10 = w \end{array}$$

$$\begin{array}{l} l = 2(10) - 3 \\ = 20 - 3 \\ = 17 \end{array}$$

Ans: width 10ft
length 17ft

Al can row his boat the 30 miles downstream to the general store in 3 hours. The return trip, against the current, will take him 5 hours. How fast is the river current?

Let x = the speed of the boat
 c = the speed of the current

	D	r	t
Down-Stream	30	$x+c$	3
UP-Stream	30	$x-c$	5

set-up $\rightarrow \left. \begin{array}{l} 30 = (x+c) \cdot 3 \\ 30 = (x-c) \cdot 5 \end{array} \right\}$ OR $\left. \begin{array}{l} 30 = 3x + 3c \\ 30 = 5x - 5c \end{array} \right\} \begin{array}{l} \times (5) \\ \times (3) \end{array}$

$$\left. \begin{array}{l} 150 = 15x + 15c \\ \textcircled{+} 90 = 15x - 15c \end{array} \right\}$$

Back substitute

$$240 = 30x$$

$$8 = x$$

$$30 = 3 \cdot 8 + 3c$$

$$30 = 24 + 3c$$

$$6 = 3c$$

$$2 = c$$

The speed of the boat is 8mph and 2mph for current.

Two angles are supplementary and one angle is 45° less than 4 times the other. Find the two angles.

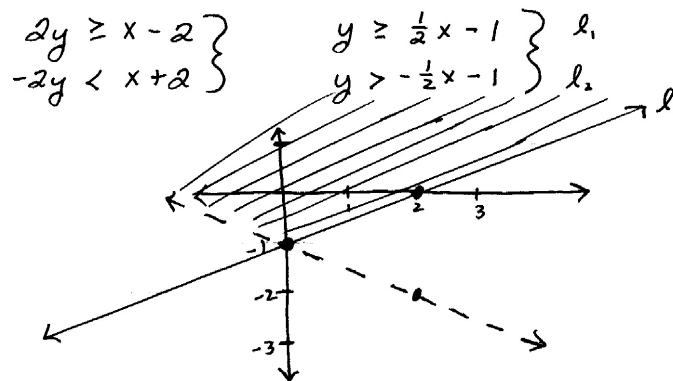
Let x = measure of one angle
 y = measure of the other

Set-up $\rightarrow \begin{cases} x + y = 180^\circ \\ y = 4x - 45^\circ \end{cases}$ Back substitute

Substitution: $x + 4x - 45 = 180$ $45 + y = 180$
 $\quad\quad\quad +45 \quad +45$ $-45 \quad -45$

$5x = 225$ $y = 135^\circ$
 $x = 45^\circ$ The two angles
 measure 45° & 135° ✓

Graph all solutions to the system of inequalities: $\begin{cases} 2y \geq x - 2 \\ -2y < x + 2 \end{cases}$





Chapter 1 - 4_Cumulative Review

Contents

Typically, chapters 1 – 4 will be the material covered on an Elementary Algebra midterm exam. At this point we will review and reinforce the techniques we have learned. The best way to do this is to partition 2 hours of your time to take the following sample midterm exam. When you are finished check your answers and go back and find problems similar to the ones you missed. When you have successfully answered all the questions you will be ready to move on to Chapter 5.

- ✓ *Sample Midterm Exam*
- ✓ *Sample Midterm Exam Answers*

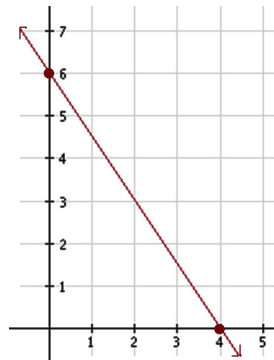
Chapters 1 – 4 _ Cumulative Review

Sample Midterm Exam

Please answer all the questions and show work where appropriate.

1. Simplify: $5 - 9 \div 3(2) - 5^2$
2. Evaluate: $b^2 - 4ac$ when $a = -1$, $b = -3$ and $c = 5$
3. Simplify: $3(2x + 5) - (5x - 4)$
4. Simplify: $\frac{2}{3}x + \frac{5}{2}y - \frac{1}{2}x + \frac{3}{5}y$
5. Solve: $3x - 7 = 8$
6. Solve: $\frac{2}{3}(3x - 12) - \frac{1}{2}(2x + 6) = 9$
7. Solve: $5(-2x + 3) + 7x < 21$
8. Solve for w : $P = 2l + 2w$
9. A taxi service charges \$5.00 for a ride plus \$0.70 per mile. How many miles can you go if you wish to spend at most \$40.00?
10. To earn a “B” in a mathematics course your average score must be at least 80. If you earn a 70, 75 and 82 on the first three exams, what must you score on the 4th exam to earn a “B”?

11. Find the slope of the line in the given graph.



12. Find the x and y – intercepts of $3x - y = -6$ and use them to graph the line.
13. Determine if the line passing through $(-3, -5)$ and $(3, 4)$ is parallel or perpendicular to $-3x + 2y = 6$. Justify your answer.
14. Find the equation of the line, in slope intercept form, perpendicular to $x + 2y = 14$ passing through the point $(-5, 1)$.
15. Solve by Graphing:
$$\left. \begin{array}{l} 3x + 4y = -4 \\ -x + 4y = 12 \end{array} \right\}$$
16. Solve by Substitution:
$$\left. \begin{array}{l} x + 5y = 15 \\ 5y = -x + 15 \end{array} \right\}$$
17. Solve by any method:
- a.
$$\left. \begin{array}{l} -2x + 3y = -12 \\ -5x + 2y = -19 \end{array} \right\}$$
- b.
$$\left. \begin{array}{l} 2x + 6y = 6 \\ x + 3y = -3 \end{array} \right\}$$
18. The perimeter of a rectangular garden measures 64 feet. If the length is 4 feet less than twice its width, what are the dimensions of the garden?
19. There are 21 quarters and dimes in a can whose value adds to \$3.60. How many of each coin are in the can?
20. An airplane flying with the wind can travel 690 miles in 3 hours. On the return trip, against the same wind, the plane can only travel 510 miles in the same amount of time. What is the speed of the wind?

Chapters 1 – 4 _ Cumulative Review
Sample Midterm Exam Answers

1. -26
2. 29
3. $x + 19$
4. $\frac{1}{6}x + \frac{31}{10}y$
5. $x = 5$
6. $x = 20$
7. $x > -2$ or $(-2, \infty)$
8. $w = \frac{P - 2l}{2}$
9. At most 50 miles
10. At least 93 on exam #4
11. $m = -\frac{3}{2}$
12. x -intercept $(-2, 0)$, y -intercept $(0, 6)$
13. Parallel
14. $y = 2x + 11$
15. $(-4, 2)$
16. Dependant System, $(x, -\frac{1}{5}x + 3)$
17.
 - a. $(3, -2)$
 - b. Inconsistent System, No Solution
18. 12 feet by 20 feet
19. 10 quarters and 11 dimes
20. 30 mph



Chapter 5 *Polynomial Operations*

Contents

So far we have limited our discussion mainly to linear expressions and equations. This chapter moves us on to non linear expressions and equations. There will be some new ideas and properties to learn but much of what we have been doing is the same.

Polynomial Operations

5.1 ... Rules of Exponents

5.2 ... Negative Exponents

5.3 ... Adding and Subtracting Polynomials

5.4 ... Multiplying Polynomials

5.5 ... Dividing Polynomials

5.6 ... Evaluating Expressions

Sample Exam

Sample Exam Solutions

Before we can learn how to add, subtract, multiply and divide polynomials we need to understand the rules of exponents. There will be many new rules to remember and you might feel overwhelmed, but with much practice you will quickly use them effortlessly. (In this section we will assume all variables are real and exponents are integers.)

$$\begin{array}{lcl} 2^3 \cdot 2^4 & & x^3 \cdot x^4 \\ = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) & \text{or} & = (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) \\ = 2^7 & & = x^7 \end{array}$$

Product Rule – For any real base b , $b^n \cdot b^m = b^{n+m}$

When multiplying two real numbers with the same base we will simply add exponents.

Multiply: $x^5 \cdot x^7$	
$\begin{array}{l} x^5 \cdot x^7 \\ = x^{5+7} \\ = x^{12} \end{array}$	$\leftarrow$ Add exponents

A. Multiply

$y^3 \cdot y^{12}$
$\begin{array}{l} y^3 \cdot y^{12} \\ = y^{3+12} \\ = y^{15} \end{array}$

$3^3 \cdot 3^5 \cdot 3^7$
$\begin{array}{l} 3^3 \cdot 3^5 \cdot 3^7 \\ = 3^{3+5+7} \\ = 3^{15} \end{array}$

$(2x+1)^3(2x+1)^5$
$\begin{array}{l} (2x+1)^3(2x+1)^5 \\ = (2x+1)^{3+5} \\ = (2x+1)^8 \end{array}$

Add exponents only if the bases are exactly the same.

Power Rule – For any real base b , $(b^n)^m = b^{n \cdot m}$

When raising an exponent to a power simply multiply the exponents.

$$\begin{array}{l} (2^3)^2 \\ = (2^3)(2^3) \\ = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2) \\ = 2^6 \end{array}$$

Simplify: $(x^5)^7$	
$\begin{array}{l} (x^5)^7 \\ = x^{5 \cdot 7} \\ = x^{35} \end{array}$	$\leftarrow$ Multiply exponents

B. Simplify

$(y^3)^{12}$
$(y^3)^{12}$ $= y^{3 \cdot 12}$ $= y^{36}$

$(x^3)^2(x^4)^3$
$(x^3)^2(x^4)^3$ $= x^6 \cdot x^{12}$ $= x^{18}$

$(3^3)^5$	<i>Never multiply a base with an exponent!</i>
$(3^3)^5$	
$= 3^{3 \cdot 5}$ $= 3^{15} \leftarrow$	

Power of a Product Rule – For any real base a and b ,

$$(ab)^n = a^n b^n$$

When we raise a product to a power we must raise each factor to that power.

$$(2x)^3$$

$$= (2x)(2x)(2x)$$

$$= 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$$

$$= 2^3 x^3$$

Simplify: $(xy^5)^7$
$(xy^5)^7$ $= x^7 y^{5 \cdot 7} \leftarrow$ $= x^7 y^{35}$

Multiply exponents

C. Simplify

$(x^2 y^3)^2$
$(x^2 y^3)^2$ $= x^{2 \cdot 2} y^{3 \cdot 2}$ $= x^4 y^6$

$(x^3 y)^2 (x^4 y^2)^3$
$(x^3 y)^2 (x^4 y^2)^3$ $= x^6 y^2 \cdot x^{12} y^6$ $= x^{18} y^8$

$(2x^2 y^3 z^5)^5$
$(2x^2 y^3 z^5)^5$ $= 2^5 x^{10} y^{15} z^{25}$ $= 32 x^{10} y^{15} z^{25}$

$(-3x^2 y^3)^4 (xy^3)^2$
$(-3x^2 y^3)^4 (xy^3)^2$ $= (-3)^4 x^8 y^{12} \cdot x^2 y^6$ $= 81 x^{10} y^{18}$

$(-2x^3 y^4)^3 (xy^3)^3$
$(-2x^3 y^4)^3 (xy^3)^3$ $= (-2)^3 x^9 y^{12} \cdot x^3 y^9$ $= -8 x^{12} y^{21}$

$(-a^2 b)^5 (-2ab^4)^2$
$(-a^2 b)^5 (-2ab^4)^2$ $= (-1)^5 a^{10} b^5 (-2)^2 a^2 b^8$ $= (-1) a^{12} b^{13} (4)$ $= -4 a^{12} b^{13}$

$-(2st^2)^4$
$-(2st^2)^4$ $= -(2)^4 s^4 t^{2 \cdot 4}$ $= -16 s^4 t^8$

$-3^3(3xy)^2$
$-3^3(3xy)^2$ $= -27 \cdot 3^2 x^2 y^2$ $= -27 \cdot 9 x^2 y^2$ $= -243 x^2 y^2$

$(x+2y^3)^2$
$(x+2y^3)^2$ ↑ Power of a Product Does not Apply.

Power of a Quotient Rule – For any real base a and $b \neq 0$,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

When raising a quotient to a power we must raise numerator and denominator to that power. Assume the variables in the denominator are nonzero. If we have zero in the denominator then the expression is undefined.

$$\begin{aligned} &\left(\frac{2}{3}\right)^3 \\ &= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \\ &= \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} \\ &= \frac{2^3}{3^3} = \frac{8}{27} \end{aligned}$$

Simplify: $\left(\frac{x}{y^2}\right)^3$

$$\begin{aligned} &\left(\frac{x}{y^2}\right)^3 = \\ &= \left(\frac{x^3}{y^{2 \cdot 3}}\right) = \frac{x^3}{y^6} \end{aligned}$$

D. Simplify

$$\left(\frac{-2x^3}{3y^2}\right)^2$$

$$\begin{aligned} &\left(\frac{-2x^3}{3y^2}\right)^2 \\ &= \frac{(-2)^2 x^6}{3^2 y^4} \\ &= \frac{4x^6}{9y^4} \end{aligned}$$

$$\left(\frac{-x^2}{y^3}\right)^3$$

$$\begin{aligned} &\left(\frac{-x^2}{y^3}\right)^3 \\ &= \frac{(-1)^3 x^6}{y^9} \\ &= \frac{-x^6}{y^9} \end{aligned}$$

$$\left(\frac{ab^2}{3c^3d^2}\right)^4$$

$$\begin{aligned} &\left(\frac{ab^2}{3c^3d^2}\right)^4 \\ &= \frac{a^4 b^8}{3^4 c^{12} d^8} = \frac{a^4 b^8}{81 c^{12} d^8} \end{aligned}$$

Tip: If we raise expressions to an even power then the quantity will be positive. If we raise negative expressions to an odd power then the quantity will be negative.



Quotient Rule – For any real base $b \neq 0$,

$$\frac{b^n}{b^m} = b^{n-m}$$

When dividing two expressions with the same base we subtract exponents. This rule is basically cancellation in disguise.

$$\begin{aligned} &\frac{2^5}{2^3} \\ &= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} \\ &= \frac{2 \cdot 2}{1} = \frac{2^2}{1} = 4. \end{aligned}$$

Simplify: $\frac{3x^8}{x^3}$

$$\begin{aligned} &\frac{3x^8}{x^3} \\ &= 3x^{8-3} \\ &= 3x^5 \end{aligned}$$

E. Simplify (Assume all variables are nonzero.)

$\frac{-2x^3y^5}{4xy^2}$	$\frac{50a^9b}{100a^2b}$	$\frac{(2x+1)^{10}}{(2x+1)^8}$
$\frac{\cancel{-2}x^3y^5}{\cancel{4}xy^2}$ $= \frac{-1x^{3-1}y^{5-2}}{2}$ $= \frac{-x^2y^3}{2}$	$\frac{\cancel{50}a^9\cancel{b}}{\cancel{100}a^2\cancel{b}}$ $= \frac{a^7}{2}$	$\frac{(2x+1)^{10}}{(2x+1)^8}$ $= (2x+1)^{10-8}$ $= (2x+1)^2$

$\left(\frac{-2x^3y}{16x^2y}\right)^2$	$\left(\frac{30ab^3}{3abc}\right)^3$	$\left(\frac{3s^3t^2}{2s^2t}\right)^? = \frac{27s^3t^3}{8}$
$\left(\frac{-2x^3y}{16x^2y}\right)^2$ $= \left(\frac{-x}{8}\right)^2$ $= \frac{x^2}{64}$	$\left(\frac{30ab^3}{3abc}\right)^3$ $= \left(\frac{10b^2}{c}\right)^3$ $= \frac{1000b^6}{c^3}$	$\left(\frac{3s^3t^2}{2s^2t}\right)^?$ $= \left(\frac{3st}{2}\right)^?$ so ? = 3 ✓

Zero Exponent Rule – For any real base $b \neq 0$, $b^0 = 1$

When an expression is raised to the zero power it will be equal to 1, unless the base is zero, zero to the zero power is undefined.

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

so $2^0 = 1$

Simplify: $(x^5)^0$
$(x^5)^0$ $= x^{5 \cdot 0} = x^0 = 1$

F. Simplify (Assume all variables are nonzero.)

$(-6x^2y^3)^0$	8675309^0	$\left(\frac{2x^2y^3z^5}{-3z^6}\right)^0$
$(-6x^2y^3)^0$ $= 1$	8675309^0 $= 1$	$\left(\frac{2x^2y^3z^5}{-3z^6}\right)^0 = 1 \quad \leftarrow$

You got to like that!

The quotient rule can be used to define a new idea in our study of Algebra. It might seem strange to think of negative exponents but we will run into them. Therefore we need to know where they come from and how to work with them.

$$\frac{1}{3} = \frac{9}{27} = \frac{3^2}{3^3} = 3^{2-3} = 3^{-1}$$

Negative Exponents – For any real base $b \neq 0$, $b^{-n} = \frac{1}{b^n}$

Factors in the numerator with a negative exponents move to the denominator.

Simplify: x^{-3}
$x^{-3} = \frac{1}{x^3}$

A. Simplify

y^{-5}
$y^{-5} = \frac{1}{y^5}$

$3x^{-3}$
$3x^{-3} = \frac{3}{x^3}$

$(3x)^{-3}$
$(3x)^{-3} = \frac{1}{(3x)^3} = \frac{1}{27x^3}$

The negative exponent applies to the whole quantity.

If we are given a factor with a negative exponent in the denominator simply move it to the numerator. We may use the following reasoning to justify this.

$$\frac{1}{5^{-2}} = \frac{1}{(\frac{1}{5^2})} = \frac{5^2}{1} = 25$$

Negative Exponents – For any real base $b \neq 0$, $\frac{1}{b^{-n}} = b^n$

B. Simplify

$\frac{3}{2y^{-4}}$
$\frac{3}{2y^{-4}} = \frac{3y^4}{2}$

$\frac{x^{-1}}{y^{-2}}$
$\frac{x^{-1}}{y^{-2}} = \frac{y^2}{x}$

$\frac{1}{(2x+1)^{-5}}$
$\frac{1}{(2x+1)^{-5}} = (2x+1)^5$

A common mistake is to multiply the base with the exponent when it is negative. For example, avoid this mistake $3^{-2} \neq -6$. The correct solution is $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

Another useful property involves a rational expression to a negative exponent.

$$\left(\frac{2}{3}\right)^{-1} = \frac{1}{\left(\frac{2}{3}\right)} = 1 \cdot \left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^1$$

Negative Exponents – For any real base $a \neq 0$ and $b \neq 0$,

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

C. Simplify

$$\left(\frac{2x}{y^2}\right)^{-3}$$

$$\left(\frac{2x}{y^2}\right)^{-3} = \left(\frac{y^2}{2x}\right)^3$$

$$= \frac{y^6}{2^3 x^3} = \frac{y^6}{8x^3}$$

$$\left(\frac{-3a^2b}{c^4}\right)^{-1}$$

$$\left(\frac{-3a^2b}{c^4}\right)^{-1} = \left(\frac{c^4}{-3a^2b}\right)^1$$

$$= \frac{c^4}{-3a^2b}$$

$$\left(\frac{-x}{y^{-1}}\right)^{-4}$$

$$\left(\frac{-x}{y^{-1}}\right)^{-4} = \left(\frac{y^{-1}}{-x}\right)^4$$

$$= \frac{y^{-4}}{(-1)^4 x^4} = \frac{1}{x^4 y^4}$$

When simplifying expressions it usually is best to simplify within the parenthesis first then apply the product and/or the quotient rule.

D. Simplify

$$(2xy^{-2}z^3)^{-3}$$

$$(2xy^{-2}z^3)^{-3}$$

$$= 2^{-3} x^{-3} y^6 z^{-9}$$

$$= \frac{y^6}{2^3 x^3 z^9}$$

$$= \frac{y^6}{8x^3 z^9}$$

$$(-5a^2b^{-3}c^0)^4$$

$$(-5a^2b^{-3}c^0)^4$$

$$= (-5)^4 a^8 b^{-12} c^0$$

$$= \frac{625a^8}{b^{12}}$$

$$(-x^{-2}y^3z^{-5})^{-3}$$

$$(-x^{-2}y^3z^{-5})^{-3}$$

$$= (-1)^{-3} x^6 y^{-9} z^{15}$$

$$= \frac{x^6 z^{15}}{(-1)^3 y^9}$$

$$= \frac{x^6 z^{15}}{-y^9} = -\frac{x^6 z^{15}}{y^9}$$

$$\left(\frac{2xy^3z^{-1}}{y^2z^3}\right)^{-3}$$

$$\left(\frac{2xy^3z^{-1}}{y^2z^3}\right)^{-3} = \left(\frac{2xy}{z^3z^1}\right)^{-3}$$

$$= \left(\frac{2xy}{z^4}\right)^{-3} = \left(\frac{z^4}{2xy}\right)^3$$

$$= \frac{z^{12}}{2^3 x^3 y^3} = \frac{z^{12}}{8x^3 y^3}$$

$$\left(\frac{-3a^2bc}{ab^0c^4}\right)^2$$

$$\left(\frac{-3a^2bc}{ab^0c^4}\right)^2 = \left(\frac{-3ab}{c^3}\right)^2$$

$$= \frac{(-3)^2 a^2 b^2}{c^6} = \frac{9a^2 b^2}{c^6}$$

$$\left(\frac{-xyz}{x^5y^{-1}z^3}\right)^{-4}$$

$$\left(\frac{-xyz}{x^5y^{-1}z^3}\right)^{-4} = \left(\frac{-y \cdot y}{x^4 z^2}\right)^{-4}$$

$$= \left(\frac{x^4 z^2}{-y^2}\right)^4 = \frac{x^{16} z^8}{y^8}$$

Scientific Notation

Scientific notation is an application of negative exponents. It is used to easily express very large or very small numbers.

Scientific Notation – Numbers in scientific notation have the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

An example of a power of ten might look like, $10^6 = 1,000,000$, so when considering the number in scientific notation $3.14 \times 10^6 = 3.14 \times 1,000,000 = 3,140,000$. Remember that we can obtain the same result by moving the decimal over six places to the right and filling in with the digit 0. Another example might look like, $10^{-4} = \frac{1}{10^4} = \frac{1}{10,000} = 0.0001$, so when considering the number in scientific notation $3.14 \times 10^{-4} = 3.14 \times .0001 = 0.000314$. Obtain the same result by moving the decimal over to the left 4 units.

E. Express the number in scientific notation

93,210,000,000
9.321×10^{10}

0.0000000718
7.18×10^{-8}

2,000,000
2×10^6

Choose the decimal so that it is between 1 and 10.

Multiplication is commutative, so when multiplying numbers in scientific notation simply multiply the decimal parts first. Next, multiply the powers of 10 using the product rule.

F. Multiply

$(2.187 \times 10^{12})(1.3 \times 10^{-6})$
$(2.187 \times 10^{12})(1.3 \times 10^{-6})$ $= (2.187)(1.3) \cdot 10^{12} \cdot 10^{-6}$ $= 2.8431 \times 10^6$

$(-1.5 \times 10^{-3})(9.81 \times 10^{-10})$
$(-1.5 \times 10^{-3})(9.81 \times 10^{-10})$ $= (-1.5)(9.81) \cdot 10^{-3} \cdot 10^{-10}$ $= -14.715 \times 10^{-13}$ $= -1.4715 \times 10^{-12}$

$(2 \times 10^5)(8 \times 10^7)$
$(2 \times 10^5)(8 \times 10^7)$ $= (2)(8) \cdot 10^5 \cdot 10^7$ $= 16 \times 10^{12}$ $= 1.6 \times 10^{13}$

Use the product rule and add exponents when multiplying.

G. Divide

$(2.187 \times 10^{12}) \div (1.2 \times 10^{-6})$
$\frac{2.187 \times 10^{12}}{1.2 \times 10^{-6}}$ $= 1.8225 \times 10^{12-(-6)}$ $= 1.8225 \times 10^{18}$

$(2.088 \times 10^{-3}) \div (9 \times 10^{-10})$
$\frac{2.088 \times 10^{-3}}{9 \times 10^{-10}}$ $= .232 \times 10^{-3-(-10)}$ $= .232 \times 10^7$ $= 2.32 \times 10^6$

$(2 \times 10^5) \div (8 \times 10^7)$
$\frac{2 \times 10^5}{8 \times 10^7}$ $= .25 \times 10^{5-7}$ $= .25 \times 10^{-2}$ $= 2.5 \times 10^{-3}$

Use the quotient rule and subtract exponents when dividing.

Before we begin this section, we must first make a few definitions. Some of the terms have already been defined but it does not hurt to go over them again. First we must recall that algebraic expressions such as $5x^2$, $-2a^2b$, $4xyz$ and $-x$ are called **terms**. A term such as $-2a^2b$ has -2 as a **coefficient** and the **variable part** is a^2b .

Polynomial – Any sum or difference of algebraic terms.

A polynomial with only one term is called a **monomial**. One with 2 terms is called a **binomial** and a 3 term polynomial is called a **trinomial**. We will not use any other special names for polynomials with more terms.

$$\begin{array}{ll} -3x^2 & \text{monomial} \\ -3x^2 + 2x & \text{binomial} \\ -3x^2 + 2x - 5 & \text{trinomial} \end{array}$$

It is common practice to express polynomials in descending order from largest exponent down to the constant term. The **constant term**, or the term with no variable part, can be thought of as the coefficient of the x^0 term.

$$\begin{array}{l} 4x - 3 + 2x^2 - 5x^3 \\ = -5x^3 + 2x^2 + 4x - 3 \end{array} \quad \leftarrow \begin{array}{l} \text{Standard Form} \\ \text{(descending order)} \end{array}$$

Degree – The largest exponent of a polynomial with a single variable.

	$9x^4 + 2x - 7$	Degree 4
Linear $\Rightarrow$	$5x - 3$	Degree 1
	$2x^3 - 7x^2 + 2x - 1$	Degree 3
	$x^2 - 7$	Degree 2
Constant Term $\Rightarrow$	12	Degree 0

Adding polynomials requires us to first identify then combine like terms. Remember that like terms are terms with the exact same variable part.

$$\begin{aligned} 2\text{apples} + 3\text{apples} &= 5\text{apples} \\ 2x^2y + 3x^2y &= 5x^2y \end{aligned}$$

A. Add the polynomials.

$$\begin{aligned} (3x^2 + 2x - 5) + (5x^2 - 3x + 2) \\ (3x^2 + 2x - 5) + (5x^2 - 3x + 2) \\ = 8x^2 - x - 3 \end{aligned}$$

$$\begin{aligned} (8a^2b - 5ab - 7ab^2) + (2a^2b - 7ab + 10ab^2) \\ (8a^2b - 5ab - 7ab^2) + (2a^2b - 7ab + 10ab^2) \\ = 10a^2b - 12ab + 3ab^2 \end{aligned}$$

When **subtracting polynomials** we must subtract all the terms. In practice, we will distribute the negative first, actually (-1) , then identify and combine like terms.

B. Subtract the polynomials.

$$\begin{aligned} (3x^2 + 2x - 5) - (5x^2 - 3x + 2) \\ (3x^2 + 2x - 5) - (5x^2 - 3x + 2) \\ = 3x^2 + 2x - 5 - 5x^2 + 3x - 2 \\ = -2x^2 + 5x - 7 \end{aligned}$$

$$\begin{aligned} (8a^2b - 5ab - 7ab^2) - (2a^2b - 7ab + 10ab^2) \\ (8a^2b - 5ab - 7ab^2) - (2a^2b - 7ab + 10ab^2) \\ = 8a^2b - 5ab - 7ab^2 - 2a^2b + 7ab - 10ab^2 \\ = 6a^2b + 2ab - 17ab^2 \end{aligned}$$

We certainly may encounter **function notation** and some new notation, such as, $(f \pm g)(x) = f(x) \pm g(x)$. In this case, just add or subtract the given polynomials as we did in the above examples.

C. For the given functions find $f(x) + g(x)$ and $f(x) - g(x)$.

$$\begin{aligned} f(x) &= 3x^4 + 5x^3 - 2x + 5 \\ g(x) &= -2x^4 + 3x^2 - 8 \\ f(x) + g(x) \\ &= (3x^4 + 5x^3 - 2x + 5) + (-2x^4 + 3x^2 - 8) \\ &= x^4 + 5x^3 + 3x^2 - 2x - 3 \\ f(x) - g(x) \\ &= (3x^4 + 5x^3 - 2x + 5) - (-2x^4 + 3x^2 - 8) \\ &= 3x^4 + 5x^3 - 2x + 5 + 2x^4 - 3x^2 + 8 \\ &= 5x^4 + 5x^3 - 3x^2 - 2x + 13 \end{aligned}$$

$$\begin{aligned} f(x) &= -x^2 - 2x + 7 \\ g(x) &= 4x^2 + 3x + 1 \\ f(x) + g(x) \\ &= (-x^2 - 2x + 7) + (4x^2 + 3x + 1) \\ &= 3x^2 + x + 8 \\ f(x) - g(x) \\ &= (-x^2 - 2x + 7) - (4x^2 + 3x + 1) \\ &= -x^2 - 2x + 7 - 4x^2 - 3x - 1 \\ &= -5x^2 - 5x + 6 \end{aligned}$$

Proper notation reminds us to distribute.

The distributive property and the product rule are the keys to multiplication of polynomials. Remember the product rule, $b^n \cdot b^m = b^{n+m}$, it says that when we multiply two numbers with the same base we must add the exponents.

✓ **Monomial $\times$ Polynomial**

Multiply: $2xy(3x^2y + 4xy - 5xy^2)$		
<i>Multiply the coefficients and add the exponents of the variables.</i>	$ \begin{aligned} & 2xy(3x^2y + 4xy - 5xy^2) \\ &= 3x^2y(2xy) + 4xy(2xy) - 5xy^2(2xy) \\ &= 6x^3y^2 + 8x^2y^2 - 10x^2y^3 \end{aligned} $	<i>In practice, we need not show this step.</i>

A. Multiply

$(-6x^4y^2)(3x^8y^4)$
$ \begin{aligned} & (-6x^4y^2)(3x^8y^4) \\ &= -18x^{12}y^6 \end{aligned} $

$2x(x^2 + 5)$
$ \begin{aligned} & 2x(x^2 + 5) \\ &= 2x^3 + 10x \end{aligned} $

$-3x^2(2x^2 + 4x - 7)$
$ \begin{aligned} & -3x^2(2x^2 + 4x - 7) \\ &= -6x^4 - 12x^3 + 21x^2 \end{aligned} $

$-x(4x^5 - 3x^3 + 2x - 1)$
$ \begin{aligned} & -x(4x^5 - 3x^3 + 2x - 1) \\ &= -4x^6 + 3x^4 - 2x^2 + x \end{aligned} $

$2xy^2(-5x^2 + 2xy - y^2)$
$ \begin{aligned} & 2xy^2(-5x^2 + 2xy - y^2) \\ &= -10x^3y^2 + 4x^2y^3 - 2xy^4 \end{aligned} $

A common mistake is to distribute when working with multiplication. The distributive property only applies when addition or subtraction separates the terms.

$$\begin{aligned}
 6(2+x) &= 12 + 6x && \leftarrow \text{Distribute here} \\
 6 \cdot (2x) &= 12x && \leftarrow \text{Do not Distribute}
 \end{aligned}$$

$-7(5xy^4)$
$ \begin{aligned} & -7(5xy^4) \\ &= -35xy^4 \end{aligned} $

$-(5x^2 + 5x - 1)$
$ \begin{aligned} & -(5x^2 + 5x - 1) \\ &= -5x^2 - 5x + 1 \end{aligned} $

$-3a^3b(2a^2 + 4b - 7b^4)$
$ \begin{aligned} & -3a^3b(2a^2 + 4b - 7b^4) \\ &= -6a^5b - 12a^3b^2 + 21a^3b^5 \end{aligned} $

✓ **Binomial** $\times$ **Polynomial**

Multiply: $(x-3)(x+2)$		
<i>Distribute the binomial then distribute again.</i>	$ \begin{aligned} & \overbrace{(x-3)(x+2)} \\ &= x \cdot (x-3) + 2(x-3) \quad \leftarrow \\ &= x^2 - 3x + 2x - 6 \\ &= x^2 - x - 6 \end{aligned} $	<i>In practice, we need not show this step.</i>

We can save a step by distributing each term in the binomial individually. Sometimes this technique is referred to as the FOIL method. (**FOIL** – Multiply the **F**irst, **O**uter, **I**nner then **L**ast terms together.)

B. Multiply

$(2x-5)(x+2)$	$(-3x+1)(x-1)$	$(x+5)(-x-1)$
$ \begin{aligned} & \overbrace{(2x-5)(x+2)} \\ & 2x^2 + 4x - 5x - 10 \\ & = 2x^2 - x - 10 \end{aligned} $	$ \begin{aligned} & (-3x+1)(x-1) \\ & = -3x^2 + 3x + x - 1 \\ & = -3x^2 + 4x - 1 \end{aligned} $	$ \begin{aligned} & (x+5)(-x-1) \\ & = -x^2 - x - 5x - 5 \\ & = -x^2 - 6x - 5 \end{aligned} $

When combining like terms, be sure that the variable parts are exactly the same. Go slow and work in an organized fashion because it is easy to make an error when many terms are involved. It is good practice to recheck your distributive step.

Multiply: $(x-5)(3x^2+4x-5)$		
<i>Multiply the 3 terms in the trinomial by x and then -5.</i>	$ \begin{aligned} & \overbrace{(x-5)(3x^2+4x-5)} \\ & = 3x^3 + 4x^2 - 5x - 15x^2 - 20x + 25 \quad \leftarrow \\ & = 3x^3 - 11x^2 - 25x + 25 \end{aligned} $	<i>Notice that there will be six terms after we distribute.</i>

$(x+y)(3xy^2-5xy+x^2y)$	$(2x-1)(3x^4-5x^3+2x^2+x-1)$
$ \begin{aligned} & (x+y)(3xy^2-5xy+x^2y) \\ & = 3x^2y^2-5x^2y+x^3y+3xy^3-5xy^2+x^2y^2 \\ & = 4x^2y^2-5x^2y+x^3y+3xy^3-5xy^2 \end{aligned} $	$ \begin{aligned} & (2x-1)(3x^4-5x^3+2x^2+x-1) \\ & = 6x^5-10x^4+4x^3+2x^2-2x \\ & \quad -3x^4+5x^3-2x^2-x+1 \\ & = 6x^5-13x^4+9x^3-3x+1 \end{aligned} $

$(x-2y)^3$
$\begin{aligned}(x-2y)^3 &= (x-2y)(x-2y)(x-2y) \\ &= (x-2y)(x^2-2xy-2xy+4y^2) \\ &= (x-2y)(x^2-4xy+4y^2) \\ &= x^3-4x^2y+4xy^2-2x^2y+8xy^2-8y^3 \\ &= x^3-6x^2y+12xy^2-8y^3\end{aligned}$

$(x+2)^3$
$\begin{aligned}(x+2)^3 &= (x+2)(x+2)(x+2) \\ &= (x+2)(x^2+2x+2x+4) \\ &= (x+2)(x^2+4x+4) \\ &= x^3+4x^2+4x+2x^2+8x+8 \\ &= x^3+6x^2+12x+8\end{aligned}$

Caution:
 $(x+2)^3 \neq x^3 + 2^3$

✓ Trinomial $\times$ Polynomial

Multiply: $(2x^2 - 3x + 5)(x^2 + 5x - 1)$	
Distribute each term then combine like terms.	$\begin{aligned}&(2x^2 - 3x + 5)(x^2 + 5x - 1) \\ &= 2x^4 + 10x^3 - 2x^2 - 3x^3 - 15x^2 + 3x + 5x^2 + 25x - 5 \\ &= 2x^4 + 7x^3 - 12x^2 + 28x - 5\end{aligned}$
	Note that there are nine terms here.

The following problem is one that we will have to be able to do before moving on to Intermediate Algebra. These are tedious, time consuming and often worked incorrectly. Use caution because $(2x-1)^4 \neq (2x)^4 - 1^4$. Take your time and work slowly.

Multiply: $(2x-1)^4$	
$\begin{aligned}(2x-1)^4 &= (2x-1)(2x-1)(2x-1)(2x-1) \\ &= (4x^2-2x-2x+1)(4x^2-2x-2x+1) \\ &= (4x^2-4x+1)(4x^2-4x+1) \\ &= 16x^4-16x^3+4x^2-16x^3+16x^2-4x+4x^2-4x+1 \\ &= 16x^4-32x^3+24x^2-8x+1\end{aligned}$	Use the fact that all the binomials are the same.

Function Notation for multiplication looks like $(f \cdot g)(x) = f(x) \cdot g(x)$.

C. Given the functions f and g find $f(x) \cdot g(x)$

$\begin{aligned}f(x) &= -6x - 1 \\ g(x) &= 3x^2\end{aligned}$
$\begin{aligned}f(x) \cdot g(x) &= (-6x-1)(3x^2) \\ &= -18x^3 - 3x^2\end{aligned}$

$\begin{aligned}f(x) &= x^2 - x - 3 \\ g(x) &= 9x + 1\end{aligned}$
$\begin{aligned}f(x) \cdot g(x) &= (x^2-x-3)(9x+1) \\ &= 9x^3-9x^2-27x+x^2-x-3 \\ &= 9x^3-8x^2-28x-3\end{aligned}$

$\begin{aligned}f(x) &= 5x^2 + 2x \\ g(x) &= -8x^7\end{aligned}$
$\begin{aligned}f(x) \cdot g(x) &= (5x^2+2x)(-8x^7) \\ &= -40x^9 - 16x^8\end{aligned}$

Special Products will simplify things if we memorize them. These are special because we use them often.

Multiply: $(a+b)(a-b)$	
$ \begin{aligned} &(a+b)(a-b) \\ &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2 \end{aligned} $	$\leftarrow$ Multiplication is commutative, $ab = ba$.

This special product is often called **difference of squares**. Notice that the middle terms cancel because one term will always be positive and the other will be negative.

$(a+b)(a-b) = a^2 - b^2$

Multiply: $(a+b)^2$	Multiply: $(a-b)^2$
$ \begin{aligned} &(a+b)(a+b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned} $	$ \begin{aligned} &(a-b)(a-b) \\ &= a^2 - ab - ba + b^2 \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2 \end{aligned} $

$ \begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \end{aligned} $

Tip: We may use these formulas as templates to save us a step when we identify problems as special products. It is a good idea to memorize them.



D. Multiply

<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="background-color: #e0e0e0; padding: 5px;"> $(2x-3)^2$ </th> </tr> <tr> <td style="padding: 10px;"> $\begin{aligned} &(2x-3)^2 \\ &\Rightarrow = (2x)^2 - 2(2x)(3) + (3)^2 \\ &= 4x^2 - 12x + 9 \end{aligned}$ </td> </tr> </table> Here $a = 2x$ and $b = 3$.	$(2x-3)^2$	$ \begin{aligned} &(2x-3)^2 \\ &\Rightarrow = (2x)^2 - 2(2x)(3) + (3)^2 \\ &= 4x^2 - 12x + 9 \end{aligned} $	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="background-color: #e0e0e0; padding: 5px;"> $(5x+1)(5x-1)$ </th> </tr> <tr> <td style="padding: 10px;"> $\begin{aligned} &(5x+1)(5x-1) \\ &= (5x)^2 - (1)^2 \\ &= 25x^2 - 1 \end{aligned}$ </td> </tr> </table> Here $a = 5x$ and $b = 1$.	$(5x+1)(5x-1)$	$ \begin{aligned} &(5x+1)(5x-1) \\ &= (5x)^2 - (1)^2 \\ &= 25x^2 - 1 \end{aligned} $
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$(-x+5)^2$					
$ \begin{aligned} &(-x+5)^2 \\ &= (-x)^2 + 2(-x)(5) + (5)^2 \\ &= x^2 - 10x + 25 \end{aligned} $					
$(x+7)(x-7)$					
$ \begin{aligned} &(x+7)(x-7) \\ &= (x)^2 - (7)^2 \\ &= x^2 - 49 \end{aligned} $					

Problems Solved! Chapter 5 *Polynomial Operations*
Dividing Polynomials

In this section we will learn how to divide polynomials. Students find this to be one of the more difficult topics in Elementary Algebra. Plan on spending some extra time reviewing the techniques and solutions presented here. (*Assume all variables that appear in a denominator are nonzero.*)

✓ **Dividing by a Monomial**

When dividing we will be using the quotient rule, $\frac{b^n}{b^m} = b^{n-m}$, which says that when dividing two numbers with the same base we must subtract exponents.

Divide: $(25x^4y^2 - 15x^3y^3 + 10x^2y^2) \div 5y^2x^2$	
First, break up the fraction using the common denominator.	$\Rightarrow \frac{25x^4y^2 - 15x^3y^3 + 10y^2x^2}{5x^2y^2}$ $= \frac{25x^4y^2}{5x^2y^2} - \frac{15x^3y^3}{5x^2y^2} + \frac{10y^2x^2}{5x^2y^2}$ $= 5x^2 - 3xy + 2$

Then cancel each term.

In fact, we are using the property for adding fractions with a common denominator in reverse, $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$. By breaking up the fraction we could then simply cancel.

A. Divide

$\frac{42x^8y^5z^{11}}{6x^3yz^5}$	$\frac{27x^3 - 30x^2 + 5x}{-5x}$	$\frac{8x^4 + 2x^3 - 16x^2 + 32x}{2x}$
$\frac{42x^8y^5z^{11}}{6x^3yz^5}$ $= 7x^{8-3}y^{5-1}z^{11-5}$ $= 7x^5y^4z^6$	$\frac{27x^3 - 30x^2 + 5x}{-5x}$ $= \frac{27x^3}{-5x} - \frac{30x^2}{-5x} + \frac{5x}{-5x}$ $= -\frac{27}{5}x^2 + 6x - 1$	$\frac{8x^4 + 2x^3 - 16x^2 + 32x}{2x}$ $= \frac{8x^4}{2x} + \frac{2x^3}{2x} - \frac{16x^2}{2x} + \frac{32x}{2x}$ $= 4x^3 + x^2 - 8x + 16$

A common mistake would be to cancel denominator with only one of the terms. We are dividing the entire expression in the numerator so every term must be cancelled with the denominator.

$$\frac{-3a^{11}b^7 + 9a^8b^6 - a^4b^2 + a^2b^2}{-a^2b^2}$$

$$= \frac{-3a^{11}b^7}{-a^2b^2} + \frac{9a^8b^6}{-a^2b^2} - \frac{a^4b^2}{-a^2b^2} + \frac{a^2b^2}{-a^2b^2}$$

$$= 3a^9b^5 - 9a^6b^4 + a^2 - 1$$

$$\frac{16m^4n^3 - 12m^6n^6 + 40m^2n^4 - 20mn^2}{4mn^2}$$

$$= \frac{16m^4n^3}{4mn^2} - \frac{12m^6n^6}{4mn^2} + \frac{40m^2n^4}{4mn^2} - \frac{20mn^2}{4mn^2}$$

$$= 4m^3n - 3m^5n^4 + 10mn^2 - 5$$

✓ Dividing by a Polynomial

Dividing by a non-monomial requires us to use **polynomial long division**. The good news is that the steps are basically the same as the regular long division algorithm we are already used to. We need long division to divide $(x^3 + 3x^2 - 8x - 4) \div (x - 2)$.

$$x-2 \overline{) \overset{?}{x^3} + 3x^2 - 8x - 4}$$

Step 1: Match the leading term.
 $(?)(x) = x^3 \dots$ the answer here is x^2

Remember that you want to match the leading term exactly, later we will subtract.

$$x-2 \overline{) \overset{x^2}{x^3} + 3x^2 - 8x - 4}$$

$$x-2 \overline{) \overset{x^2}{x^3} + 3x^2 - 8x - 4}$$

$$\underline{x^3 - 2x^2}$$

Step 2: Distribute.
 $x^2(x-2) = x^3 - 2x^2$

Step 3: Subtract.
 Remember to distribute the negative which changes the signs.

$$x-2 \overline{) \overset{x^2}{x^3} + 3x^2 - 8x - 4}$$

$$\underline{-(x^3 - 2x^2)} \quad \downarrow$$

$$5x^2 - 8x - 4$$

Step 4: Repeat.
 Start at step 1 and match the leading term again.

Divide: $(x^3 + 3x^2 - 8x - 4) \div (x - 2)$

$$x-2 \overline{) \overset{x^2+5x+2}{x^3} + 3x^2 - 8x - 4}$$

$$\underline{-(x^3 - 2x^2)}$$

$$5x^2 - 8x - 4$$

$$\underline{-(5x^2 - 10x)}$$

$$2x - 4$$

$$\underline{-(2x - 4)}$$

$$0$$

This is the answer.

The remainder is zero.

Division does not always work out so evenly, sometimes we will have a **remainder**.

When left with a remainder, simply add to the answer a fraction with the remainder as the numerator and divisor as the denominator.	Divide: $\frac{3x^3 - 13x^2 + 7x + 1}{3x - 1}$ $ \begin{array}{r} x^2 - 4x + 1 \\ 3x-1 \overline{) 3x^3 - 13x^2 + 7x + 1} \\ \underline{-(3x^3 - x^2)} \\ -12x^2 + 7x + 1 \\ \underline{-(-12x^2 + 4x)} \\ 3x + 1 \\ \underline{-(3x - 1)} \\ 2 \end{array} $ ← Remainder Ans: $= x^2 - 4x + 1 + \frac{2}{3x-1}$
--	---

B. Divide

$\frac{-10x^2 + 19x + 15}{5x + 3}$
$ \begin{array}{r} -2x + 5 \\ 5x+3 \overline{) -10x^2 + 19x + 15} \\ \underline{-(-10x^2 - 6x)} \\ 25x + 15 \\ \underline{-(25x + 15)} \\ 0 \end{array} $ Ans: $= -2x + 5$

$\frac{-4x^2 + 19x - 26}{x - 3}$
$ \begin{array}{r} -4x + 7 \\ x-3 \overline{) -4x^2 + 19x - 26} \\ \underline{-(-4x^2 + 12x)} \\ 7x - 26 \\ \underline{-(7x - 21)} \\ -5 \end{array} $ Ans: $= -4x + 7 - \frac{5}{x-3}$

$\frac{2x^4 - 7x^3 + 7x^2 - 5x + 6}{x - 2}$
$ \begin{array}{r} 2x^3 - 3x^2 + x - 3 \\ x-2 \overline{) 2x^4 - 7x^3 + 7x^2 - 5x + 6} \\ \underline{-(2x^4 - 4x^3)} \\ -3x^3 + 7x^2 - 5x + 6 \\ \underline{-(-3x^3 + 6x^2)} \\ x^2 - 5x + 6 \\ \underline{-(x^2 - 2x)} \\ -3x + 6 \\ \underline{-(-3x + 6)} \\ 0 \end{array} $ Ans: $= 2x^3 - 3x^2 + x - 3$

$\frac{5x^4 - 11x^3 + 7x^2 - 16x + 5}{5x - 1}$
$ \begin{array}{r} x^3 - 2x^2 + x - 3 \\ 5x-1 \overline{) 5x^4 - 11x^3 + 7x^2 - 16x + 5} \\ \underline{-(5x^4 - x^3)} \\ -10x^3 + 7x^2 - 16x + 5 \\ \underline{-(-10x^3 + 2x^2)} \\ 5x^2 - 16x + 5 \\ \underline{-(5x^2 - x)} \\ -15x + 5 \\ \underline{-(-15x + 3)} \\ 2 \end{array} $ Ans: $= x^3 - 2x^2 + x - 3 + \frac{2}{5x-1}$

Some of the polynomials that we will be dividing will be missing terms. In other words, not all the exponents will be there. When first learning, it really is best to use placeholders and include the **missing terms** using zero as coefficients.

Divide: $\frac{2x^3 - 10}{x - 2}$	
$ \begin{array}{r} 2x^2 + 4x + 8 \\ x-2 \overline{) 2x^3 + 0x^2 + 0x - 10} \\ \underline{-(2x^3 - 4x^2)} \\ 4x^2 + 0x - 10 \\ \underline{-(4x^2 - 8x)} \\ 8x - 10 \\ \underline{-(8x - 16)} \\ 6 \end{array} $	Use placeholders.
Ans: $2x^2 + 4x + 8 + \frac{6}{x-2}$	

C. Divide

$\frac{x^3 - 27}{x - 3}$
$ \begin{array}{r} x^2 + 3x + 9 \\ x-3 \overline{) x^3 + 0x^2 + 0x - 27} \\ \underline{-(x^3 - 3x^2)} \\ 3x^2 + 0x - 27 \\ \underline{-(3x^2 - 9x)} \\ 9x - 27 \\ \underline{-(9x - 27)} \\ 0 \end{array} $
Answer: $x^2 + 3x + 9$

$\frac{x^3 - 5x - 3}{x - 2}$
$ \begin{array}{r} x^2 + 2x - 1 \\ x-2 \overline{) x^3 + 0x^2 - 5x - 3} \\ \underline{-(x^3 - 2x^2)} \\ 2x^2 - 5x - 3 \\ \underline{-(2x^2 - 4x)} \\ -x - 3 \\ \underline{-(-x + 2)} \\ -5 \end{array} $
Ans: $x^2 + 2x - 1 - \frac{5}{x-2}$

$\frac{x^4 - 10,000}{x - 10}$
$ \begin{array}{r} x^3 + 10x^2 + 100x + 1,000 \\ x-10 \overline{) x^4 + 0x^3 + 0x^2 + 0x - 10,000} \\ \underline{-(x^4 - 10x^3)} \\ 10x^3 + 0x^2 + 0x - 10,000 \\ \underline{-(10x^3 - 100x^2)} \\ 100x^2 + 0x - 10,000 \\ \underline{-(100x^2 - 1,000x)} \\ 1,000x - 10,000 \\ \underline{-(1,000x - 10,000)} \\ 0 \end{array} $
Ans: $x^3 + 10x^2 + 100x + 1,000$

$\frac{2x^4 + 1 - 2x - x^3}{2x - 1}$
$ \begin{array}{r} x^3 - 1 \\ 2x-1 \overline{) 2x^4 - x^3 + 0x^2 - 2x + 1} \\ \underline{-(2x^4 - x^3)} \\ 0 \downarrow \\ -2x + 1 \\ \underline{-(-2x + 1)} \\ 0 \end{array} $
Ans: $x^3 - 1$

Polynomial long division takes some practice to master. Be patient, do lots of problems and soon you will find them to be enjoyable.

D. Divide

$$\begin{array}{r} 4x^5 - x^4 - 4x^3 + x^2 + 12x - 3 \\ \underline{4x - 1} \\ x^4 - x^2 + 3 \\ 4x - 1 \overline{) 4x^5 - x^4 - 4x^3 + x^2 + 12x - 3} \\ \underline{-(4x^5 - x^4)} \\ 0 - 4x^3 + x^2 + 12x - 3 \\ \underline{-(-4x^3 + x^2)} \\ 0 + 12x - 3 \\ \underline{(12x - 3)} \\ 0 \end{array}$$

$$\begin{array}{r} 2x^4 - 5x^3 + 7x^2 - 7x + 5 \\ \underline{2x^2 - x + 3} \\ 2x^4 - x^3 + 3x^2 - 7x + 5 \\ \underline{-(2x^4 - x^3 + 3x^2)} \\ -4x^3 + 4x^2 - 7x + 5 \\ \underline{-(-4x^3 + 2x^2 - 6x)} \\ 2x^2 - x + 5 \\ \underline{-(2x^2 - x + 3)} \\ 2 \end{array}$$

Ans: $x^2 - 2x + 1 + \frac{2}{2x^2 - x + 3}$

$$\begin{array}{r} x^3 - 4x^2y + 7xy^2 - 4y^3 \\ \hline x - y \end{array}$$

$$\begin{array}{r} x^2 - 3xy + 4y^2 \\ x - y \overline{) x^3 - 4x^2y + 7xy^2 - 4y^3} \\ \underline{-(x^3 - x^2y)} \\ -3x^2y + 7xy^2 - 4y^3 \\ \underline{-(-3x^2y + 3xy^2)} \\ 4xy^2 - 4y^3 \\ \underline{-(4xy^2 - 4y^3)} \\ 0 \end{array}$$

$$\frac{a^4 + 4a^3b + a^2b^2 + 3a^2b + 12ab^2 + 3b^3}{a^2 + 3b}$$

$$\begin{array}{r} \frac{-7a^4 + 4a^3 - a^2 - 4a + 8}{a^2 - 1} \\ a^2 - 1 \overline{) -7a^4 + 4a^3 - a^2 - 4a + 8} \\ \underline{-(-7a^4 + 7a^2)} \\ 4a^3 - 8a^2 - 4a + 8 \\ \underline{-(4a^3 - 4a)} \\ -8a^2 + 8 \\ \underline{-(-8a^2 + 8)} \\ 0 \end{array}$$

To check your answers multiply the quotient by the divisor to see that we get the dividend.

$$\begin{array}{r} x \\ \overline{a^2 - 1 \over -7a^2 + 4a - 8} \\ \underline{7a^2 - 4a + 8} \\ -7a^4 + 4a^3 - a^2 - 4a + 8 \end{array}$$

check: $(a^2 - 1)(-7a^2 + 4a - 8)$
 $= -7a^4 + 4a^3 - 8a^2 + 7a^2 - 4a + 8$
 $= -7a^4 + 4a^3 - a^2 - 4a + 8 \quad \checkmark$

Problems Solved! Chapter 5 Polynomial Operations
Evaluating Expressions

Evaluating expressions involves replacing the variable with the appropriate numerical value. In other words, plug in the values and use the order of operations to calculate the answer.

Evaluate: $2xy(3x^2y + 4xy - 5xy^2)$ when $x = -1$ and $y = 2$	
Avoid order of operation errors by placing a blank parenthesis wherever we see a variable then plug in.	$2xy(3x^2y + 4xy - 5xy^2)$ $2(x)(3(\quad)^2(\quad) + 4(\quad)(\quad) - 5(\quad)(\quad)^2) \leftarrow$ $2(-1)(2)(3(-1)^2(2) + 4(-1)(2) - 5(-1)(2)^2)$ $= -4(6 - 8 + 20)$ $= -4(18)$ $= -72$
	We do not need to show this step.

A. Evaluate

$3x^2 + 2x - 7$ when $x = -3$	$x^2y^2 - 2xy + 4$ when $x = 2$ and $y = -1$	$x^2 - y^2$ when $x = -1$ and $y = -2$
$3x^2 + 2x - 7$ $3(-3)^2 + 2(-3) - 7$ $= 3 \cdot 9 - 6 - 7$ $= 27 - 6 - 7$ $= 14$	$x^2y^2 - 2xy + 4$ $(2)^2(-1)^2 - 2(2)(-1) + 4$ $= 4 + 4 + 4$ $= 12$	$x^2 - y^2$ $(-1)^2 - (-2)^2$ $= 1 - 4$ $= -3$
$\frac{-4x^2 + 2x - 1}{x - 1}$ when $x = 3$	$\frac{18a^3bc^2}{-2abc}$ when $a = 1$, $b = -2$ and $c = -3$	$\frac{x^2 - 100}{x - 10}$ when $x = 10$
$\frac{-4x^2 + 2x - 1}{x - 1}$ $= \frac{-4(3)^2 + 2(3) - 1}{(3) - 1}$ $= \frac{-36 + 6 - 1}{2} = \frac{-31}{2}$	$\frac{18a^3bc^2}{-2abc} = -9a^2c$ $-9(1)^2(-3)$ $= 27$	$\frac{x^2 - 100}{x - 10}$ $\frac{(10)^2 - 100}{10 - 10}$ $= \frac{100 - 100}{10 - 10} = \frac{0}{0}$ undefined.

Tip: Simplifying expressions first saves steps when evaluating and the results will be the same.



$-x(4x^5 - 3x^3 + 2x - 1)$ when $x = 0$	$2xy^2(-5x^2 + 2xy - y^2)$ when $x = 4$ and $y = 0$
$\begin{aligned} & -x(4x^5 - 3x^3 + 2x - 1) \\ & -(0)(4(0)^5 - 3(0)^3 + 2(0) - 1) \\ & = 0(-1) \\ & = 0 \end{aligned}$	$\begin{aligned} & 2xy^2(-5x^2 + 2xy - y^2) \\ & 2(4)(0)^2(-5(4)^2 + 2(4)(0) - (0)^2) \\ & = 0(-80) \\ & = 0 \end{aligned}$

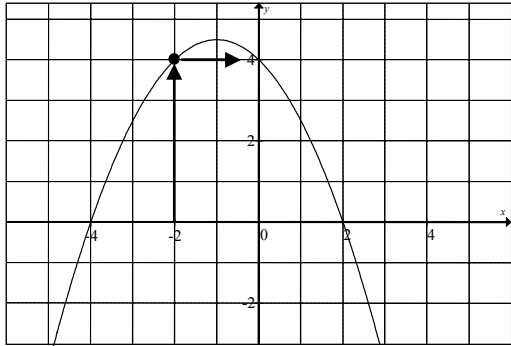
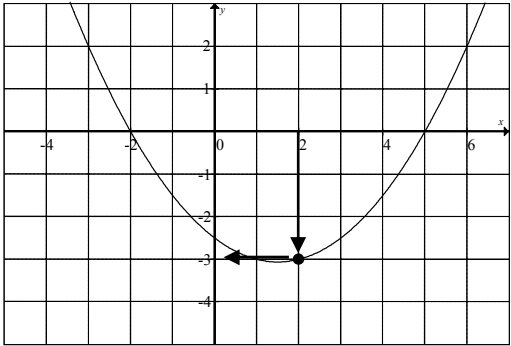
We can use function notation to evaluate. Do not let the notation get in the way of your ability to solve these. The idea is the same, just plug in the appropriate values.

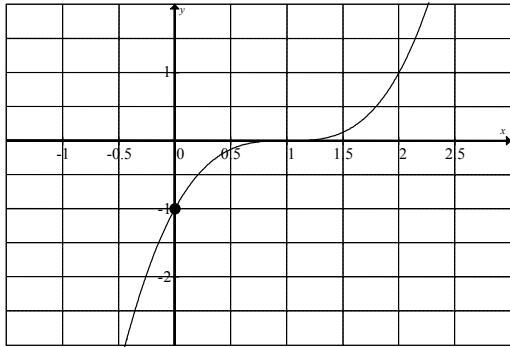
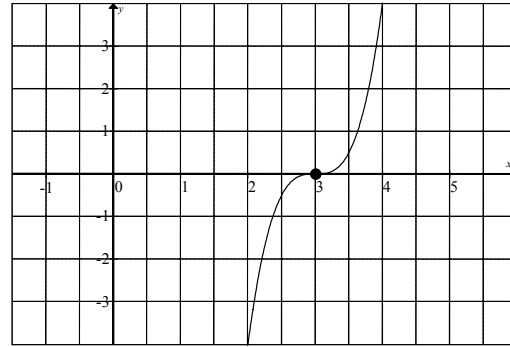
B. Evaluate

$f(x) = \frac{x^4 - x^2 + 7}{x^2 + 1}$ find $f(-2)$	$g(x) = \frac{(x-h)^2 - 5}{x-h}$ find $g(-1)$	$h(x) = 4x^4 - 10x^2 - 8$ find $h(0)$
$\begin{aligned} f(x) &= \frac{x^4 - x^2 + 7}{x^2 + 1} \\ f(-2) &= \frac{(-2)^4 - (-2)^2 + 7}{(-2)^2 + 1} \\ &= \frac{16 - 4 + 7}{4 + 1} \\ &= \frac{19}{5} \end{aligned}$	$\begin{aligned} g(x) &= \frac{(x-h)^2 - 5}{x-h} \\ g(-1) &= \frac{(-1-h)^2 - 5}{-1-h} \\ &= \frac{1 + 2h + h^2 - 5}{-1-h} \\ &= \frac{h^2 + 2h - 4}{-1-h} \end{aligned}$	$\begin{aligned} h(x) &= 4x^4 - 10x^2 - 8 \\ h(0) &= 4(0)^4 - 10(0)^2 - 8 \\ &= 0 - 0 - 8 \\ &= -8 \end{aligned}$

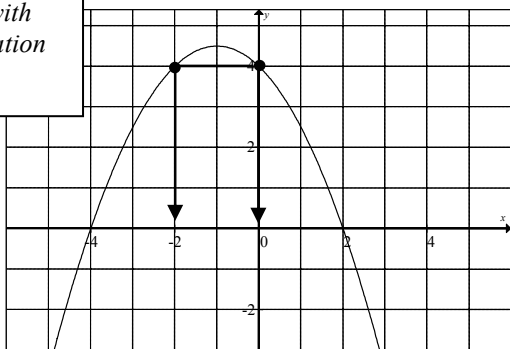
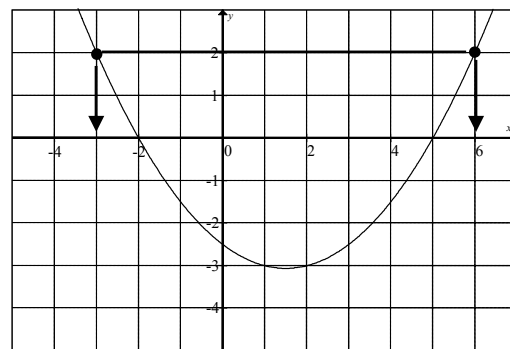
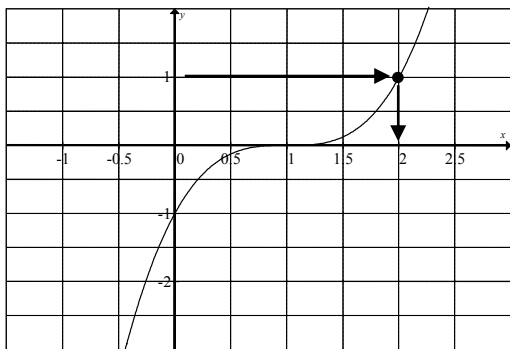
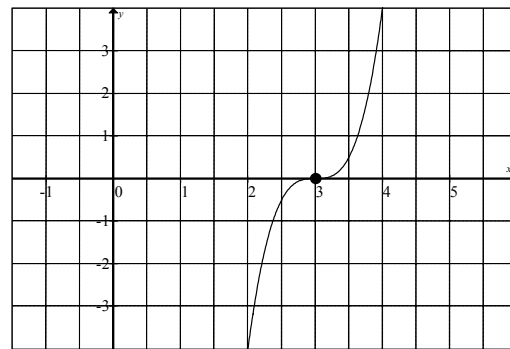
Often we will run into problems where the graph is given instead of the algebraic equation. In this case we must read the graph.

C. Given the graph of the function

Find $f(-2)$  Answer: $f(-2) = 4$	Find $f(2)$  Answer: $f(2) = -3$
--	--

Find $f(0)$ Answer: $f(0) = -1$ Find $f(3)$ Answer: $f(3) = 0$ **D. Given the graph of the function**Find the x -values where $f(x) = 4$

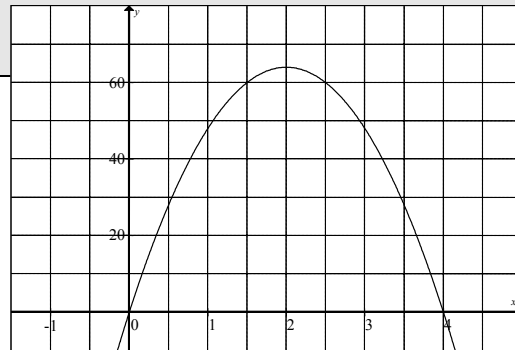
Recall that with
function notation
 $y = f(x)$.

Answers: $x = -2$ and $x = 0$ Find the x -values where $f(x) = 2$ Answers: $x = -3$ and $x = 6$ Find the x -value where $f(x) = 1$ Answer: $x = 2$ Find the x -value where $f(x) = 0$ Answer: $x = 3$

E. Typical Word Problems

A projectile is fired from the ground with an initial velocity of 64 feet per second. The height of the projectile in feet after t seconds is given by the function $h(t) = -16t^2 + 64t$ with the following graph.

- Use the graph to determine how long it takes to reach the maximum height.
- How long will it take to hit the ground?
- What times will the projectile be at 60ft?
- Use the function $h(t) = -16t^2 + 64t$ to determine the height of the projectile at $t = 1$ sec.



- $t = 2 \text{ sec.}$
- $t = 4 \text{ sec.}$
- $t = 1.5 \text{ sec}$ and $t = 2.5 \text{ sec.}$
- $$h(t) = -16t^2 + 64t$$

$$h(1) = -16(1)^2 + 64(1)$$

$$= -16 + 64$$

$$= 48$$

At $t = 1 \text{ sec.}$ the height is 48 ft.

The function $v(r) = \frac{4}{3}\pi r^3$ gives the volume of a sphere given its radius r . Use the function to calculate the volume of a sphere of radius 6 centimeters. Use $\pi = 3.14$ as an approximation for pi.

$$v(r) = \frac{4}{3}\pi r^3$$

$$v(6) \approx \frac{4}{3}(3.14)(6)^3$$

$$= \frac{4(3.14)(216)}{3}$$

$$= 904.32$$

The volume is about 904.32 cm^3

A projectile is fired upwards from the roof of a 256 ft building with an initial velocity of 96 ft per second. The height of the projectile in feet after t seconds is given by

$$h(t) = -16t^2 + 96t + 256.$$

- How long does it take to a maximum height?
- What is the maximum height?
- How long does it take for the projectile to hit the ground?
- Use the function to determine the height after 6 seconds.

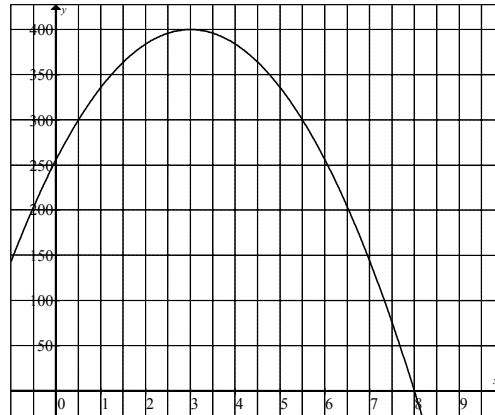
a. $t = 3 \text{ sec}$

b. $h(3) = 400 \text{ ft}$

c. $t = 8 \text{ sec}$

d. $h(t) = -16t^2 + 96t + 256$

$$\begin{aligned} h(6) &= -16(6)^2 + 96(6) + 256 \\ &= -576 + 576 + 256 \\ &= 256 \text{ ft} \end{aligned}$$



The height of a box is 2 less than twice its length, and the length is 2 more than its width. Express the volume in terms of length.

Let w = the width of the box.

$l = w + 2$ the length of the box $\Rightarrow w = l - 2$

$h = 2l - 2$ the height of the box

$$\text{Volume} = l \cdot w \cdot h$$

$$= l(l-2)(2l-2)$$

$$= (l^2 - 2l)(2l - 2)$$

$$= 2l^3 - 2l^2 - 4l^2 + 4l$$

$$= 2l^3 - 6l^2 + 4l \quad \checkmark$$

Chapter 5 _ Polynomial Operations

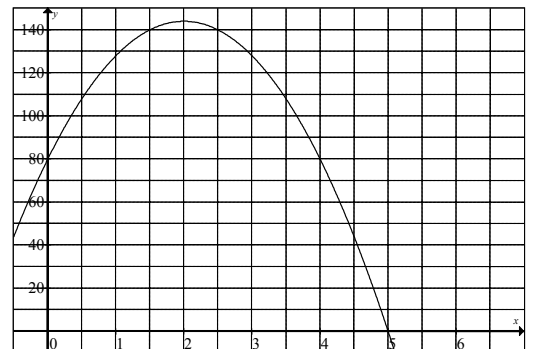
Sample Exam

Please answer all the questions and show work where appropriate.

1. Simplify: $\left(\frac{2x^3}{3y^2}\right)^3$
2. Simplify: $(5xy^{-3}z^4)^{-3}$
3. Given $f(x) = -3x^2 + 2x - 9$ find $f(-2)$.
4. Add: $(3x^2 + 2x - 9) + (-5x^2 - 7x + 7)$
5. Subtract: $(-5x^2 - 3x + 4) - (2x^2 - 3x - 5)$
6. Multiply: $-2x^2(3x^3 + 2x^2 - 5x + 1)$
7. Multiply: $(2x - 3)^3$
8. Divide: $\frac{20x^6 + 18x^5 - 4x^3}{2x^3}$
9. Divide: $\frac{3x^4 + 4x^3 - 13x^2 + 9x + 3}{3x - 2}$

10. A projectile is fired from the top of an 80 ft building with an initial velocity of 64 feet per second. The height of the projectile, in feet, after t seconds is given by the function $h(t) = -16t^2 + 64t + 80$ graphed here.

- a. Use the graph to determine how long it takes to reach the maximum height.
- b. How long will it take to hit the ground?
- c. What is the height of the projectile after 4 seconds?



Chapter 5 *Polynomial Operations*
Sample Exam Answers

1. $\frac{8x^9}{27y^6}$

2. $\frac{y^9}{125x^3z^{12}}$

3. -25

4. $-2x^2 - 5x - 2$

5. $-7x^2 + 9$

6. $-6x^5 - 4x^4 + 10x^3 - 2x^2$

7. $8x^3 - 36x^2 + 54x - 27$

8. $10x^3 + 9x^2 - 2$

9. $x^3 + 2x^2 - 3x + 1 + \frac{5}{3x - 2}$

10. a. $t = 2$ seconds
b. $t = 5$ seconds
c. $h(4) = 80$ feet

Simplify: $\left(\frac{2x^3}{3y^2}\right)^3$

$$\begin{aligned}\left(\frac{2x^3}{3y^2}\right)^3 &= \frac{2^3 x^9}{3^3 y^6} \\ &= \frac{8x^9}{27y^6}\end{aligned}$$

Simplify: $(5xy^{-3}z^4)^{-3}$

$$\begin{aligned}(5xy^{-3}z^4)^{-3} &= 5^{-3} x^{-3} y^9 z^{-12} \\ &= \frac{y^9}{5^3 x^3 z^{12}} \\ &= \frac{y^9}{125x^3z^{12}}\end{aligned}$$

Given $f(x) = -3x^2 + 2x - 9$
find $f(-2)$.

$$\begin{aligned}f(x) &= -3x^2 + 2x - 9 \\ f(-2) &= -3(-2)^2 + 2(-2) - 9 \\ &= -3 \cdot 4 - 4 - 9 \\ &= -12 - 4 - 9 \\ &= -25\end{aligned}$$

Add: $(3x^2 + 2x - 9) + (-5x^2 - 7x + 7)$

$$\begin{aligned}(3x^2 + 2x - 9) + (-5x^2 - 7x + 7) \\ = -2x^2 - 5x - 2\end{aligned}$$

Subtract: $(-5x^2 - 3x + 4) - (2x^2 - 3x - 5)$

$$\begin{aligned}(-5x^2 - 3x + 4) - (2x^2 - 3x - 5) \\ = -5x^2 - 3x + 4 - 2x^2 + 3x + 5 \\ = -7x^2 + 9\end{aligned}$$

Multiply: $-2x^2(3x^3 + 2x^2 - 5x + 1)$

$$\begin{aligned}-2x^2(3x^3 + 2x^2 - 5x + 1) \\ = -6x^5 - 4x^4 + 10x^3 - 2x^2\end{aligned}$$

Multiply: $(2x - 3)^3$

$$\begin{aligned}(2x - 3)^3 &= (2x - 3)(2x - 3)(2x - 3) \\ &= (4x^2 - 6x - 6x + 9)(2x - 3) \\ &= (4x^2 - 12x + 9)(2x - 3) \\ &= 8x^3 - 12x^2 - 24x^2 + 36x + 18x - 27 \\ &= 8x^3 - 36x^2 + 54x - 27\end{aligned}$$

Divide: $\frac{20x^6 + 18x^5 - 4x^3}{2x^3}$

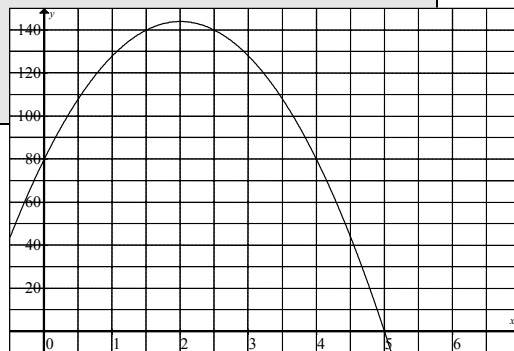
$$\begin{aligned}\frac{20x^6 + 18x^5 - 4x^3}{2x^3} \\ = \frac{20x^6}{2x^3} + \frac{18x^5}{2x^3} - \frac{4x^3}{2x^3} \\ = 10x^3 + 9x^2 - 2\end{aligned}$$

Divide: $\frac{3x^4 + 4x^3 - 13x^2 + 9x + 3}{3x - 2}$

$$\begin{array}{r}x^3 + 2x^2 - 3x + 1 + \frac{5}{3x-2} \\ 3x-2 \overline{) 3x^4 + 4x^3 - 13x^2 + 9x + 3} \\ \underline{-(3x^4 - 2x^3)} \\ 6x^3 - 13x^2 \\ \underline{-(6x^3 - 4x^2)} \\ -9x^2 + 9x \\ \underline{-(-9x^2 + 6x)} \\ 3x + 3 \\ \underline{-(3x - 2)} \\ 5\end{array}$$

A projectile is fired from the top of an 80 ft building with an initial velocity of 64 feet per second. The height of the projectile in feet after t seconds is given by the function $h(t) = -16t^2 + 64t + 80$ with the following graph.

- Use the graph to determine how long it takes to reach the maximum height.
- How long will it take to hit the ground?
- What is the height of the projectile after 4 seconds?



A. $t = 2 \text{ sec}$

B. $t = 5 \text{ sec}$

C. $h(t) = -16t^2 + 64t + 80$

$$h(4) = -16(4)^2 + 64(4) + 80$$

$$= -16 \cdot 16 + 256 + 80$$

$$= -256 + 256 + 80$$

$$= 80 \text{ ft}$$

The next step up from linear equations is quadratic equations. This entire chapter leads to solving quadratic equations, $ax^2 + bx + c = 0$, and their applications. But before we can do this we must learn how to factor. We have spent much time multiplying polynomials using the distributive property, now we will need to learn how to undo this operation or factor. The factoring step will be the key step in many of the techniques that we will learn in our study of Algebra.

Factoring and Quadratic Equations*GCF and Factoring by Grouping ...6.1**Factoring Trinomials ...6.2**Factoring Binomials ...6.3**General Factoring ...6.4**Solving by Factoring ...6.5**Solving with the Quadratic Formula ...6.6**Word Problems ...6.7**Sample Exam**Sample Exam Solutions*

Factoring is one of the more important skills that we will learn in Beginning Algebra. The idea is to work the distributive property in reverse. The goal is to write polynomials as products of simpler polynomials.

GCF – The **Greatest Common Factor** of a polynomial must be a factor of each term.

To decide what the GCF is, look for the largest factor that divides into all the terms. In other words, what common term will divide into all the given terms evenly?

Find the GCF: $27x^2y^3z^2$, $3x^4y^2z$, $9x^3y$	
Answer: $3x^2y$	Notice that 3 is the GCF of the coefficients.

In the above example, the variable z is not common to all the terms so it is not included in the GCF. Use the smallest exponent for the common variables x and y .

A. Identify the GCF

$10x^2y^4$, $25x^3y^7$, $5xy^5$	$49ab^2$, $7ab$, a^2b^2	$4x(x-5)^3$, $8x^3(x-5)^5$
Answer: $5xy^4$	Answer: ab	Answer: $4x(x-5)^3$

The GCF will be used when we are asked to **factor**. Once we identify the GCF of all the terms in a polynomial we will write a blank parenthesis, then after deciding what will be left by dividing we will fill in the blanks.

Factor: $6x^4y^2 + 9x^2y^3 - 3xy^4$		Check by using the distributive property to multiply the polynomials. check: $3xy^2(2x^3 + 3xy - y^2)$ $= 6x^4y^2 + 9x^2y^3 - 3xy^4$
Identify the GCF	$6x^4y^2 + 9x^2y^3 - 3xy^4$ $\Rightarrow = 3xy^2(\quad)$ $= 3xy^2(2x^3 + 3xy - y^2) \leftarrow$	
	Divide each term	

The check is not really necessary but it is good practice to at least check the factoring mentally.

B. Factor the GCF out of the expression.

$7x^2 - 14$
$= 7(x^2 - 2)$

$5x^2 - 25x - 5$
$= 5(x^2 - 5x - 1)$

$4x^2y^2 + 6x^2y + 8xy^2 - 2xy$
$= 2xy(2xy + 3x + 4y - 1)$

Tip: Be careful to use a 1 when the entire term factors out. A common error is to leave it blank. This is where a mental check is important. Be sure that if we were to distribute we would get back to the original expression.



$-5x^5 - 15x^3 - 5x^2$
$= -5x^2(x^3 + 3x + 1)$

$-49a^2b + 7b^3$
$= -7b(7a^2 - b^2)$

$-27m^3n^6 - 9m^3n^9$
$= -9m^3n^6(3 + n^3)$

$5x(4x - 1) - (4x - 1)$
$= (4x - 1)(5x - 1)$

$3x(x + 2) - 5(x + 2)$
$= (x + 2)(3x - 5)$

$12xy(3x - 4)^3 - 4x^2y^3(3x - 4)^4$
$= 4xy(3x - 4)^3[3 - xy^2(3x - 4)]$

All of the above problems require only one step. The hard part is to identify the GCF. Here is the check for the last solved problem.

$$\begin{aligned}
 & \text{Handwritten check: } 4xy(3x-4)^3[3 - xy^2(3x-4)] \\
 & = 12xy(3x-4)^3 - 4x^2y^3(3x-4)^4
 \end{aligned}$$

✓ Factoring 4 Term Polynomials

Now we will use the idea of factoring out the GCF in a technique called **factoring by grouping** for four term polynomials. The steps are as follows.

Step 1: Group the first and second two terms then identify and factor out the GCF of each grouping.	Factor: $6x^2 + 9x - 8xy - 12y$ $\Rightarrow \underline{6x^2 + 9x} - \underline{8xy - 12y}$ $= 3x(\quad) - 4y(\quad)$ $= 3x(2x + 3) - 4y(2x + 3) \leftarrow$ $= (2x + 3)(\quad)$ $= (2x + 3)(3x - 4y)$
---	--

Check: $(2x+3)(3x-4y)$

$= 6x^2 - 8xy + 9x - 12y$

$= 6x^2 + 9x - 8xy - 12y$

Step 2: If the binomials are the same, factor it out as a GCF.

C. Factor by grouping.

$2x^2 + 6x - x - 3$ $2x^2 + 6x - x - 3$ $= 2x(x+3) - (x+3)$ $= (x+3)(2x-1)$
--

$10x^2 + 5x + 4x + 2$ $10x^2 + 5x + 4x + 2$ $= 5x(2x+1) + 2(2x+1)$ $= (2x+1)(5x+2)$
--

$2a^2 - 2ab - ab^2 + b^3$ $2a^2 - 2ab - ab^2 + b^3$ $= 2a(a-b) - b^2(a-b)$ $= (a-b)(2a-b^2)$

$x^2 - x + x - 1$ $x^2 - x + x - 1$ $= x(x-1) + (x-1)$ $= (x-1)(x+1)$
--

$x^2 - xy + 2xy - 2y^2$ $x^2 - xy + 2xy - 2y^2$ $= x(x-y) + 2y(x-y)$ $= (x-y)(x+2y)$

$3a^2 + ab^2 - 3ab - b^3$ $3a^2 + ab^2 - 3ab - b^3$ $= a(3a+b^2) - b(3a+b^2)$ $= (3a+b^2)(a-b)$
--

Sometimes we will encounter 4 term polynomials where factoring by grouping does not seem to work because the terms are in a different order. When factoring by grouping and the binomials are not exactly the same try again with the terms in a different order.

For example, try to factor $ab - 2a^2b + a^3 - 2b^3$ by grouping.

$$\begin{aligned}
 & ab - 2a^2b + a^3 - 2b^3 \quad \longleftrightarrow \quad a^3 - 2a^2b + ab - 2b^3 \quad \longleftarrow \\
 \Rightarrow & = ab(1-2a) + (a^3-2b^2) \quad = a^2(a-2b) + b(a-2b) \\
 & \quad \quad \quad = (a-2b)(a^2+b)
 \end{aligned}$$

Rearrange the terms and we can factor this polynomial.

Dead end, the binomials are not the same so there is no GCF.

$x^3 + y^3 - xy^2 - x^2y$ $x^3 + y^3 - xy^2 - x^2y$ $= \underline{x^3 - xy^2} - \underline{x^2y + y^3}$ $= x(x^2 - y^2) - y(x^2 - y^2)$ $= (x^2 - y^2)(x - y)$
--

$m^3 - n^3 + m^3n^3 - 1$ $m^3 - n^3 + m^3n^3 - 1$ $= m^3n^3 - n^3 + m^3 - 1$ $= n^3(m^3 - 1) + (m^3 - 1)$ $= (m^3 - 1)(n^3 + 1)$
--

$16x^3y^3 + 16 - 8y^3 - 32x^3$ $16x^3y^3 + 16 - 8y^3 - 32x^3$ $= 16x^3y^3 - 8y^3 - 32x^3 + 16$ $= 8y^3(2x^3 - 1) - 16(2x^3 - 1)$ $= (2x^3 - 1)(8y^3 - 16)$
--

Problems Solved! Chapter 6 *Factoring and Quadratic Equations*
Factoring Trinomials

In this section we will factor trinomials, or polynomials with three terms. Students find this difficult at first but after a while, and with some practice, we will see that factoring these will become routine. If a trinomial factors, then it will factor into two binomials.

Factor: $x^2 + 7x + 12$		
Step 1: Factor the first term first ($x^2 = x \cdot x$).	$ \begin{aligned} &x^2 + 7x + 12 \quad 12 = 1 \cdot 12 \\ \Rightarrow &= (x \quad)(x \quad) \quad = 2 \cdot 6 \\ &\quad \quad \quad = 3 \cdot 4 \\ &\Rightarrow = (x \quad 3)(x \quad 4) \\ &\quad \quad \quad \begin{array}{c} +3x \\ +4x \\ \hline +7x \end{array} \\ &\Rightarrow = (x + 3)(x + 4) \end{aligned} $	Step 4: Check your answer.
Step 2: Factor the last term, choose the factors that add or subtract to obtain the middle term.	Step 3: Determine the signs ($\pm$) by multiplying the inner and outer terms and adding or subtracting them to obtain the middle term.	Check: $(x + 3)(x + 4)$ $= x^2 + 4x + 3x + 12$ $= x^2 + 7x + 12$

Tip: Rather than trying all possible combinations of the factors that make up the last term spend some time looking at the factors before starting step 2. Look for combinations that will produce the middle term. Here is the thought process in choosing 3 and 4 in step two above:



“Can I add or subtract 1 and 12 to obtain 7?” – NO

“Can I add or subtract 2 and 6 to obtain 7?” – NO

“Can I add or subtract 3 and 4 to obtain 7?” – YES

$$\begin{aligned}
 12 &= 1 \cdot 12 \\
 &= 2 \cdot 6 \\
 &= \boxed{3 \cdot 4}
 \end{aligned}$$

A. Factor the trinomials.

$x^2 + 8x - 20$
$ \begin{aligned} &x^2 + 8x - 20 \quad 20 = 1 \cdot 20 \\ &\quad \quad \quad = 2 \cdot 10 \\ &\quad \quad \quad = 4 \cdot 5 \\ &= (x - 2)(x + 10) \\ &\quad \quad \quad \begin{array}{c} -2x \\ +10x \\ \hline 8x \end{array} \end{aligned} $

$x^2 - 11x - 42$
$ \begin{aligned} &x^2 - 11x - 42 \quad 42 = 1 \cdot 42 \\ &\quad \quad \quad = 2 \cdot 21 \\ &\quad \quad \quad = 3 \cdot 14 \\ &\quad \quad \quad = 6 \cdot 7 \\ &= (x - 14)(x + 3) \\ &\quad \quad \quad \begin{array}{c} -14x \\ +3x \\ \hline -11x \end{array} \end{aligned} $

$x^2 + 26x + 25$
$ \begin{aligned} &x^2 + 26x + 25 \\ &= (x + 1)(x + 25) \end{aligned} $
In practice, this is all we need to show for our work.

This process, sometimes called **guess and check**, for factoring allows us to skip some steps on the check. The biggest problem occurs when the signs are improperly chosen. With this in mind, we can save some time by only checking the sign of the last term. Also, since multiplication is commutative **order does not matter**, in other words $(x + 1)(x + 25) = (x + 25)(x + 1)$

$$\begin{aligned}
 &x^2 - 18x + 81 \\
 &x^2 - 18x + 81 \\
 &= (x \quad 9)(x \quad 9) \\
 &\quad \quad \quad \begin{array}{r} -9x \\ -9x \\ \hline -18x \end{array} \\
 &= (x-9)(x-9) \\
 &= (x-9)^2
 \end{aligned}$$

$$\begin{aligned}
 &x^2 + 2xy + y^2 \\
 &x^2 + 2xy + y^2 \\
 &= (x \quad y)(x \quad y) \\
 &\quad \quad \quad \begin{array}{r} +xy \\ +xy \\ \hline 2xy \end{array} \\
 &= (x+y)(x+y) \\
 &= (x+y)^2
 \end{aligned}$$

$$\begin{aligned}
 &a^2 - 10ab - 56b^2 \\
 &a^2 - 10ab - 56b^2 \quad \left| \begin{array}{l} 56 = 2 \cdot 28 \\ \quad \quad \quad = 4 \cdot 14 \\ \quad \quad \quad = 7 \cdot 8 \\ \quad \quad \quad = 1 \cdot 56 \end{array} \right. \\
 &= (a \quad 4b)(a \quad 14b) \\
 &\quad \quad \quad \begin{array}{r} +4ab \\ +4ab \\ \hline -14ab \\ -10ab \end{array} \\
 &= (a+4b)(a-14b)
 \end{aligned}$$

If the trinomial has a GCF we should factor that out first. In fact, we could still factor it if we did not factor out the GCF first, but the process is greatly simplified if we do this first.

B. Factor the trinomial.

$$\begin{aligned}
 &3x^2 + 39x - 90 \\
 &3x^2 + 39x - 90 \quad \left| \begin{array}{l} 30 = 1 \cdot 30 \\ \quad \quad \quad = 2 \cdot 15 \\ \quad \quad \quad = 3 \cdot 10 \\ \quad \quad \quad = 5 \cdot 6 \end{array} \right. \\
 &= 3(x^2 + 13x - 30) \\
 &= 3(x-2)(x+15)
 \end{aligned}$$

$$\begin{aligned}
 &-2x^2 + 26x + 28 \\
 &-2x^2 + 26x + 28 \\
 &= -2(x^2 - 13x - 14) \\
 &= -2(x+1)(x-14)
 \end{aligned}$$

$$\begin{aligned}
 &-x^2 + 5x + 6 \\
 &-x^2 + 5x + 6 \\
 &= -1(x^2 - 5x - 6) \\
 &= -(x+1)(x-6)
 \end{aligned}$$

$$\begin{aligned}
 &2x^5 + 44x^4 + 144x^3 \\
 &2x^5 + 44x^4 + 144x^3 \quad \left| \begin{array}{l} 72 = 1 \cdot 72 \\ \quad \quad \quad = 2 \cdot 36 \\ \quad \quad \quad = 3 \cdot 24 \\ \quad \quad \quad = 4 \cdot 18 \\ \quad \quad \quad = 6 \cdot 12 \\ \quad \quad \quad = 8 \cdot 9 \end{array} \right. \\
 &= 2x^3(x^2 + 22x + 72) \\
 &= 2x^3(x+4)(x+18)
 \end{aligned}$$

$$\begin{aligned}
 &-3x^2y^2 + 18xy^2 - 24y^2 \\
 &-3x^2y^2 + 18xy^2 - 24y^2 \\
 &= -3y^2(x^2 - 6x + 8) \\
 &= -3y^2(x-2)(x-4)
 \end{aligned}$$

$$\begin{aligned}
 &5x^3 - 65x^2 + 60x \\
 &5x^3 - 65x^2 + 60x \\
 &= 5x(x^2 - 13x + 12) \\
 &= 5x(x-1)(x-12)
 \end{aligned}$$

$$\begin{aligned}
 &x^2y^2 + 6xy + 5 \\
 &x^2y^2 + 6xy + 5 \\
 &= (xy+5)(xy+1)
 \end{aligned}$$

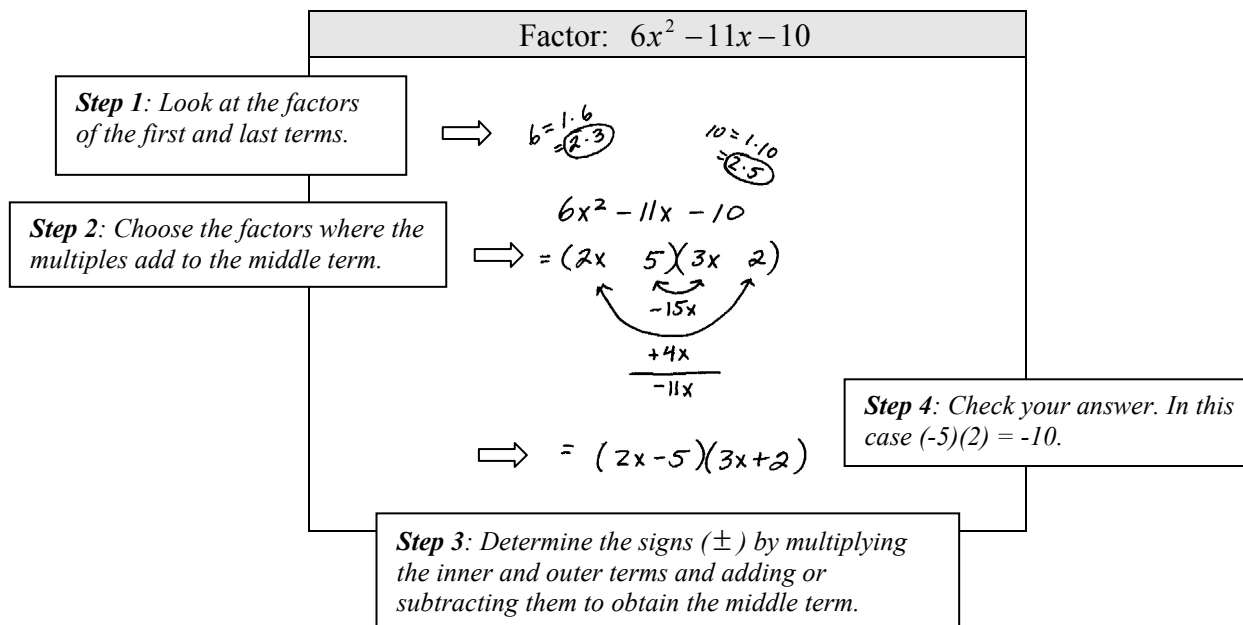
$$\begin{aligned}
 &x^2 - 6xy + 9y^2 \\
 &x^2 - 6xy + 9y^2 \\
 &= (x-3y)(x-3y)
 \end{aligned}$$

$$\begin{aligned}
 &a^2b^2 + 20ab + 100 \\
 &a^2b^2 + 20ab + 100 \\
 &= (ab+10)(ab+10)
 \end{aligned}$$

We can see that there are two ways to obtain a middle term with variable part xy

Take care to do the check. Most of the problems that we will encounter factor nicely but be sure to watch out for something like this $x^2 - 5x - 6 \neq (x-2)(x-3)$. The middle term works but the last term does not $(-2)(-3) = +6$.

So far we have looked at trinomials with leading coefficients of 1. Now we will see how the process is changed when the leading coefficients are something other than 1.



In this example the factors $\{2, 3\}$ and $\{2, 5\}$ can be multiplied in such a way, $3 \cdot 5 = 15$ and $2 \cdot 2 = 4$, to obtain -11 , the coefficient of the middle term. As we can see there are many more combinations to consider when the leading coefficient is not 1. These take time but become routine with practice.

C. Factor the trinomial.

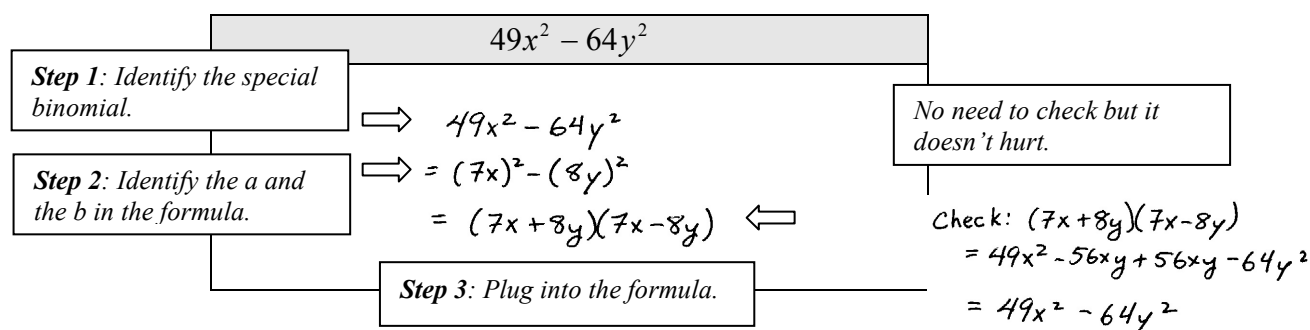
$2x^2 + 9x - 5$ $= (2x - 1)(x + 5)$	$5x^2 - 28x - 12$ $= (5x + 2)(x - 6)$	$7x^2 - 46x - 21$ $= (7x + 3)(x - 7)$
$10x^2 + 11x - 6$ $= (2x + 3)(5x - 2)$	$12x^2 + 17x - 5$ $= (3x + 5)(4x - 1)$	$6x^2 - 17x - 28$ $= (6x + 7)(x - 4)$

Problems Solved! Chapter 6 Factoring and Quadratic Equations
Factoring Binomials

Here we will be factoring special binomials. We have seen some of the formulas in a previous section but we will be using them in a different manner here. These formulas will be used as a template for factoring.

$a^2 - b^2 = (a + b)(a - b)$	- Difference of Squares
$a^2 + b^2$ does not factor	- Sum of Squares
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	- Difference of Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	- Sum of Cubes

The idea is to identify the a and the b then plug in to the formulas above.



The same steps can be used for the sum and difference of cubes formula.

$27x^3 - 8y^3$	
$ \begin{aligned} &27x^3 - 8y^3 \\ &= (3x)^3 - (2y)^3 \\ &= (3x - 2y)((3x)^2 + (3x)(2y) + (2y)^2) \\ &= (3x - 2y)(9x^2 + 6xy + 4y^2) \end{aligned} $	Difference of cubes Check: $(3x - 2y)(9x^2 + 6xy + 4y^2)$ $= 27x^3 + 18x^2y + 12xy^2 - 18x^2y - 12xy^2 - 8y^3$ $= 27x^3 - 8y^3$

At this point it is nice to review the perfect cubes that we will be likely to run into when working problems in this section. $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$ and $10^3 = 1000$. Many times the coefficients will give a clue as to what special binomial formula is to be used.

A. Factor

$x^2y^2 - 1$	$36m^2 - 9$	$81x^2 + y^2$
$x^2y^2 - 1$ $= (xy + 1)(xy - 1)$	$36m^2 - 9$ $= (6m + 3)(6m - 3)$	$81x^2 + y^2$ Does not Factor

$8x^3 + 1$	$125x^3y^3 - 64z^3$
$8x^3 + 1$ $= (2x + 1)((2x)^2 - (2x)(1) + (1)^2)$ $= (2x + 1)(4x^2 - 2x + 1)$	$125x^3y^3 - 64z^3$ $= (5xy - 4z)((5xy)^2 + (5xy)(4z) + (4z)^2)$ $= (5xy - 4z)(25x^2y^2 + 20xyz + 16z^2)$

At this point we will look a bit more closely at the process in which we are factoring sum of cubes using the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$$\begin{aligned}
 a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\
 m^3 + 8n^3 &= (m)^3 + (2n)^3 = (m + 2n)(m^2 - (m)(2n) + (2n)^2) \\
 &= (m + 2n)(m^2 - 2mn + 4n^2)
 \end{aligned}$$

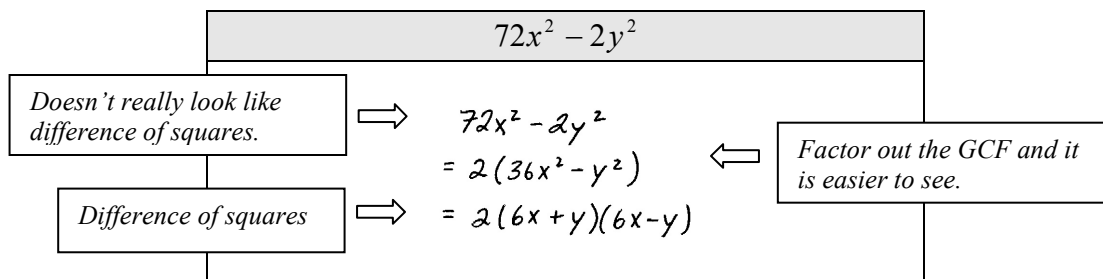
Once we identify the a and the b in the formula it really is just a matter of plugging in to the formula or using it as a template. Most of this is done mentally, so it is sufficient to present your solution without the above steps.

$$\begin{aligned}
 m^3 + 8n^3 \\
 = (m + 2n)(m^2 - 2mn + 4n^2)
 \end{aligned}$$

Rest assured that with much practice you will be able to jump straight to the answer too. The first step to this ability, of course, is to memorize the formulas.

$1000x^3 + 1$	$64m^3n^3 + 8$
$1000x^3 + 1$ $= (10x + 1)(100x^2 - 10x + 1)$	$64m^3n^3 + 8$ $= (4mn + 2)(16m^2n^2 - 8mn + 4)$
$216 + y^3$	$x^3 + 3$
$216 + y^3$ $= (6 + y)(36 - 6y + y^2)$ $= (y + 6)(y^2 - 6y + 36)$	$x^3 + 3$ does not factor (3 is <u>not</u> a cube)

As we have seen before, we will often run into polynomials with a GCF. It is important with special binomials to factor out the GCF first.

**B. Factor**

$-5x^3y^3 + 40$
$ \begin{aligned} &-5x^3y^3 + 40 \\ &= -5(x^3y^3 - 8) \\ &= -5(xy - 2)(x^2y^2 + 2xy + 4) \end{aligned} $

$128 + 2m^3$
$ \begin{aligned} &128 + 2m^3 \\ &= 2(64 + m^3) \\ &= 2(4 + m)(16 - 4m + m^2) \\ &= 2(m + 4)(m^2 - 4m + 16) \end{aligned} $

$-18x^2 + 32y^2$
$ \begin{aligned} &-18x^2 + 32y^2 \\ &= -2(9x^2 - 16y^2) \\ &= -2(3x + 4y)(3x - 4y) \end{aligned} $

$-2x^2 - 18$
$ \begin{aligned} &-2x^2 - 18 \\ &= -2(x^2 + 9) \end{aligned} $

Sum of squares does not factor for us.

Factoring binomials is a bit more complicated when larger exponents are involved. It is difficult to recognize that x^6 , for example, is a perfect cube. We can think of $x^6 = (x^2)^3$ or the cube of x squared. Also, recall the rule of exponents $(b^m)^n = b^{mn}$.

$8x^6 + 27y^6$		
Sum of cubes	$\Rightarrow$ $ \begin{aligned} &8x^6 + 27y^6 \\ &= (2x^2)^3 + (3y^2)^3 \\ &= (2x^2 + 3y^2)((2x^2)^2 - (2x^2)(3y^2) + (3y^2)^2) \\ &= (2x^2 + 3y^2)(4x^4 - 6x^2y^2 + 9y^4) \end{aligned} $	Here $a = 2x^2$ and $b = 3y^2$ in the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

It is not always necessary to show all the steps shown above. Ask your instructor what he or she wants to see in the way of steps when presenting your solutions in this case.

C. Factor

$1 + m^6$	$x^9 - y^6$
$ \begin{aligned} &1 + m^6 \\ &= (1)^3 + (m^2)^3 \\ &= (1 + m^2)(1^2 - (1)(m^2) + (m^2)^2) \\ &= (1 + m^2)(1 - m^2 + m^4) \end{aligned} $	$ \begin{aligned} &x^9 - y^6 \\ &= (x^3)^3 - (y^2)^3 \\ &= (x^3 - y^2)((x^3)^2 + (x^3)(y^2) + (y^2)^2) \\ &= (x^3 - y^2)(x^6 + x^3y^2 + y^4) \end{aligned} $

$n^{12} + 216$	$x^4 - y^4$
$ \begin{aligned} &n^{12} + 216 \\ &= (n^4)^3 + (6)^3 \\ &= (n^4 + 6)((n^4)^2 - (n^4)(6) + (6)^2) \\ &= (n^4 + 6)(n^8 - 6n^4 + 36) \end{aligned} $	$ \begin{aligned} &x^4 - y^4 \\ &= (x^2)^2 - (y^2)^2 \\ &= (x^2 + y^2)(x^2 - y^2) \leftarrow \text{Factor using difference of squares again.} \\ &= (x^2 + y^2)(x + y)(x - y) \end{aligned} $

Tip: Look out for problems that require us to factor the factors. In other words, look to continue factoring until all factors are completely factored. Also, the trinomials that we get when using the sum and difference of squares will not factor for us so do not even try.



$64x^6 - y^6$	$m^8 - n^8$
$ \begin{aligned} &64x^6 - y^6 \\ &= (8x^3)^2 - (y^3)^2 \\ &= (8x^3 + y^3)(8x^3 - y^3) \\ &= (2x + y)(4x^2 - 2xy + y^2)(2x - y)(4x^2 + 2xy + y^2) \end{aligned} $	$ \begin{aligned} &m^8 - n^8 \\ &= (m^4)^2 - (n^4)^2 \\ &= (m^4 + n^4)(m^4 - n^4) \\ &= (m^4 + n^4)(m^2 + n^2)(m^2 - n^2) \\ &= (m^4 + n^4)(m^2 + n^2)(m + n)(m - n) \end{aligned} $

ALWAYS factor difference of squares first.

$y^{12} - 1$	$-5x^3 + 625y^6$
$ \begin{aligned} &y^{12} - 1 \\ &= (y^6)^2 - (1)^2 \\ &= (y^6 + 1)(y^6 - 1) \\ &= [(y^3)^2 + (1)^2][(y^3)^2 - (1)^2] \\ &= (y^2 + 1)(y^4 - y^2 + 1)(y^3 + 1)(y^3 - 1) \\ &= (y^2 + 1)(y^4 - y^2 + 1)(y + 1)(y^2 - y + 1)(y - 1)(y^2 + y + 1) \end{aligned} $	$ \begin{aligned} &-5x^3 + 625y^6 \\ &= -5(x^3 - 125y^6) \\ &= -5(x - 5y^2)(x^2 + 5xy^2 + 25y^4) \end{aligned} $

If we are confronted with a polynomial that is both a difference of squares and a difference of cubes we must factor it as a difference of squares first. Doing this will ensure that our formulas will achieve the best possible factorization.

Now that we have learned techniques for factoring 4, 3, and 2 term polynomials we are ready to practice by mixing up the problems. The challenge is to first identify the type of factoring problem then decide which method to apply. Below are the basic guidelines for factoring.

Guidelines for Factoring Polynomials:

1. Factor out the GCF first.
2. Four term polynomials – Factor by Grouping.
3. Trinomials – Factor by “guess and check.”
4. Binomials – Use the special products in this order:

$a^2 - b^2 = (a + b)(a - b)$	- Difference of Squares
$a^2 + b^2$ does not factor	- Sum of Squares
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	- Difference of Cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	- Sum of Cubes

** If a binomial is both difference of squares and cubes we must factor it as difference of squares first to obtain a more complete factorization.*

** Not all polynomials factor over the real numbers, in this case, Beginning Algebra students can simply write “does not factor.”*

A. Factor

$25x - 125$	$121 - 64x^2y^2$	$x^2 - 8x + 16$
$25x - 125$ $= 25(x - 5)$	$121 - 64x^2y^2$ $= (11)^2 - (8xy)^2$ $= (11 + 8xy)(11 - 8xy)$	$x^2 - 8x + 16$ $= (x - 4)(x - 4)$ $= (x - 4)^2$
$-54x^3 + 16y^3$	$3x^3 + x^2y - 12xy - 4y^2$	
$-54x^3 + 16y^3$ $= -2(27x^3 - 8y^3)$ $= -2(3x - 2y)((3x)^2 + (3x)(2y) + (2y)^2)$ $= -2(3x - 2y)(9x^2 + 6xy + 4y^2)$		
	$3x^3 + x^2y - 12xy - 4y^2$ $= x^2(3x + y) - 4y(3x + y)$ $= (3x + y)(x^2 - 4y)$	

Tip: Make some note cards to aid in helping memorize the special products.



Look out for factoring problems that continue to factor. Sometimes factoring once is not enough. Continue to factor the factors until the expression is completely factored.

B. Factor

$$\begin{aligned}
 &2m^3 - m^2n - 8mn^2 + 4n^3 \\
 &= m^2(2m - n) - 4n^2(2m - n) \\
 &= (2m - n)(m^2 - 4n^2) \\
 &= (2m - n)(m + 2n)(m - 2n)
 \end{aligned}$$

$$\begin{aligned}
 &a^4 - 16b^4 \\
 &= (a^2)^2 - (4b^2)^2 \\
 &= (a^2 + 4b^2)(a^2 - 4b^2) \\
 &= (a^2 + 4b^2)(a + 2b)(a - 2b)
 \end{aligned}$$

$$\begin{aligned}
 &x^6 - y^6 \\
 &= (x^3)^2 - (y^3)^2 \\
 &= (x^3 + y^3)(x^3 - y^3) \\
 &= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)
 \end{aligned}$$

$$\begin{aligned}
 &x^6 + y^6 \\
 &= (x^2)^3 + (y^2)^3 \quad \leftarrow \\
 &= (x^2 + y^2)((x^2)^2 - (x^2)(y^2) + (y^2)^2) \\
 &= (x^2 + y^2)(x^4 - x^2y^2 + y^4)
 \end{aligned}$$

Sum of squares does not factor therefore we will apply the sum of cubes formula first.

Take some time to understand the difference between the last two solved problems. Notice that $x^6 - y^6$ is both difference of squares and cubes at the same time. Here we chose to apply the difference of squares formula first. On the other hand, $x^6 + y^6$ we chose to apply the sum of cubes formula first because the sum of squares does not factor.

C. Factor

$$\begin{aligned}
 &8x^5 - x^2 - 8x^3y^2 + y^2 \\
 &= x^2(8x^3 - 1) - y^2(8x^3 - 1) \\
 &= (8x^3 - 1)(x^2 - y^2) \\
 &= (2x - 1)(4x^2 + 2x + 1)(x + y)(x - y)
 \end{aligned}$$

$$\begin{aligned}
 &x^8 - 1 \\
 &= (x^4)^2 - (1)^2 \\
 &= (x^4 + 1)(x^4 - 1) \\
 &= (x^4 + 1)(x^2 + 1)(x^2 - 1) \\
 &= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)
 \end{aligned}$$

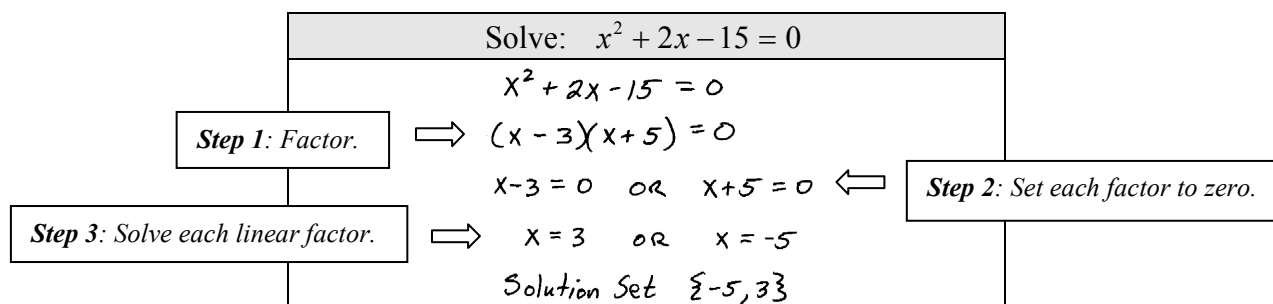
$$\begin{aligned}
 &-20x^2 + 20x - 5 \\
 &= -5(4x^2 - 4x + 1) \\
 &= -5(2x - 1)(2x - 1) \\
 &= -5(2x - 1)^2
 \end{aligned}$$

$$\begin{aligned}
 &x^5 + 8x^2 \\
 &= x^2(x^3 + 8) \\
 &= x^2(x + 2)(x^2 - 2x + 4)
 \end{aligned}$$

Previously we learned how to solve linear equations, now we will outline a technique to solve factorable quadratic equations that look like $ax^2 + bx + c = 0$. In addition, we will revisit function notation and apply the techniques in this section to quadratic functions.

Zero Factor Property – If $a \cdot b = 0$ then either $a = 0$ or $b = 0$.

The above property is the key to solving quadratic equations by factoring. So far we have been solving linear equations which usually had only one solution. We will see that quadratic equations can have up to two solutions.



This technique requires the zero factor property to work so make sure the quadratic is set equal to zero before factoring in step 1.

Check: $x^2 + 2x - 15$
 $x = 3$ $(3)^2 + 2(3) - 15$
 $= 9 + 6 - 15$
 $= 15 - 15 = 0 \checkmark$

Check: $x^2 + 2x - 15$
 $x = -5$ $(-5)^2 + 2(-5) - 15$
 $= 25 - 10 - 15$
 $= 25 - 25 = 0 \checkmark$

Tip: We can always see if we solved correctly by checking our answer.

On an exam it is useful to know if got the correct solutions or not.



A. Solve

$x^2 + 8x + 7 = 0$
$x^2 + 8x + 7 = 0$
$(x + 7)(x + 1) = 0$
$x + 7 = 0$ OR $x + 1 = 0$
$x = -7$ $x = -1$
Solution Set $\{-7, -1\}$

$6x^2 + x - 2 = 0$
$6x^2 + x - 2 = 0$
$(2x - 1)(3x + 2) = 0$
$2x - 1 = 0$ OR $3x + 2 = 0$
$2x = 1$ $3x = -2$
$x = \frac{1}{2}$ $x = -\frac{2}{3}$
Solution Set $\{-\frac{2}{3}, \frac{1}{2}\}$

$x^2 + 12x + 36 = 0$
$x^2 + 12x + 36 = 0$
$(x + 6)(x + 6) = 0$
$x + 6 = 0$ OR $x + 6 = 0$
$x = -6$ $x = -6$
Solution Set $\{-6\}$

One solution here, we can say -6 is a double root.

Actually there are four steps for solving quadratic equations by factoring. The first step is to put the equation in standard form, $ax^2 + bx + c = 0$, by adding all terms to one side.

Solve: $x^2 - x = 56$		
Put in standard form...	$\begin{array}{r} x^2 - x = 56 \\ -56 \quad -56 \\ \hline x^2 - x - 56 = 0 \\ (x + 7)(x - 8) = 0 \quad \leftarrow \text{...then factor} \\ x + 7 = 0 \quad \text{or} \quad x - 8 = 0 \\ x = -7 \quad \quad x = 8 \\ \text{Solution Set } \{-7, 8\} \end{array}$	Check to see if the solution set satisfies the original equation. <u>check $x = -7$:</u> $x^2 - x \stackrel{?}{=} 56$ $(-7)^2 - (-7)$ $= 49 + 7 = 56 \checkmark$ <u>check $x = 8$:</u> $x^2 - x = 56$ $(8)^2 - (8)$ $= 64 - 8 = 56 \checkmark$

A common mistake is to set each factor to 56 here but that is not what the zero factor property says. We must have zero on one side of our equation for this technique to work.

B. Solve

$x^2 = 5x$ $\begin{array}{r} x^2 = 5x \\ -5x \quad -5x \\ \hline x^2 - 5x = 0 \\ x(x - 5) = 0 \\ x = 0 \quad \text{or} \quad x - 5 = 0 \\ \quad \quad \quad x = 5 \\ \text{solution Set } \{0, 5\} \end{array}$	$x(2x + 5) = 3$ $\begin{array}{r} x(2x + 5) = 3 \\ 2x^2 + 5x = 3 \\ 2x^2 + 5x - 3 = 0 \\ (2x - 1)(x + 3) = 0 \\ 2x - 1 = 0 \quad \text{or} \quad x + 3 = 0 \\ \quad \quad \quad 2x = 1 \quad \quad \quad -3 \quad -3 \\ \quad \quad \quad x = \frac{1}{2} \quad \text{or} \quad x = -3 \\ \text{solution Set } \{-3, \frac{1}{2}\} \end{array}$	$x^2 - 49 = 0$ $\begin{array}{r} x^2 - 49 = 0 \\ (x + 7)(x - 7) = 0 \\ x + 7 = 0 \quad \text{or} \quad x - 7 = 0 \\ x = -7 \quad \quad x = 7 \\ \text{solution Set } \{-7, 7\} \end{array}$
$x(x + 5) = 2(x + 2)$ $\begin{array}{r} x(x + 5) = 2(x + 2) \\ x^2 + 5x = 2x + 4 \\ x^2 + 3x - 4 = 0 \\ (x - 1)(x + 4) = 0 \\ x - 1 = 0 \quad \text{or} \quad x + 4 = 0 \\ \quad \quad \quad x = 1 \quad \quad x = -4 \\ \text{solution Set } \{-4, 1\} \end{array}$	$x^2 + 11x = 0$ $\begin{array}{r} x^2 + 11x = 0 \\ x(x + 11) = 0 \\ x = 0 \quad \text{or} \quad x + 11 = 0 \\ \quad \quad \quad x = -11 \\ \text{solution Set } \{0, -11\} \end{array}$	$4x^2 = 9$ $\begin{array}{r} 4x^2 = 9 \\ 4x^2 - 9 = 0 \\ (2x + 3)(2x - 3) = 0 \\ 2x + 3 = 0 \quad \text{or} \quad 2x - 3 = 0 \\ 2x = -3 \quad \quad 2x = 3 \\ x = -\frac{3}{2} \quad \quad x = \frac{3}{2} \\ \text{solution Set } \{-\frac{3}{2}, \frac{3}{2}\} \end{array}$

We have seen before that **clearing fractions** from our equations can be done by multiplying both sides of our equation by the LCM of the denominators. In doing this the equation becomes a bit easier to solve.

Solve: $\frac{1}{2}x^2 + \frac{7}{6}x + \frac{1}{3} = 0$		
To clear the fractions multiply both sides by 6.	$\frac{1}{2}x^2 + \frac{7}{6}x + \frac{1}{3} = 0$ $\Rightarrow 6\left(\frac{1}{2}x^2 + \frac{7}{6}x + \frac{1}{3}\right) = 6(0)$ $3x^2 + 7x + 2 = 0$ $(3x+1)(x+2) = 0$ $3x+1=0 \text{ OR } x+2=0$ $3x=-1$ $x=-\frac{1}{3} \quad x=-2$ $\text{Solution Set } \left\{-\frac{1}{3}, -2\right\}$	No more fractions solve like normal.

C. Solve

$\frac{1}{16}x^2 = \frac{1}{4}$
$\frac{1}{16}x^2 = \frac{1}{4}$ $16\left(\frac{1}{16}x^2\right) = 16\left(\frac{1}{4}\right)$ $x^2 = 4$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $x+2=0 \text{ OR } x-2=0$ $x=-2 \quad x=2$ $\text{Solution Set } \left\{-2, 2\right\}$

$\frac{1}{5}x(x - \frac{26}{5}) = -\frac{1}{5}$
$\frac{1}{5}x(x - \frac{26}{5}) = -\frac{1}{5}$ $\frac{1}{5}x^2 - \frac{26}{25}x + \frac{1}{5} = 0$ $25\left(\frac{1}{5}x^2 - \frac{26}{25}x + \frac{1}{5}\right) = 25(0)$ $5x^2 - 26x + 5 = 0$ $(5x-1)(x-5) = 0$ $5x-1=0 \text{ OR } x-5=0$ $5x=1$ $x=\frac{1}{5} \quad x=5$ $\text{Solution Set } \left\{\frac{1}{5}, 5\right\}$

$\frac{2}{15}x^2 - \frac{7}{10}x + \frac{2}{3} = 0$
$\frac{2}{15}x^2 - \frac{7}{10}x + \frac{2}{3} = 0$ $30\left(\frac{2}{15}x^2 - \frac{7}{10}x + \frac{2}{3}\right) = 30(0)$ $4x^2 - 21x + 20 = 0$ $(4x-5)(x-4) = 0$ $4x-5=0 \text{ OR } x-4=0$ $4x=5$ $x=\frac{5}{4} \quad x=4$ $\text{Solution Set } \left\{\frac{5}{4}, 4\right\}$

Finding equations given the solutions requires us to work the entire process backwards. Given the solutions we can find factors that we can multiply to obtain an equation.

D. Find a quadratic equation with given solution set.

$\{-1, 3\}$
$x = -1 \text{ OR } x = 3$ $x+1=0 \quad x-3=0$ $(x+1)(x-3) = 0$ $x^2 - 3x + x - 3 = 0$ $x^2 - 2x - 3 = 0$

$\{-7, 7\}$
$x = -7 \text{ OR } x = 7$ $x+7=0 \quad x-7=0$ $(x+7)(x-7) = 0$ $x^2 - 7x + 7x - 49 = 0$ $x^2 - 49 = 0$

$\{-\frac{1}{2}, \frac{2}{3}\}$
$x = -\frac{1}{2} \text{ OR } x = \frac{2}{3}$ $2x = -1 \quad 3x = 2$ $2x+1=0 \quad 3x-2=0$ $(2x+1)(3x-2) = 0$ $6x^2 - 4x + 3x - 2 = 0$ $6x^2 - x - 2 = 0$

The notation for quadratic functions is very much the same as linear functions.
Remember that $y = f(x)$ reads y is a function of x .

E. Evaluate the given function.

Given $f(x) = 2x^2 + x - 21$,
find $f(-2)$.

$$\begin{aligned} f(x) &= 2x^2 + x - 21 \\ f(-2) &= 2(-2)^2 + (-2) - 21 \\ &= 8 - 2 - 21 \\ &= -15 \end{aligned}$$

Given $g(x) = x^2 - 64$,
find $g(0)$.

$$\begin{aligned} g(x) &= x^2 - 64 \\ g(0) &= (0)^2 - 64 \\ &= -64 \end{aligned}$$

Given $h(x) = 2x^2 + 9x - 5$,
find $h(-1)$.

$$\begin{aligned} h(x) &= 2x^2 + 9x - 5 \\ h(-1) &= 2(-1)^2 + 9(-1) - 5 \\ &= 2 - 9 - 5 \\ &= -12 \end{aligned}$$

Given $f(x) = 2x^2 + x - 21$,
find x where $f(x) = 0$.

$$\begin{aligned} f(x) &= 2x^2 + x - 21 \\ 0 &= 2x^2 + x - 21 \\ 0 &= (2x+7)(x-3) \\ 2x+7 &= 0 \text{ or } x-3=0 \\ 2x &= -7 \\ x &= -\frac{7}{2} \quad x=3 \\ \text{Solution Set } &\left\{-\frac{7}{2}, 3\right\} \end{aligned}$$

Given $g(x) = x^2 - 64$,
find x where $g(x) = 0$.

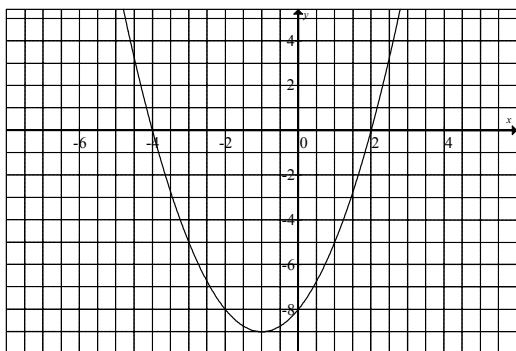
$$\begin{aligned} g(x) &= x^2 - 64 \\ 0 &= x^2 - 64 \\ 0 &= (x+8)(x-8) \\ x+8 &= 0 \text{ or } x-8=0 \\ x &= -8 \quad x=8 \\ \text{Solution Set } &\{-8, 8\} \end{aligned}$$

Given $h(x) = 2x^2 + 9x - 5$,
find x where $h(x) = -5$.

$$\begin{aligned} h(x) &= 2x^2 + 9x - 5 \\ -5 &= 2x^2 + 9x - 5 \\ +5 & \quad +5 \\ 0 &= 2x^2 + 9x \\ 0 &= x(2x+9) \\ x &= 0 \text{ or } 2x+9=0 \\ & \quad 2x=-9 \\ & \quad x=-\frac{9}{2} \\ \text{Solution Set } &\left\{-\frac{9}{2}, 0\right\} \end{aligned}$$

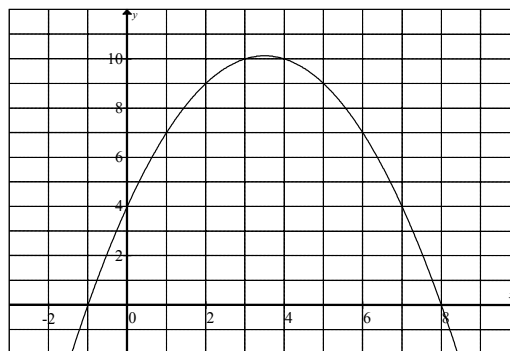
F. Given the graph of the quadratic function x - and y -intercepts.

Use the graph to find the intercepts.



y-intercept $\{(0, -8)\}$
x-intercepts $\{(-4, 0), (2, 0)\}$

Use the graph to find the intercepts.



y-intercept $\{(0, 4)\}$
x-intercepts $\{(-1, 0), (8, 0)\}$

Problems Solved! Chapter 6 Factoring and Quadratic Equations
Solving with the Quadratic Formula

If solving a quadratic equation and it factors, then certainly, we should solve it by factoring. The problem is that not all quadratic equations factor. When this is the case we could use the quadratic formula to find the solution. (A derivation of the formula will come later and we will limit our **introduction** to equations where $b^2 - 4ac \geq 0$)

Quadratic Formula: If $ax^2 + bx + c = 0$ where $a \neq 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve: $2x^2 + 7x + 4 = 0$		
Step 1: Identify a , b and c . Step 2: Write down the formula.	$2x^2 + 7x + 4 = 0$ $\Rightarrow a=2 \quad b=7 \quad c=4$ $\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4(2)(4)}}{2(2)}$ $= \frac{-7 \pm \sqrt{49 - 32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$ $\text{Solution Set } \left\{ \frac{-7 - \sqrt{17}}{4}, \frac{-7 + \sqrt{17}}{4} \right\}$	Step 3: Plug in the appropriate values and evaluate.

A. Solve

$x^2 - 2x - 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$ $= \frac{2 \pm \sqrt{4 + 20}}{2}$ $= \frac{2 \pm \sqrt{24}}{2}$ $= \frac{2 \pm \sqrt{4 \cdot 6}}{2}$ $= \frac{2 \pm 2\sqrt{6}}{2}$ $= 1 \pm \sqrt{6}$ $\text{Solution Set } \{1 - \sqrt{6}, 1 + \sqrt{6}\}$	$-2x^2 + x + 7 = 0$ $a=-2 \quad b=1 \quad c=7$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(-2)(7)}}{2(-2)}$ $= \frac{-1 \pm \sqrt{1 + 56}}{-4}$ $= \frac{-1 \pm \sqrt{57}}{-4}$ $= \frac{1 \pm \sqrt{57}}{4}$ $\text{Solution Set } \left\{ \frac{1 - \sqrt{57}}{4}, \frac{1 + \sqrt{57}}{4} \right\}$	$x^2 - 5x + 6 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$ $= \frac{5 \pm \sqrt{25 - 24}}{2}$ $= \frac{5 \pm \sqrt{1}}{2}$ $= \frac{5 \pm 1}{2}$ $x = \frac{5-1}{2} \text{ or } x = \frac{5+1}{2}$ $x = 2 \quad x = 3$ $\text{Solution Set } \{2, 3\}$
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The applications in this chapter will involve multiplication. Look for the keyword “product” which indicates that we will have to multiply in our set-up. Remember to read the question several times, identify the variables and use algebra to solve the problems. Whenever possible we will try to set them up using only one variable.

A. Number Problems

One positive number is 4 less than twice another positive number and their product is 96. Set-up an algebraic equation and solve it to find the two numbers.

Let x = One of the positive numbers
 $2x - 4$ = The other positive number.

Set-up $\rightarrow x(2x - 4) = 96$
 $2x^2 - 4x = 96$
 $2x^2 - 4x - 96 = 0$
 $2(x^2 - 2x - 48) = 0$

$2(x + 6)(x - 8) = 0$
 $x + 6 = 0$ or $x - 8 = 0$
 $x = -6$ or $x = 8$
 BACK Substitute:
 $2x - 4 = 2(8) - 4 = 12$

The two numbers are 8 and 12.

The difference between two positive integers is 5 and their product is 126. Find the two integers.

Let x = The larger integer (positive).
 $x - 5$ = The smaller integer.

Set-up $\rightarrow x(x - 5) = 126$
 $x^2 - 5x = 126$
 $x^2 - 5x - 126 = 0$
 $(x + 9)(x - 14) = 0$

$x + 9 = 0$ or $x - 14 = 0$
 $x = -9$ or $x = 14$
 BACK Substitute:
 $x - 5 = 14 - 5 = 9$

The two integers are 9 and 14.

Sarah is 2 years older than her brother Ryan. If the product of their ages is 15 how old are they. (Set up an algebraic equation and solve it)

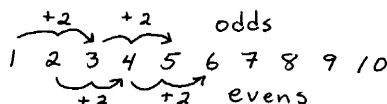
Let x = age of brother, Ryan.
 $x + 2$ = age of sister, Sarah.

Set-up $\rightarrow x(x + 2) = 15$
 $x^2 + 2x = 15$
 $x^2 + 2x - 15 = 0$
 $(x - 3)(x + 5) = 0$

$x - 3 = 0$ or $x + 5 = 0$
 $x = 3$ or $x = -5$
 BACK Substitute:
 $x + 2 = 3 + 2 = 5$

Sarah is 5 yrs old and Ryan is 3 yrs old.

Consecutive integers and consecutive even and odd integer problems pretty much **always show up on the final exam**. We need to be comfortable with them. Recall that consecutive integers are separated by 1 and consecutive even and odds are separated by 2 units.



B. Consecutive Integer Problems

The product of two consecutive odd positive integers is 99. Find the integers.

Let x = The first odd integer (positive)

$x+2$ = The next odd integer.

Set-up $\rightarrow x(x+2) = 99$

$$x^2 + 2x = 99$$

$$x^2 + 2x - 99 = 0$$

$$(x+11)(x-9) = 0$$

$$x+11=0 \quad \text{or} \quad x-9=0$$

$$x=-11 \quad \text{or} \quad \boxed{x=9}$$

BACK SUBSTITUTE: $x+2 = 9+2 = 11$

The two odd positive integers are 9 and 11.

The product of two consecutive even positive integers is 168. Find the integers

Let x = The first even positive integer.

$x+2$ = The next even integer.

$\Rightarrow$ Set-up $\rightarrow x(x+2) = 168$

$$x^2 + 2x = 168$$

$$x^2 + 2x - 168 = 0$$

$$(x-12)(x+14) = 0$$

$$x-12=0 \quad \text{or} \quad x+14=0$$

$$\boxed{x=12}$$

$$x=-14$$

BACK SUBSTITUTE: $x+2 = 12+2 = 14$

The two even integers are 12 and 14.

Notice that the set-up is the same as odds.

The product of two consecutive integers is 182. Find the integers.

Let x = the first integer.

$x+1$ = the next integer.

Set-up $\rightarrow x(x+1) = 182$

$$x^2 + x = 182$$

$$x^2 + x - 182 = 0$$

$$(x-13)(x+14) = 0$$

$$x-13=0 \quad \text{or} \quad x+14=0$$

$$x=13$$

$$x=-14$$

Two solutions:

If $x=13$ $x+1=14$

If $x=-14$ $x+1=-13$

Positive Solution $\{13, 14\}$, Negative Solution $\{-14, -13\}$

Since this problem did not specify negative or positive integers we must provide both answers.

Basic Guidelines for Solving Word Problems:

1. Read the problem several times and organize the given information.
2. Identify the variables by assigning a letter to the unknown quantity.
3. Set up an algebraic equation.
4. Solve the equation.
5. Finally, answer the question and make sure it makes sense.



With geometry type problems it sometimes helps if you draw a picture. Here are some **area formulas** that you are expected to know. ($\pi \approx 3.14$)

Area of a Rectangle: $A = l \cdot w$

Area of a Triangle: $A = \frac{1}{2}bh$

Perimeter of a Circle: $A = \pi r^2$

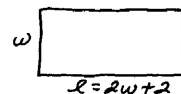
Area of a Square: $A = s^2$

C. Geometry Problems

The length of a rectangular room is 2 feet more than twice its width. If the total area in the room is 220 square feet find the dimensions of the room.

Let w = the width of the room.

$2w + 2$ = the length of the room.



Set-up $\rightarrow (2w + 2) \cdot w = 220$

$$2w^2 + 2w = 220$$

$$2w^2 + 2w - 220 = 0$$

$$2(w^2 + w - 110) = 0$$

$$2(w - 10)(w + 11) = 0$$

$$w - 10 = 0 \text{ OR } w + 11 = 0$$

$$\boxed{w = 10} \quad w = -11$$

BACK substitute:

$$l = 2w + 2 = 2(10) + 2 = 22$$

The room measures 10 ft by 22 ft.

A square cement patio is to be surrounded by a 2 ft brick border. If the total area is to be 121 ft² what will the dimensions of the cement slab be?

Let s = length of each side of the cement slab.

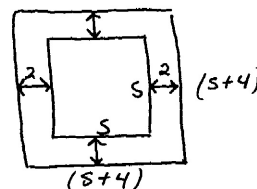
Set-up $\rightarrow (s + 4)(s + 4) = 121$

$$s^2 + 4s + 4s + 16 = 121$$

$$\quad \quad \quad -121 \quad -121$$

$$s^2 + 8s - 105 = 0$$

$$(s + 15)(s - 7) = 0$$



$$s + 15 = 0 \text{ OR } s - 7 = 0$$

$$s = -15 \quad \boxed{s = 7}$$

The dimensions of the slab will be 7 ft by 7 ft.

A pig pen is to be built adjacent to a barn with 16 feet of fencing. If the length is to be twice that of the width what will the area of the pen be?

Let w = width of the pen

$2w$ = length of the pen

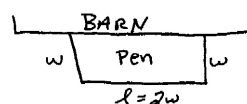
Set-up $\rightarrow w + 2w + w = 16$

$$4w = 16$$

$$w = 4$$

$$\text{so } l = 2w = 2 \cdot 4 = 8$$

$$\text{Total Area} = l \cdot w = 8 \cdot 4 = 32 \text{ ft}^2.$$



The height of a triangle is 3in less than twice the length of its base. If the total area of the triangle is 52 in² find the lengths of the base and height.

Let b = the length of the base

$2b-3$ = the height of the triangle

Set-up $\rightarrow A = \frac{1}{2}b \cdot h$

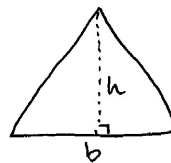
$$52 = \frac{1}{2}b(2b-3)$$

$$2(52) = 2\left(\frac{1}{2}b(2b-3)\right)$$

$$104 = b(2b-3)$$

$$104 = 2b^2 - 3b$$

$$0 = 2b^2 - 3b - 104$$



$$0 = (2b+13)(b-8)$$

$$2b+13=0 \text{ or } b-8=0$$

$$2b = -13$$

$$b = -13/2$$

$$\boxed{b=8}$$

Back subst: $h = 2b-3 = 16-3=13$

The base measures 8in and the height measures 13in.

Pythagorean Theorem – For any *right triangle* with hypotenuse c and legs a and b .

$$a^2 + b^2 = c^2$$

The hypotenuse of a right triangle is 2cm more than the smallest leg. If the other leg measures 4cm find the lengths of the other two sides.

Let x = the length of the small leg.

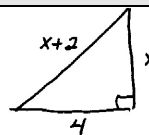
$x+2$ = the length of the hypotenuse.

Set-up $\rightarrow (4)^2 + x^2 = (x+2)^2$

$$16 + x^2 = x^2 + 4x + 4$$

$$12 = 4x$$

$$3 = x$$



Hypotenuse measures
 $x+2 = 3+2 = 5\text{cm}$

The length of the legs
are 3cm and 4cm.

The hypotenuse of a right triangle measures 10 in. If the small leg is 2in less than the longer leg, find the lengths of the legs.

Let x = the length of the longer leg.

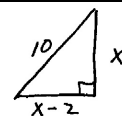
$x-2$ = the length of the small leg.

Set-up $\rightarrow (x)^2 + (x-2)^2 = 10^2$

$$x^2 + x^2 - 4x + 4 = 100$$

$$2x^2 - 4x - 96 = 0$$

$$2(x^2 - 2x - 48) = 0$$



$$2(x+6)(x-8) = 0$$

$$x+6=0 \text{ or } x-8=0$$

$$x=-6$$

$$\boxed{x=8}$$

Back subst: $x-2 = 6$

The legs measure 6in and 8in.

For the following projectile problems we will use the formula for the height $h(t) = -16t^2 + v_0t + s$ where t is time in seconds, v_0 is the initial velocity and s is the initial height.

D. Projectile Problems

A projectile is launched from a roof 80ft high at an initial velocity of 64 feet per second. How long will it take the projectile to land on the ground?

Using $h(t) = -16t^2 + v_0t + s$ where $s = 80$ and $v_0 = 64$
 $h(t) = -16t^2 + 64t + 80$ find t when $h(t) = 0$
 $0 = -16t^2 + 64t + 80$
 $0 = -16(t^2 - 4t - 5)$
 $0 = -16(t - 5)(t + 1)$

$t - 5 = 0$ or $t + 1 = 0$
 $t = 5$ or $t = -1$

It will take 5 seconds to hit the ground.

Old fashioned big caliber handguns (45 Colt, 44 S+W) shoot a heavy bullet without a lot of gun powder behind it. Muzzle velocities for these weapons average 1000 feet per second. If a handgun is shot up into the air from the ground how long will it take to bullet to come back to the ground? (Wind and air resistance are not part of this calculation)

Using $h(t) = -16t^2 + v_0t + s$ where $s = 0$ and $v_0 = 1000$
 $h(t) = -16t^2 + 1000t + 0$ find t when $h(t) = 0$
 $0 = -16t^2 + 1000t$
 $0 = -8t(2t - 125)$
 $-8t = 0$ or $2t - 125 = 0$
 $t = 0$ or $2t = 125$
 $t = 125/2 = 62.5$ seconds.

It will take over a minute to fall back.

From the previous problem, how high is the bullet at $t = 30$ seconds.

Using $h(t) = -16t^2 + 1000t$ find $h(30)$ (here $t = 30$ seconds)
 $h(30) = -16(30)^2 + 1000(30)$
 $= -14,400 + 30,000$
 $= 15,600$ ft

$15,600 \text{ ft} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}}$
 $\approx 2.95 \text{ miles}$

It will be about 2.95 miles up at 30 seconds.

If a rocket is shot straight up with an initial velocity of 48 ft/sec then its height h , in feet, after t seconds is given by $h(t) = 48t - 16t^2$.

a. How high is the rocket $\frac{1}{2}$ seconds after it is launched?

b. When is the rocket exactly 32 feet above the ground?

c. When will the rocket hit the ground?

$$\begin{aligned} \text{a. } h(t) &= 48t - 16t^2 \\ h\left(\frac{1}{2}\right) &= 48\left(\frac{1}{2}\right) - 16\left(\frac{1}{2}\right)^2 \\ &= 24 - 16 \cdot \frac{1}{4} \\ &= 24 - 4 = 20 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{b. } h(t) &= 48t - 16t^2 \\ 32 &= 48t - 16t^2 \\ +16t^2 - 48t &\quad -48t + 16t^2 \\ 16t^2 - 48t + 32 &= 0 \\ 16(t^2 - 3t + 2) &= 0 \\ 16(t-1)(t-2) &= 0 \end{aligned}$$

$t-1=0$ or $t-2=0$
 $t=1$ $t=2$
 The height is 32 ft
 when $t=1$ sec and $t=2$ sec

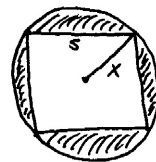
$$\begin{aligned} \text{c. } h(t) &= 48t - 16t^2 \\ 0 &= 48t - 16t^2 \\ 0 &= 16t(3-t) \\ 16t &= 0 \quad \text{or} \quad 3-t=0 \\ t &= 0 \quad \text{or} \quad 3=t \end{aligned}$$

The rocket will hit the ground in 3 sec.

Find a polynomial that gives the area of a circle of radius x with a square, whose diagonal measures $2x$, cut from it.

$$\begin{aligned} \text{Area} &= \text{Area Circle} - \text{Area Square} \\ &= \pi r^2 - s^2 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \pi x^2 - 2x^2 \\ &= x^2(\pi - 2) \quad \checkmark \end{aligned}$$



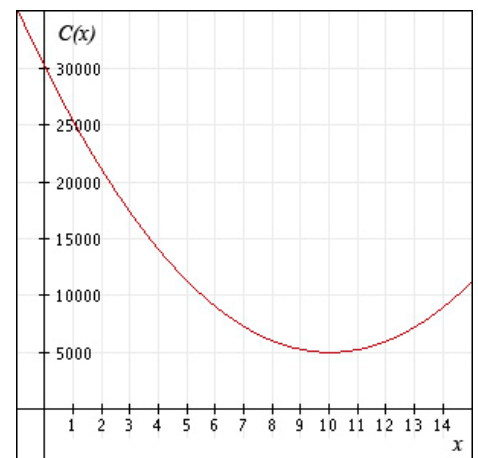
$$\begin{aligned} s^2 &= x^2 + x^2 \\ &= 2x^2 \end{aligned}$$

Chapter 6_ Factoring and Quadratic Equations

Sample Exam

Please answer all the questions and show work where appropriate.

1. Factor: $2x^2 + 6x - 3xy - 9y$
2. Factor: $6x^2 + 25x - 9$
3. Factor: $2x^6 - 128$
4. Solve: $(x - 3)^2 = 2(5 - 3x)$
5. Solve: $-3x^2 + 300 = 0$
6. Given $f(x) = 2x^2 - 9x - 5$
 - a. Calculate $f(0)$.
 - b. Find all values of x for which $f(x) = 0$.
7. Find a quadratic equation with integer coefficients that has $\{-3, \frac{2}{3}\}$ as a solution set.
8. The product of two consecutive odd positive integers is eleven less than 10 times the larger. Find the integers.
9. The length of a rectangle is 6 cm longer than twice its width. If the area is 140 cm^2 , find the dimensions of the rectangle.
10. The cost of a particular car can be approximated by the function $C(x) = 250x^2 - 5,000x + 30,000$ where x is the age of the car in years.
 - a. Use the graph to determine the cost of the car when it was new?
 - b. How old will the car be when it reaches its minimum cost?
 - c. How much is this car worth when it reaches 5 years old?



Chapter 6 *Factoring and Quadratic Equations*

Sample Exam Answers

1. $(x + 3)(2x - 3y)$
2. $(2x + 9)(3x - 1)$
3. $2(x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)$
4. $\{-1, 1\}$
5. $\{-10, 10\}$
6.
 - a. $f(0) = -5$
 - b. $x = -\frac{1}{2}$ or $x = 5$
7. $3x^2 + 7x - 6 = 0$
8. 9 and 11
9. 7cm by 20cm
10.
 - a. \$30,000
 - b. 10 years
 - c. \$11,250

Factor: $2x^2 + 6x - 3xy - 9y$	Factor: $6x^2 + 25x - 9$	Solve: $-3x^2 + 300 = 0$
$2x^2 + 6x - 3xy - 9y$ $= 2x(x+3) - 3y(x+3)$ $= (x+3)(2x-3y)$	$6x^2 + 25x - 9$ $= (2x+9)(3x-1)$	$-3x^2 + 300 = 0$ $-3(x^2 - 100) = 0$ $-3(x+10)(x-10) = 0$ $x+10=0 \quad \text{or} \quad x-10=0$ $x=-10 \quad \quad \quad x=10$

Solve: $(x-3)^2 = 2(5-3x)$	Factor: $2x^6 - 128$
$(x-3)^2 = 2(5-3x)$ $x^2 - 6x + 9 = 10 - 6x$ $x^2 - 1 = 0$ $(x+1)(x-1) = 0$ $x+1=0 \quad \text{or} \quad x-1=0$ $x=-1 \quad \quad \quad x=1$	$2x^6 - 128$ $= 2(x^6 - 64)$ $= 2(x^3 + 8)(x^3 - 8)$ $= 2(x+2)(x^2 - 2x + 4)(x-2)(x^2 + 2x + 4)$

Given $f(x) = 2x^2 - 9x - 5$	
a. Calculate $f(0)$.	
b. Find all values of x for which $f(x) = 0$.	
A. $f(x) = 2x^2 - 9x - 5$ $f(0) = 2 \cdot 0^2 - 9 \cdot 0 - 5$ $= -5$	B.) $f(x) = 2x^2 - 9x - 5$ $0 = 2x^2 - 9x - 5$ $0 = (2x+1)(x-5)$ $2x+1=0 \quad \text{or} \quad x-5=0$ $2x=-1 \quad \quad \quad x=5$ $x=-\frac{1}{2} \quad \quad \quad x=5$

Find a quadratic equation with integer coefficients that has $\{-3, \frac{2}{3}\}$ as a solution set.
$x = -3 \quad \text{or} \quad x = \frac{2}{3}$ $x+3=0 \quad \quad \quad 3x=2$ $3x-2=0$ $(x+3)(3x-2)=0$ $3x^2 - 2x + 9x - 6 = 0$ $3x^2 + 7x - 6 = 0$

The product of two consecutive odd positive integers is eleven less than 10 times the larger. Find the integers.

Let x = the first odd (pos.) integer

$x+2$ = the next odd integer

Set-up $\rightarrow x(x+2) = 10(x+2) - 11$

$$x^2 + 2x = 10x + 20 - 11$$

$$x^2 + 2x = 10x + 9$$

$$x^2 - 8x - 9 = 0$$

$$(x - 9)(x + 1) = 0$$

$$x - 9 = 0 \text{ or } x + 1 = 0$$

$$x = 9$$

$$x = -1$$

pos. only

The two integers are 9 and 11.

The length of a rectangle is 6 cm longer than twice its width. If the area is 140 cm^2 , find the dimensions of the rectangle.

Let w = width of the rectangle

$2w + 6$ = length of the rectangle

$$A = l \cdot w$$

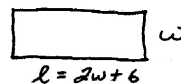
$$140 = (2w + 6) \cdot w \quad \leftarrow \text{set-up}$$

$$140 = 2w^2 + 6w$$

$$0 = 2w^2 + 6w - 140$$

$$0 = 2(w^2 + 3w - 70)$$

$$0 = 2(w - 7)(w + 10)$$



$$w - 7 = 0 \text{ or } w + 10 = 0$$

$$w = 7 \text{ cm} \quad w = -10 \text{ not pos.}$$

$$l = 2(7) + 6 = 20 \text{ cm}$$

The width is 7 cm and the length measures 20 cm.

The cost of a particular car can be approximated by the function

$C(x) = 250x^2 - 5,000x + 30,000$ where x is the age of the car in years.

- Use the graph to determine the cost of the car when it was new?
- How old will the car be when it reaches its minimum cost?
- How much is this car worth when it reaches 5 years old?

A. When $x = 0$ yrs the cost = \$30,000

B. Minimum cost when $x = 10$ yrs

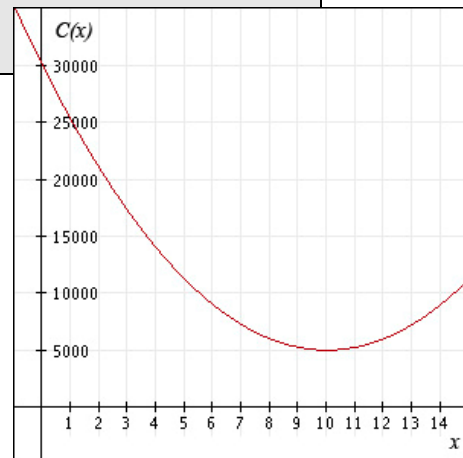
$$C. \quad C(x) = 250x^2 - 5,000x + 30,000$$

$$C(5) = 250(5)^2 - 5000(5) + 30,000$$

$$= 250(25) - 25,000 + 30,000$$

$$= 6,250 + 5,000 = 11,250$$

Value after 5 yrs is \$11,250.



Rational expressions can be very intimidating at first glance. But once the techniques of this section are learned students generally find them to be actually fun to work out. There are some new keywords and new types of word problems presented in this chapter. This is the chapter in which factoring skills are truly honed. In fact, most of the techniques that we have learned throughout the course will be used in this chapter. With this in mind, we can think of this chapter, our last chapter in Beginning Algebra, as a good warm up to the final exam.

Rational Expressions and Equations

*7.1... Simplifying Rational Expressions**7.2... Multiplying and Dividing Rational Expressions**7.3... Adding and Subtracting Rational Expressions**7.4... Simplifying Complex Fractions**7.5... Solving Rational Equations**7.6... Word Problems**Sample Exam**Sample Exam Solutions****Cumulative Review****Sample Final Exam**Sample Final Exam Answers*

Problems Solved! Chapter 7 *Rational Expressions and Equations*
Simplifying Rational Expressions

The steps for simplifying in this section are to **factor then cancel**. Given a rational expression, the quotient of two polynomials, we will factor the numerator and denominator if we can and cancel factors that are exactly the same.

Rational Expression – The quotient of two polynomials $\frac{p(x)}{q(x)}$ where $q(x) \neq 0$.

When evaluating rational expressions we can plug in the appropriate values either before we simplify or after, the result will be the same. Although, it is more efficient to simplify first then evaluate.

Evaluate $\frac{25x^2 - 9}{5x^2 + 7x - 6}$ when $x = -1$ $\frac{25x^2 - 9}{5x^2 + 7x - 6} = \frac{25(-1)^2 - 9}{5(-1)^2 + 7(-1) - 6}$ $= \frac{25 - 9}{5 - 7 - 6} = \frac{16}{-8} = -2$	Evaluate $\frac{25x^2 - 9}{5x^2 + 7x - 6}$ when $x = -1$ $\frac{25x^2 - 9}{5x^2 + 7x - 6} = \frac{(5x+3)(5x-3)}{(5x-3)(x+2)}$ $= \frac{(5x+3)\cancel{(5x-3)}}{\cancel{(5x-3)}(x+2)} = \frac{5x+3}{x+2}$ Now evaluate when $x = -1$. $\frac{5x+3}{x+2} = \frac{5(-1)+3}{(-1)+2} = \frac{-2}{1} = -2$
--	--

Factor the numerator and denominator first then cancel.

We can see that when evaluating, the result will be the same whether or not we simplify first. It turns out that not all numbers can be used when we evaluate.

Evaluate $\frac{25x^2 - 9}{5x^2 + 7x - 6}$ when $x = -2$ $\frac{25x^2 - 9}{5x^2 + 7x - 6} = \frac{25(-2)^2 - 9}{5(-2)^2 + 7(-2) - 6}$ $= \frac{25(4) - 9}{5 \cdot 4 - 14 - 6} = \frac{91}{0} \text{ undefined}$	This indicates that we have a restriction, or that $x \neq -2$
--	---

To find restrictions
 $5x - 3 \neq 0$ and $x + 2 \neq 0$
 $5x \neq 3$
 $x \neq \frac{3}{5}$ $x \neq -2$

The point is that not all real numbers will be defined in the above rational expression. In fact there are two **restrictions to the domain**, $x \neq -2$ and $x \neq \frac{3}{5}$. These values, when plugged in, will result in zero in the denominator. Another way to say this is that the domain consists of all real numbers except for -2 and $\frac{3}{5}$. **Tip:** To find the restrictions, set each factor in the denominator “not equal” to zero and solve. The factors in the numerator do not contribute to the list of restrictions.



A. Simplify and state the restrictions to the domain.

$$\frac{x^2 - x - 6}{3x^2 - 8x - 3}$$

$$\frac{x^2 - x - 6}{3x^2 - 8x - 3} = \frac{(x+2)(x-3)}{(3x+1)(x-3)}$$

$$= \frac{(x+2)\cancel{(x-3)}}{(3x+1)\cancel{(x-3)}} = \frac{x+2}{3x+1}$$

Restrictions: $x \neq 3$ and $x \neq -\frac{1}{3}$

$$\frac{16y^2 - 1}{(4y+1)^2}$$

$$\frac{16y^2 - 1}{(4y+1)^2} = \frac{(4y+1)(4y-1)}{(4y+1)(4y+1)}$$

$$= \frac{\cancel{(4y+1)}(4y-1)}{\cancel{(4y+1)}(4y+1)} = \frac{4y-1}{4y+1}$$

Restrictions: $y \neq -\frac{1}{4}$

$$\frac{3x^2 + 4x - 15}{x^2 - 9}$$

$$\frac{3x^2 + 4x - 15}{x^2 - 9} = \frac{(3x-5)(x+3)}{(x+3)(x-3)}$$

$$= \frac{(3x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{3x-5}{x-3}$$

Restrictions: $x \neq 3$ and $x \neq -3$

$$\frac{4x^2 - 4x + 1}{(2x-1)^2}$$

$$\frac{4x^2 - 4x + 1}{(2x-1)^2} = \frac{(2x-1)(2x-1)}{(2x-1)(2x-1)}$$

$$= \frac{\cancel{(2x-1)}\cancel{(2x-1)}}{\cancel{(2x-1)}\cancel{(2x-1)}} = 1$$

Restrictions: $x \neq \frac{1}{2}$

Even if the factor cancels it still contributes to the list of restrictions. Basically, it is important to remember the domain of the original expression when simplifying. Also, we must use caution when simplifying, please do not try to take obviously incorrect short cuts like this:

$$\frac{x^2 - 25}{x^2 - 9} \neq \frac{\cancel{x^2} - 25}{\cancel{x^2} - 9} \neq \frac{-25}{-9}$$

Incorrect Cancellation $\uparrow$

$$\frac{x^2 - 49}{x^2 + 7x}$$

$$\frac{x^2 - 49}{x^2 + 7x} = \frac{(x+7)(x-7)}{x(x+7)}$$

$$= \frac{(x+7)\cancel{(x-7)}}{x\cancel{(x+7)}} = \frac{x-7}{x}$$

Restrictions: $x \neq 0$ and $x \neq -7$

$$\frac{6x^2 - 25x + 25}{3x^2 + 16x - 35}$$

$$\frac{6x^2 - 25x + 25}{3x^2 + 16x - 35} = \frac{(2x-5)(3x-5)}{(3x-5)(x+7)}$$

$$= \frac{(2x-5)\cancel{(3x-5)}}{\cancel{(3x-5)}(x+7)} = \frac{2x-5}{x+7}$$

Restrictions: $x \neq -7$ and $x \neq \frac{5}{3}$

$$\frac{x^3 - 1}{x^2 - 1}$$

$$\frac{x^3 - 1}{x^2 - 1} = \frac{(x-1)(x^2 + x + 1)}{(x+1)(x-1)}$$

$$= \frac{\cancel{(x-1)}(x^2 + x + 1)}{(x+1)\cancel{(x-1)}} = \frac{x^2 + x + 1}{x+1}$$

Restrictions: $x \neq \pm 1$

$$\frac{x^4 - 16}{x^2 + 4}$$

$$\frac{x^4 - 16}{x^2 + 4} = \frac{(x^2 + 4)(x^2 - 4)}{x^2 + 4}$$

$$= \frac{\cancel{(x^2 + 4)}(x+2)(x-2)}{\cancel{x^2 + 4}} = (x+2)(x-2)$$

Restrictions: None.

$x^2 + 4$ will never equal zero so there are no real restrictions.

Since subtraction is not commutative we must be alert to factors that are opposites. For example notice that $5 - 3 = 2$ and $3 - 5 = -2$. In general, $a - b = -(-a + b) = -(b - a)$.

Opposite Binomial Property: $a - b = -(b - a)$

Alternatively, $\frac{(a - b)}{(b - a)} = -1$

B. Simplify and state the restrictions to the domain.

$$\frac{(x-3)(2x-5)}{(x+3)(5-2x)}$$

$$\frac{(x-3)(2x-5)}{(x+3)(5-2x)} = \frac{(x-3)(2x-5)^{-1}}{(x+3)(5-2x)}$$

$$= \frac{(x-3)(-1)}{(x+3)} = -\frac{x-3}{x+3}$$

Restrictions: $x \neq -3$ and $x \neq \frac{5}{2}$

$$\frac{9y^2 - 64}{(8-3y)^2}$$

$$\frac{9y^2 - 64}{(8-3y)^2} = \frac{(3y+8)(3y-8)}{(8-3y)(8-3y)}$$

$$= \frac{(3y+8)(3y-8)^{-1}}{(8-3y)(8-3y)} = -\frac{3y+8}{8-3y}$$

Restrictions: $y \neq \frac{8}{3}$

Since addition is commutative, $x+3 = 3+x$, these factors cancel.

$$\frac{(3+x)^2}{x^2-9}$$

$$\frac{(3+x)^2}{x^2-9} = \frac{(3+x)(3+x)}{(x+3)(x-3)}$$

$$\Rightarrow = \frac{(3+x)(3+x)}{(x+3)(x-3)} = \frac{3+x}{x-3}$$

Restrictions: $x \neq \pm 3$

$$\frac{x^2 - 5x - 14}{7 - 15x + 2x^2}$$

$$\frac{x^2 - 5x - 14}{7 - 15x + 2x^2} = \frac{(x-7)(x+2)}{(7-x)(1-2x)}$$

$$= \frac{(x-7)(x+2)^{-1}}{(7-x)(1-2x)} = -\frac{x+2}{1-2x}$$

Restrictions: $x \neq 7$ and $x \neq \frac{1}{2}$

At this point we will do the same type of problems using function notation.

C. Simplify the function and evaluate.

Given $f(x) = \frac{10-9x-x^2}{x^2-100}$ find $f(1)$

$$f(x) = \frac{10-9x-x^2}{x^2-100} = \frac{(10+x)(1-x)}{(x+10)(x-10)}$$

$$= \frac{(10+x)(1-x)}{(x+10)(x-10)} = \frac{1-x}{x-10}$$

So $f(x) = \frac{1-x}{x-10}$ where $x \neq \pm 10$

$$f(1) = \frac{1-(1)}{(1)-10} = \frac{0}{-9} = 0$$

Given $g(x) = \frac{49-x^2}{2x^2-14x}$ find $g(-1)$

$$g(x) = \frac{49-x^2}{2x^2-14x} = \frac{(7+x)(7-x)}{2x(x-7)}$$

$$= \frac{(7+x)(7-x)^{-1}}{2x(x-7)} = -\frac{7+x}{2x}$$

So $g(x) = -\frac{7+x}{2x}$ when $x \neq 0$ and $x \neq 7$

$$g(-1) = -\frac{7+(-1)}{2(-1)} = -\frac{6}{-2} = 3$$

Multiplying rational expressions requires basically the same steps as simplifying them. We will simply **factor then cancel**. Only cancel factors that are exactly the same, or opposites, in the numerator and denominator.

Recall the property for multiplying fractions, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. When multiplying fractions there is no need for a common denominator, just multiply the numerators and denominators.

A. Multiply

$$\frac{(2x+3)}{x(x+5)} \cdot \frac{(x+5)}{(2x+3)(2x-1)}$$

$$\frac{(2x+3)}{x(x+5)} \cdot \frac{(x+5)}{(2x+3)(2x-1)}$$

$$= \frac{\cancel{(2x+3)}\cancel{(x+5)}}{x\cancel{(x+5)}\cancel{(2x+3)}(2x-1)}$$

$$= \frac{1}{x(2x-1)}$$

$$\frac{x^2-x}{2x^2+13x-7} \cdot \frac{2x^2+5x-3}{x^2+2x-3}$$

$$\frac{x^2-x}{2x^2+13x-7} \cdot \frac{2x^2+5x-3}{x^2+2x-3}$$

$$= \frac{x(x-1)}{(2x-1)(x+7)} \cdot \frac{(2x-1)(x+3)}{(x-1)(x+3)}$$

$$= \frac{x\cancel{(x-1)}\cancel{(2x-1)}\cancel{(x+3)}}{\cancel{(2x-1)}(x+7)\cancel{(x-1)}\cancel{(x+3)}}$$

$$= \frac{x}{x+7}$$

$$\frac{49y^2-1}{7y^2+29y+4} \cdot \frac{8+6y+y^2}{1-6y-7y^2}$$

$$\frac{49y^2-1}{7y^2+29y+4} \cdot \frac{8+6y+y^2}{1-6y-7y^2}$$

$$= \frac{(7y+1)(7y-1)}{(7y+1)(y+4)} \cdot \frac{(4+y)(2+y)}{(1+y)(1-7y)}$$

$$= \frac{\cancel{(7y+1)}\cancel{(7y-1)}(4+y)(2+y)}{\cancel{(7y+1)}(y+4)(1+y)\cancel{(1-7y)}}$$

$$= -\frac{2+y}{1+y} \left(\text{or } = -\frac{y+2}{y+1} \right)$$

$$\frac{x^2-4x-45}{x^2+10x+25} \cdot \frac{x^2+3x-10}{x^2-11x+18}$$

$$\frac{x^2-4x-45}{x^2+10x+25} \cdot \frac{x^2+3x-10}{x^2-11x+18}$$

$$= \frac{(x+5)(x-9)}{(x+5)(x+5)} \cdot \frac{(x-2)(x+5)}{(x-2)(x-9)}$$

$$= \frac{\cancel{(x+5)}\cancel{(x-9)}\cancel{(x-2)}\cancel{(x+5)}}{\cancel{(x+5)}\cancel{(x+5)}\cancel{(x-2)}\cancel{(x-9)}}$$

$$= 1$$

Restrictions:
 $x \neq -5, x \neq 2 \text{ or } x \neq 9$

The previous question did not ask for the restrictions but we certainly can list them anyway. Look at the factors in the denominator to see what values for x will make it zero. The list of restrictions is $\{-5, 2, 9\}$.

Remember that when using function notation $(f \cdot g)(x) = f(x) \cdot g(x)$ implies multiplication.

B. For the given functions find $(f \cdot g)(x)$.

$$f(x) = \frac{5x^2}{3x+2}, g(x) = \frac{3x+2}{25x^2-25x}$$

$$\begin{aligned} f(x) \cdot g(x) &= \frac{5x^2}{3x+2} \cdot \frac{3x+2}{25x^2-25x} \\ &= \frac{5x^2}{(3x+2)} \cdot \frac{(3x+2)}{25x(x-1)} \\ &= \frac{\cancel{5x^2}^5(3x+2)}{(3x+2)\cancel{25}x(x-1)} \\ &= \frac{x}{5(x-1)} \end{aligned}$$

$$f(x) = \frac{2x^2+5x-7}{x^2-1}, g(x) = \frac{x^2-x-2}{6x^2+31x+35}$$

$$\begin{aligned} f(x) \cdot g(x) &= \frac{2x^2+5x-7}{x^2-1} \cdot \frac{x^2-x-2}{6x^2+31x+35} \\ &= \frac{(2x+7)(x-1)}{(x+1)(x-1)} \cdot \frac{(x-2)(x+1)}{(2x+7)(3x+5)} \\ &= \frac{(2x+7)\cancel{(x-1)}(x-2)\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-1)}(2x+7)(3x+5)} \\ &= \frac{x-2}{3x+5} \end{aligned}$$

When dividing fractions, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, there is no need for a common

denominator. But the property reminds us that when dividing by a fraction the result will be the same as multiplying by the reciprocal of that fraction. At this point we will add a step when dividing, we need to **reciprocate, factor then cancel**.

C. Divide

$$\frac{(4x-1)(x+2)}{25x^2} \div \frac{(1-4x)}{100x^4}$$

$$\begin{aligned} &\frac{(4x-1)(x+2)}{25x^2} \div \frac{(1-4x)}{100x^4} \\ &= \frac{(4x-1)(x+2)}{25x^2} \cdot \frac{100x^4}{(1-4x)} \\ &= \frac{\cancel{4x-1}^{-1}(x+2) \cdot \cancel{100}^4x^4}{25x^2(1-4x)} \\ &= -4x^2(x+2) \end{aligned}$$

$$\frac{36x^2-1}{6x^2+4x} \div \frac{6x^2-5x-1}{10x^2-10x}$$

$$\begin{aligned} &\frac{36x^2-1}{6x^2+4x} \div \frac{6x^2-5x-1}{10x^2-10x} \\ &= \frac{36x^2-1}{6x^2+4x} \cdot \frac{10x^2-10x}{6x^2-5x-1} \\ &= \frac{(6x+1)(6x-1)}{2x(3x+2)} \cdot \frac{10x(x-1)}{(6x+1)(x-1)} \\ &= \frac{\cancel{6x+1}(6x-1) \cdot \cancel{10}^5x(x-1)}{2x(3x+2)\cancel{6x+1}(x-1)} \\ &= \frac{5(6x-1)}{3x+2} \end{aligned}$$

Restrictions: $x \neq -\frac{2}{3}$,
 $x \neq -\frac{1}{6}$, $x \neq 0$ or $x \neq 1$

The list of restrictions in the previous problem is a bit more involved. As we did before, look at all the factors in the denominator, even if it was cancelled, to find the values that make it zero $\{-2/3, -1/6, 0, 1\}$. **Tip:** Look at the denominators in each step to identify the restrictions.



$\frac{x^4 - y^4}{5x - 5y} \div \frac{x^2 + y^2}{25xy}$	$\frac{x^2 - 2x - 15}{x^2 + 10x + 21} \div \frac{2x^2 - 11x + 5}{2x^2 + 13x - 7}$
$\begin{aligned} & \frac{x^4 - y^4}{5x - 5y} \div \frac{x^2 + y^2}{25xy} \\ &= \frac{x^4 - y^4}{5x - 5y} \cdot \frac{25xy}{x^2 + y^2} \\ &= \frac{(x^2 + y^2)(x^2 - y^2)}{5(x - y)} \cdot \frac{25xy}{(x^2 + y^2)} \\ &= \frac{\cancel{(x^2 + y^2)}(x + y)(x - y) \cdot 5xy}{5(x - y)\cancel{(x^2 + y^2)}} \\ &= 5xy(x + y) \end{aligned}$	$\begin{aligned} & \frac{x^2 - 2x - 15}{x^2 + 10x + 21} \div \frac{2x^2 - 11x + 5}{2x^2 + 13x - 7} \\ &= \frac{x^2 - 2x - 15}{x^2 + 10x + 21} \cdot \frac{2x^2 + 13x - 7}{2x^2 - 11x + 5} \\ &= \frac{(x + 3)(x - 5)}{(x + 3)(x + 7)} \cdot \frac{(2x - 1)(x + 7)}{(2x - 1)(x - 5)} \\ &= \frac{\cancel{(x + 3)}\cancel{(x - 5)}\cancel{(2x - 1)}(x + 7)}{\cancel{(x + 3)}\cancel{(x + 7)}\cancel{(2x - 1)}\cancel{(x - 5)}} \\ &= 1 \end{aligned}$

Remember that when using function notation $(f \div g)(x) = f(x) \div g(x)$ implies division.

D. For the given functions find $(f \div g)(x)$.

$f(x) = \frac{81 - x^2}{9x + 18}, g(x) = \frac{x^2 - 4x - 45}{x^2 + 7x + 10}$	$f(x) = \frac{12x^2 + 23x - 9}{x^2 + 6x - 40}, g(x) = \frac{6x^2 + x - 1}{12 - 31x + 7x^2}$
$\begin{aligned} & f(x) \div g(x) \\ &= \frac{81 - x^2}{9x + 18} \div \frac{x^2 - 4x - 45}{x^2 + 7x + 10} \\ &= \frac{81 - x^2}{9x + 18} \cdot \frac{x^2 + 7x + 10}{x^2 - 4x - 45} \\ &= \frac{(9 + x)(9 - x)}{9(x + 2)} \cdot \frac{(x + 5)(x + 2)}{(x - 9)(x + 5)} \\ &= \frac{(9 + x)\cancel{(9 - x)}\cancel{(x + 5)}\cancel{(x + 2)}}{9\cancel{(x + 2)}\cancel{(x - 9)}\cancel{(x + 5)}} \\ &= -\frac{9 + x}{9} \end{aligned}$	$\begin{aligned} & f(x) \div g(x) \\ &= \frac{12x^2 + 23x - 9}{x^2 + 6x - 40} \div \frac{6x^2 + x - 1}{12 - 31x + 7x^2} \\ &= \frac{12x^2 + 23x - 9}{x^2 + 6x - 40} \cdot \frac{12 - 31x + 7x^2}{6x^2 + x - 1} \\ &= \frac{(3x - 1)(4x + 9)}{(x - 4)(x + 10)} \cdot \frac{(3 - 7x)(4 - x)}{(2x + 1)(3x - 1)} \\ &= \frac{\cancel{(3x - 1)}(4x + 9)(3 - 7x)\cancel{(4 - x)}}{\cancel{(x - 4)}(x + 10)(2x + 1)\cancel{(3x - 1)}} \\ &= -\frac{(4x + 9)(3 - 7x)}{(x + 10)(2x + 1)} \end{aligned}$

When adding or subtracting rational expressions we will need a common denominator.

The property that we will be using is $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$. There is an extra step, however, after we add or subtract we then need to simplify the resulting rational expression.

A. Add or subtract

$$\frac{x}{2x+3} + \frac{5}{2x+3}$$

$$\frac{x}{2x+3} + \frac{5}{2x+3}$$

$$= \frac{x+5}{2x+3}$$

$$\frac{2x+7}{(x+2)(x-5)} - \frac{x+5}{(x+2)(x-5)}$$

$$\frac{2x+7}{(x+2)(x-5)} - \frac{x+5}{(x+2)(x-5)}$$

Distribute the -1.

$$= \frac{2x+7-(x+5)}{(x+2)(x-5)} = \frac{2x+7-x-5}{(x+2)(x-5)} \leftarrow$$

$$= \frac{x+2}{(x+2)(x-5)} = \frac{1}{x-5}$$

$$\frac{x}{x^2-25} + \frac{5}{x^2-25}$$

$$\frac{x}{x^2-25} + \frac{5}{x^2-25}$$

$$= \frac{x+5}{x^2-25}$$

$$= \frac{(x+5)}{(x+5)(x-5)} = \frac{1}{x-5}$$

$$\frac{3x+4}{x-8} - \frac{2}{8-x}$$

$$\frac{3x+4}{x-8} - \frac{2}{8-x} \leftarrow$$

Use the fact that $8-x = -(x-8)$

$$= \frac{3x+4}{x-8} + \frac{2}{x-8}$$

$$= \frac{3x+4+2}{x-8} = \frac{3x+6}{x-8}$$

$$\frac{x^2+2}{x^2+3x-28} + \frac{x-22}{x^2+3x-28}$$

$$\frac{x^2+2}{x^2+3x-28} + \frac{x-22}{x^2+3x-28}$$

$$= \frac{x^2+2+x-22}{x^2+3x-28}$$

$$= \frac{x^2+x-20}{x^2+3x-28} = \frac{(x-4)(x+5)}{(x+7)(x-4)}$$

$$= \frac{(x-4)(x+5)}{(x+7)(x-4)} = \frac{x+5}{x+7}$$

$$\frac{2x^2}{x^2-9} + \frac{x+15}{9-x^2}$$

$$\frac{2x^2}{x^2-9} + \frac{x+15}{9-x^2}$$

$$= \frac{2x^2}{x^2-9} - \frac{x+15}{x^2-9}$$

$$= \frac{2x^2-x-15}{x^2-9} = \frac{(2x+5)(x-3)}{(x+3)(x-3)}$$

$$= \frac{(2x+5)(x-3)}{(x+3)(x-3)} = \frac{2x+5}{x+3}$$

If the rational expressions that we are adding or subtracting have **unlike denominators** then we will need to find the equivalent fractions with the same denominator. To do this multiply both the numerator and denominator of each expression by the factors needed to obtain a common denominator. To help determine the LCD, first factor the denominators.

B. Add or subtract

$\frac{1}{x} - \frac{1}{y}$ $\frac{1}{x} - \frac{1}{y} = \frac{1}{x} \left(\frac{y}{y} \right) - \frac{1}{y} \left(\frac{x}{x} \right)$ $= \frac{y}{xy} - \frac{x}{xy} = \frac{y-x}{xy}$	$\frac{3}{x+4} + 2$ $\frac{3}{x+4} + 2 = \frac{3}{x+4} + \frac{2}{1} \left(\frac{x+4}{x+4} \right)$ $= \frac{3+2(x+4)}{x+4} = \frac{3+2x+8}{x+4} = \frac{2x+11}{x+4}$
$\frac{-3x-2}{x^2-2x-3} + \frac{2}{x+1}$ $\frac{-3x-2}{x^2-2x-3} + \frac{2}{x+1}$ $= \frac{-3x-2}{(x+1)(x-3)} + \frac{2}{(x+1)} \cdot \frac{(x-3)}{(x-3)}$ $= \frac{-3x-2+2(x-3)}{(x+1)(x-3)}$ $= \frac{-3x-2+2x-6}{(x+1)(x-3)} = \frac{-x-8}{(x+1)(x-3)}$	$\frac{3x}{9x^2-16} - \frac{1}{3x+4}$ $\frac{3x}{9x^2-16} - \frac{1}{3x+4}$ $= \frac{3x}{(3x+4)(3x-4)} - \frac{1}{(3x+4)} \cdot \frac{(3x-4)}{(3x-4)}$ $= \frac{3x-1(3x-4)}{(3x+4)(3x-4)}$ $= \frac{3x-3x+4}{(3x+4)(3x-4)} = \frac{4}{(3x+4)(3x-4)}$
$\frac{2x}{x^2-1} + \frac{1}{x^2+x}$ $\frac{2x}{x^2-1} + \frac{1}{x^2+x}$ $= \frac{2x}{(x+1)(x-1)} + \frac{1}{x(x+1)}$ $= \frac{2x}{(x+1)(x-1)} \cdot \frac{x}{x} + \frac{1}{x(x+1)} \cdot \frac{(x-1)}{(x-1)}$ $= \frac{2x^2+1 \cdot (x-1)}{x(x+1)(x-1)}$ $= \frac{2x^2+x-1}{x(x+1)(x-1)} = \frac{(2x-1)(x+1)}{x(x+1)(x-1)}$ $= \frac{(2x-1)\cancel{(x+1)}}{x\cancel{(x+1)}(x-1)} = \frac{2x-1}{x(x-1)}$	$\frac{x}{x^2+4x+3} - \frac{3}{x^2-4x-5}$ $\frac{x}{x^2+4x+3} - \frac{3}{x^2-4x-5}$ $= \frac{x}{(x+1)(x+3)} - \frac{3}{(x+1)(x-5)}$ $= \frac{x}{(x+1)(x+3)} \cdot \frac{(x-5)}{(x-5)} - \frac{3}{(x+1)(x-5)} \cdot \frac{(x+3)}{(x+3)}$ $= \frac{x(x-5)-3(x+3)}{(x+1)(x+3)(x-5)} = \frac{x^2-5x-3x-9}{(x+1)(x+3)(x-5)}$ $= \frac{x^2-8x-9}{(x+1)(x+3)(x-5)} = \frac{(x+1)(x-9)}{(x+1)(x+3)(x-5)}$ $= \frac{\cancel{(x+1)}(x-9)}{\cancel{(x+1)}(x+3)(x-5)} = \frac{x-9}{(x+3)(x-5)}$

C. Add or subtract and state the restrictions.

$\frac{1}{x} + \frac{1}{(x+1)} + \frac{2}{x^2-1}$
$\begin{aligned} \frac{1}{x} + \frac{1}{(x+1)} + \frac{2}{x^2-1} &= \frac{1}{x} + \frac{1}{(x+1)} + \frac{2}{(x+1)(x-1)} \\ &= \frac{1}{x} \cdot \frac{(x+1)(x-1)}{(x+1)(x-1)} + \frac{1}{(x+1)} \cdot \frac{x(x-1)}{x(x-1)} + \frac{2}{(x+1)(x-1)} \cdot \frac{x}{x} \\ &= \frac{(x+1)(x-1) + x(x-1) + 2x}{x(x+1)(x-1)} = \frac{x^2 - x + x - 1 + x^2 - x + 2x}{x(x+1)(x-1)} \\ &= \frac{2x^2 + x - 1}{x(x+1)(x-1)} = \frac{(2x-1)(x+1)}{x(x+1)(x-1)} = \frac{(2x-1)\cancel{(x+1)}}{x\cancel{(x+1)}(x-1)} = \frac{2x-1}{x(x-1)} \end{aligned}$ Restrictions: $x \neq 0$, $x \neq -1$ and $x \neq 1$

$\frac{3}{x^2-9} - \frac{x}{x^2-6x+9} + \frac{1}{x+3}$
$\begin{aligned} \frac{3}{x^2-9} - \frac{x}{x^2-6x+9} + \frac{1}{x+3} &= \frac{3}{(x+3)(x-3)} - \frac{x}{(x-3)(x-3)} + \frac{1}{(x+3)} \\ &= \frac{3}{(x+3)(x-3)} \cdot \frac{(x-3)}{(x-3)} - \frac{x}{(x-3)(x-3)} \cdot \frac{(x+3)}{(x+3)} + \frac{1}{(x+3)} \cdot \frac{(x-3)(x-3)}{(x-3)(x-3)} \\ &= \frac{3(x-3) - x(x+3) + (x-3)(x-3)}{(x-3)(x-3)(x+3)} = \frac{3x-9-x^2-3x+x^2-3x+9}{(x-3)^2(x+3)} \\ &= \frac{-6x}{(x-3)^2(x+3)} \end{aligned}$ Restrictions: $x \neq 3$ and $x \neq -3$

D. For the given functions find $f(x) - g(x)$ and state the restrictions to the domain.

$f(x) = \frac{1}{x+7} \text{ and } g(x) = \frac{x-2}{x-7}$	$f(x) = \frac{1}{2} + \frac{2}{x} \text{ and } g(x) = \frac{1}{2} - \frac{1}{x}$
$\begin{aligned} f(x) - g(x) &= \frac{1}{x+7} - \frac{x-2}{x-7} \\ &= \frac{(x-7) - (x+2)(x+7)}{(x+7)(x-7)} \\ &= \frac{x-7 - (x^2+2x+7x+14)}{(x+7)(x-7)} \\ &= \frac{x-7-x^2-9x-14}{(x+7)(x-7)} \\ &= \frac{-x^2-8x-21}{(x+7)(x-7)} \\ &= \frac{-(x^2+8x+21)}{(x+7)(x-7)} \end{aligned}$ Restrictions: $x \neq -7$ and $x \neq 7$	$\begin{aligned} f(x) - g(x) &= \left(\frac{1}{2} + \frac{2}{x}\right) - \left(\frac{1}{2} - \frac{1}{x}\right) \\ &= \left(\frac{x+4}{2x}\right) - \left(\frac{x-2}{2x}\right) \\ &= \frac{x+4-x+2}{2x} \\ &= \frac{6}{2x} = \frac{3}{x} \end{aligned}$ Restriction: $x \neq 0$

Problems Solved! Chapter 7 *Rational Expressions and Equations*
Simplifying Complex Fractions

It turns out that we have all the tools needed to simplify what are called **complex fractions**. The numerator and denominator of these rational expressions contain fractions and look very intimidating. We will outline two methods for simplifying them.

Method 1: Obtain a common denominator for the numerator and denominator, multiply by the reciprocal of the denominator then factor and cancel.

Simplify $\frac{\frac{1}{3a} - \frac{1}{3b}}{\frac{a}{b} - \frac{b}{a}}$	
Multiply by the reciprocal of the denominator.	$\frac{\frac{1}{3a} - \frac{1}{3b}}{\frac{a}{b} - \frac{b}{a}} = \frac{\left(\frac{b-a}{3ab}\right)}{\left(\frac{a^2-b^2}{ab}\right)} \quad \leftarrow \text{Common denominators first.}$ $\Rightarrow = \frac{b-a}{3ab} \cdot \frac{ab}{a^2-b^2} = \frac{(b-a)}{3ab} \cdot \frac{ab}{(a+b)(a-b)} \quad \leftarrow \text{Factor then cancel.}$ $= \frac{\cancel{(b-a)}}{3\cancel{a}b} \cdot \frac{\cancel{a}b}{(a+b)\cancel{(a-b)}} = -\frac{1}{3(a+b)}$

Method 2: Multiply the numerator and denominator of the complex fraction by the LCD of all the simple fractions then factor and cancel.

Simplify $\frac{\frac{1}{3a} - \frac{1}{3b}}{\frac{a}{b} - \frac{b}{a}}$	
Factor then cancel.	$\frac{\frac{1}{3a} - \frac{1}{3b}}{\frac{a}{b} - \frac{b}{a}} = \frac{\left(\frac{1}{3a} - \frac{1}{3b}\right) 3ab}{\left(\frac{a}{b} - \frac{b}{a}\right) 3ab} \quad \leftarrow \text{Multiply numerator and denominator by the LCD}$ $= \frac{b-a}{3a^2-3b^2} = \frac{(b-a)}{3(a^2-b^2)}$ $\Rightarrow = \frac{\cancel{(b-a)}}{3(a+b)\cancel{(a-b)}} = -\frac{1}{3(a+b)}$

To illustrate what happened after we multiplied by the LCD we could add an extra step.

$$\frac{\left(\frac{1}{3a} - \frac{1}{3b}\right) 3ab}{\left(\frac{a}{b} - \frac{b}{a}\right) 3ab} = \frac{\frac{1}{\cancel{3}a} \cdot \cancel{3}ab - \frac{1}{\cancel{3}b} \cdot \cancel{3}ab}{\frac{a}{\cancel{b}} \cdot \cancel{3}ab - \frac{b}{\cancel{a}} \cdot \cancel{3}ab} = \frac{b-a}{3a^2-3b^2} = \dots$$

For the following solved problems both methods are used for each problem. Choose whichever method feels most comfortable for you.

A. Simplify using method 1.

$$\frac{\frac{1}{x^2} - \frac{1}{64}}{\frac{1}{8} + \frac{1}{x}}$$

$$\frac{\frac{1}{x^2} - \frac{1}{64}}{\frac{1}{8} + \frac{1}{x}} = \frac{\left(\frac{64-x^2}{64x^2}\right)}{\left(\frac{x+8}{8x}\right)}$$

$$= \frac{64-x^2}{64x^2} \cdot \frac{8x}{x+8} = \frac{(8+x)(8-x)}{64x^2} \cdot \frac{8x}{x+8}$$

$$= \frac{(8+x)(8-x) \cdot 8x}{64x^2(x+8)} = \frac{8-x}{8x}$$

B. Simplify using method 2.

$$\frac{\frac{1}{x^2} - \frac{1}{64}}{\frac{1}{8} + \frac{1}{x}}$$

$$\frac{\frac{1}{x^2} - \frac{1}{64}}{\frac{1}{8} + \frac{1}{x}} = \frac{\left(\frac{1}{x^2} - \frac{1}{64}\right) 64x^2}{\left(\frac{1}{8} + \frac{1}{x}\right) 64x^2}$$

$$= \frac{64-x^2}{8x^2+64x} = \frac{(8+x)(8-x)}{8x(x+8)}$$

$$= \frac{(8+x)(8-x)}{8x(x+8)} = \frac{8-x}{8x}$$

$$\frac{1 - \frac{4}{x} - \frac{21}{x^2}}{1 - \frac{2}{x} - \frac{15}{x^2}}$$

$$\frac{1 - \frac{4}{x} - \frac{21}{x^2}}{1 - \frac{2}{x} - \frac{15}{x^2}} = \frac{\left(\frac{x^2-4x-21}{x^2}\right)}{\left(\frac{x^2-2x-15}{x^2}\right)}$$

$$= \frac{x^2-4x-21}{x^2} \cdot \frac{x^2}{x^2-2x-15} = \frac{(x-7)(x+3)}{x^2} \cdot \frac{x^2}{(x+3)(x-5)}$$

$$= \frac{(x-7)(x+3) \cdot x^2}{x^2(x+3)(x-5)} = \frac{x-7}{x-5}$$

$$\frac{1 - \frac{4}{x} - \frac{21}{x^2}}{1 - \frac{2}{x} - \frac{15}{x^2}}$$

$$\frac{1 - \frac{4}{x} - \frac{21}{x^2}}{1 - \frac{2}{x} - \frac{15}{x^2}} = \frac{\left(1 - \frac{4}{x} - \frac{21}{x^2}\right) \cdot x^2}{\left(1 - \frac{2}{x} - \frac{15}{x^2}\right) \cdot x^2}$$

$$= \frac{x^2-4x-21}{x^2-2x-15} = \frac{(x+3)(x-7)}{(x+3)(x-5)}$$

$$= \frac{(x+3)(x-7)}{(x+3)(x-5)} = \frac{x-7}{x-5}$$

$$\frac{\frac{1}{x} - \frac{1}{x-2}}{\frac{4}{x^2-2x}}$$

$$\frac{\frac{1}{x} - \frac{1}{x-2}}{\frac{4}{x^2-2x}} = \frac{\frac{x-2-x}{x(x-2)}}{\frac{4}{x^2-2x}}$$

$$= \frac{\frac{x-2-x}{x(x-2)}}{\frac{4}{x^2-2x}} = \frac{-2}{x(x-2)} \cdot \frac{x^2-2x}{4} = \frac{-2}{x(x-2)} \cdot \frac{x(x-2)}{4}$$

$$= \frac{-2 \cdot x(x-2)}{x(x-2) \cdot 4} = -\frac{1}{2}$$

$$\frac{\frac{1}{x} - \frac{1}{x-2}}{\frac{4}{x^2-2x}}$$

$$\frac{\frac{1}{x} - \frac{1}{x-2}}{\frac{4}{x^2-2x}} = \frac{\left(\frac{1}{x} - \frac{1}{x-2}\right) x(x-2)}{\left(\frac{4}{x^2-2x}\right) x(x-2)}$$

$$= \frac{x-2-x}{4} = \frac{-2}{4} = -\frac{1}{2}$$

$$\frac{1 - \frac{1}{x^2}}{\frac{1}{x} - 1}$$

$$\frac{1 - \frac{1}{x^2}}{\frac{1}{x} - 1} = \frac{\left(\frac{x^2-1}{x^2}\right)}{\left(\frac{1-x}{x}\right)} = \frac{x^2-1}{x^2} \cdot \frac{x}{1-x}$$

$$= \frac{(x+1)(x-1) \cdot x}{x^2(1-x)} = -\frac{x+1}{x}$$

$$\frac{1 - \frac{1}{x^2}}{\frac{1}{x} - 1}$$

$$\frac{1 - \frac{1}{x^2}}{\frac{1}{x} - 1} = \frac{\left(1 - \frac{1}{x^2}\right) \cdot x^2}{\left(\frac{1}{x} - 1\right) \cdot x^2}$$

$$= \frac{x^2-1}{x-x^2} = \frac{(x+1)(x-1)}{x(1-x)} = -\frac{x+1}{x}$$

Problems Solved! Chapter 7 *Rational Expressions and Equations*
Solving Rational Equations

Rational equations are simply equations with rational expressions in them. We can use the technique outlined earlier to clear the fractions of a rational equation. After clearing the fractions we will be left with either a linear or quadratic equation that can be solved as usual.

Solve $\frac{1}{x-4} + \frac{x}{x-2} = \frac{2}{x^2 - 6x + 8}$		
Step 1: Factor the denominators.	$\frac{1}{x-4} + \frac{x}{x-2} = \frac{2}{x^2 - 6x + 8}$ $\Rightarrow \frac{1}{(x-4)} + \frac{x}{(x-2)} = \frac{2}{(x-2)(x-4)}$	Step 2: Identify the restrictions. Restrictions: $x \neq 2$ and $x \neq 4$
	$(x-2)(x-4) \left(\frac{1}{(x-4)} + \frac{x}{(x-2)} \right) = \left(\frac{2}{(x-2)(x-4)} \right) (x-2)(x-4)$	Step 3: Multiply both sides by the LCD.
Step 4: Solve like normal. In this case solve the resulting quadratic equation.	$\Rightarrow (x-2) + x(x-4) = 2$ $x-2 + x^2 - 4x = 2$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x-4=0 \quad \text{or} \quad x+1=0$ $x=4 \quad \text{or} \quad x=-1$ Restriction $x=-1$ Solution set: $\{-1\}$	
		Step 5: Check answers against the restrictions.

This process sometimes produces answers that do not solve the original equation, so it is extremely important to check them. **Tip:** It suffices to check that the answers are not restrictions to the domain of the original equation.



A. Solve

$\frac{x}{2x+3} + \frac{5}{2x+3} = 7$
$\frac{x}{2x+3} + \frac{5}{2x+3} = 7 \quad \text{Restriction } x \neq -\frac{3}{2}$ $(2x+3) \left(\frac{x}{2x+3} + \frac{5}{2x+3} \right) = 7(2x+3)$ $x + 5 = 7(2x+3)$ $x + 5 = 14x + 21$ $-13x = 16$ $x = -\frac{16}{13}$

$\frac{2x+7}{(x+2)(x-5)} = \frac{x+5}{(x+2)(x-5)}$
$\frac{2x+7}{(x+2)(x-5)} = \frac{x+5}{(x+2)(x-5)} \quad \text{Restrictions: } x \neq -2, x \neq 5$ $(x+2)(x-5) \left(\frac{2x+7}{(x+2)(x-5)} \right) = \left(\frac{x+5}{(x+2)(x-5)} \right) (x+2)(x-5)$ $2x+7 = x+5$ $x = -2$ Restriction $\rightarrow$ No solution or $\emptyset$

Determining the LCD is the step that most students have difficulty with. When deciding what the LCD is, use one of each factor, make sure that when we multiply it cancels with all the denominators and clears the fractions. If it does not, the LCD is incorrect.

B. Solve

$$\frac{1}{8} + \frac{1}{x} = \frac{1}{4}$$

$$\frac{1}{8} + \frac{1}{x} = \frac{1}{4}, \quad x \neq 0$$

$$8x \left(\frac{1}{8} + \frac{1}{x} \right) = 8x \left(\frac{1}{4} \right)$$

$$\begin{array}{r} x + 8 = 2x \\ -x \quad -x \\ \hline 8 = x \end{array}$$

Solution Set: $\{8\}$

$$\frac{3x+4}{x-8} - \frac{2}{8-x} = 1$$

$$\frac{3x+4}{x-8} - \frac{2}{8-x} = 1, \quad x \neq 8$$

$$\frac{3x+4}{x-8} + \frac{2}{x-8} = 1 \quad \leftarrow \text{Use the fact that } 8-x = -(x-8)$$

$$(x-8) \left(\frac{3x+4}{x-8} + \frac{2}{x-8} \right) = 1 \cdot (x-8)$$

$$3x+4+2 = x-8$$

$$3x+6 = x-8$$

$$2x = -14$$

$$x = -7$$

If we multiply both sides by the LCD we would next distribute it through the expression. We can save this step and simply multiply every term of the rational expression by the LCD as illustrated below.

$$\frac{2x}{x^2-9} + \frac{3}{x+3} = \frac{1}{x-3}$$

$$\frac{2x}{x^2-9} + \frac{3}{x+3} = \frac{1}{x-3}, \quad x \neq 3, \quad x \neq -3$$

$$\frac{2x}{(x+3)(x-3)} + \frac{3}{(x+3)} = \frac{1}{(x-3)}$$

$$\frac{2x(x+3)(x-3)}{(x+3)(x-3)} + \frac{3(x+3)(x-3)}{(x+3)} = \frac{1(x+3)(x-3)}{(x-3)}$$

$$2x + 3(x-3) = (x+3)$$

$$2x + 3x - 9 = x + 3$$

$$5x - 9 = x + 3$$

$$4x = 12$$

$$x = 3$$

Restriction
No Solution, $\emptyset$

$$\frac{x}{x-1} + \frac{1}{6x-1} = \frac{x}{(x-1)(6x-1)}$$

$$\frac{x}{x-1} + \frac{1}{6x-1} = \frac{x}{(x-1)(6x-1)}, \quad x \neq 1, \quad x \neq \frac{1}{6}$$

$$\frac{x(x-1)(6x-1)}{(x-1)} + \frac{1(x-1)(6x-1)}{(6x-1)} = \frac{x(x-1)(6x-1)}{(x-1)(6x-1)} \quad \leftarrow$$

$$x(6x-1) + (x-1) = x$$

$$6x^2 - x + x - 1 = x$$

$$6x^2 - x - 1 = 0$$

$$(2x-1)(3x+1) = 0$$

$$2x-1=0 \text{ or } 3x+1=0$$

$$2x=1 \quad 3x=-1$$

$$x=\frac{1}{2} \quad x=-\frac{1}{3}$$

Alternatively, we can multiply each term by the LCD.

Solution Set: $\{-\frac{1}{3}, \frac{1}{2}\}$

It is tempting to simply multiply all the factors of the denominator together to obtain a common multiple. This will work, but usually leaves us with an equation that is too cumbersome to work with. If we use the Least Common multiple of all the factors in the denominators then we will have less factoring in the end. Bottom line, it is worth spending the extra time to find the LCD.

$\frac{6}{x-5} + \frac{6}{x^2-11x+30} = \frac{2}{x-6}$ $\frac{6}{x-5} + \frac{6}{x^2-11x+30} = \frac{2}{x-6}$ $\frac{6}{(x-5)} + \frac{6}{(x-6)(x-5)} = \frac{2}{(x-6)} \quad \begin{matrix} x \neq 6 \\ x \neq 5 \end{matrix}$ $\frac{6(x-5)(x-6)}{(x-5)} + \frac{6(x-5)(x-6)}{(x-6)(x-5)} = \frac{2(x-5)(x-6)}{(x-6)}$ $6(x-6) + 6 = 2(x-5)$ $6x - 36 + 6 = 2x - 10$ $6x - 30 = 2x - 10$ $4x = 20$ $x = 5$ Restriction No Solution, $\emptyset$	$\frac{x+7}{x-1} + \frac{x-1}{x+1} = \frac{4}{x^2-1}$ $\frac{x+7}{x-1} + \frac{x-1}{x+1} = \frac{4}{x^2-1}, \quad x \neq \pm 1$ $\frac{(x+7)}{(x-1)} + \frac{(x-1)}{(x+1)} = \frac{4}{(x+1)(x-1)}$ $\frac{(x+7)(x+1)(x-1)}{(x-1)} + \frac{(x-1)(x+1)(x-1)}{(x+1)} = \frac{4(x+1)(x-1)}{(x+1)(x-1)}$ $(x+7)(x+1) + (x-1)(x-1) = 4$ $x^2 + x + 7x + 7 + x^2 - x - x + 1 = 4$ $2x^2 + 6x + 8 = 4$ $2x^2 + 6x + 4 = 0$ $2(x^2 + 3x + 2) = 0$ $2(x+2)(x+1) = 0$ $x+2=0 \text{ OR } x+1=0$ $\boxed{x=-2} \quad \begin{matrix} x=-1 \\ \text{Restriction} \end{matrix}$ Solution set $\{-2\}$
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Some **literal equations**, often referred to as formulas, are also rational equations. Use the techniques of this section and clear the fractions before solving for the particular variable.

C. Solve for the specified variable

Solve $\frac{D}{r} = t$ for D. $\frac{D}{r} = t$ $r\left(\frac{D}{r}\right) = r \cdot t$ $D = rt$	Solve $\frac{2A}{b} = h$ for A. $\frac{2A}{b} = h$ $b\left(\frac{2A}{b}\right) = b \cdot h \quad \int \quad \frac{2A}{2} = \frac{bh}{2}$ $2A = bh \quad \int \quad A = \frac{bh}{2}$
Solve $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ for a. $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ $\frac{1abc}{a} + \frac{1abc}{b} = \frac{1abc}{c}$ $bc + ac = ab - ac$ $bc = ab - ac$ $bc = a(b-c)$ $\frac{bc}{(b-c)} = \frac{a(b-c)}{(b-c)}$ $\frac{bc}{b-c} = a$	Solve $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ $\frac{1RR_1R_2}{R} = \frac{1RR_1R_2}{R_1} + \frac{1RR_1R_2}{R_2}$ $R_1R_2 = RR_2 + RR_1$ $R_1R_2 = R(R_2 + R_1)$ $\frac{R_1R_2}{(R_1+R_2)} = \frac{R(R_2+R_1)}{(R_2+R_1)}$ $\frac{R_1R_2}{(R_1+R_2)} = R$

Solve $m = \frac{y-y_1}{x-x_1}$ for y .	Solve $\frac{x}{a} + \frac{y}{b} = 1$ for a .
$m = \frac{y-y_1}{x-x_1}$ $m(x-x_1) = \frac{y-y_1}{(x-x_1)}(x-x_1)$ $m(x-x_1) = y-y_1$ $m(x-x_1) + y_1 = y$	$\frac{x}{a} + \frac{y}{b} = 1$ $xb + ya = ab$ $xb = ab - ya$ $xb = a(b-y)$ $\frac{xb}{(b-y)} = a$

The **reciprocal** of a number is the number we obtain by dividing 1 by that number. Often we think of the reciprocal as an interchange of the numerator and denominator but this definition fails us when the number to be reciprocated is a variable.

Typical Number Problem

The reciprocal of the larger of two consecutive positive odd integers is subtracted from twice the reciprocal of the smaller and the result is $\frac{9}{35}$. Find the two integers.

Let x = the smaller positive odd integer.

then $x+2$ = the larger positive odd integer.

The reciprocal of x is $\frac{1}{x}$ and the reciprocal of $x+2$ is $\frac{1}{x+2}$

$$\text{Set-up} \rightarrow 2\left(\frac{1}{x}\right) - \frac{1}{x+2} = \frac{9}{35}$$

$$\text{OR} \quad \frac{2}{x} - \frac{1}{(x+2)} = \frac{9}{35}$$

$$35x(x+2)\left(\frac{2}{x} - \frac{1}{(x+2)}\right) = \left(\frac{9}{35}\right) \cdot 35x(x+2)$$

$$70(x+2) - 35x = 9x(x+2)$$

$$70x + 140 - 35x = 9x^2 + 18x$$

$$\begin{array}{rcl} 35x + 140 & = & 9x^2 + 18x \\ -35x & -140 & -35x - 140 \end{array}$$

$$0 = 9x^2 - 17x - 140$$

$$0 = (9x + 28)(x - 5)$$

$$9x + 28 = 0 \quad \text{OR} \quad x - 5 = 0$$

$$9x = -28$$

$$x = -\frac{28}{9}$$

negative

$$\boxed{x = 5}$$

The two positive odd integers are 5 and 7.

The set-up for the applications in this chapter will be rational equations. The keyword, **reciprocal**, appears often. Remember that the reciprocal of a number is 1 divided by that number.

A. Number Problems (w/ keyword reciprocal)

One positive integer is 5 more than the other. When the reciprocal of the larger number is subtracted from the reciprocal of the smaller the result is $\frac{5}{14}$. Find the two integers.

Let x = the smaller positive integer

then $x+5$ = the larger integer.

Set-up $\rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$$\frac{1 \cdot x(x+5) \cdot 14}{x} - \frac{1 \cdot x(x+5) \cdot 14}{x+5} = \frac{5 \cdot x(x+5) \cdot 14}{14}$$

$$14(x+5) - 14x = 5x(x+5)$$

$$14x + 70 - 14x = 5x^2 + 25x$$

$$70 = 5x^2 + 25x$$

$$0 = 5x^2 + 25x - 70$$

$$0 = (x+7)(5x-10)$$

$$\begin{array}{l} x+7=0 \text{ or } 5x-10=0 \\ x=-7 \quad 5x=10 \\ \text{negative} \quad \boxed{x=2} \end{array}$$

The two integers are 2 and 7.

The difference between two integers is 5. If the reciprocal of the smaller is added to twice the reciprocal of the larger the result is $\frac{23}{66}$. Find the two integers.

Let x = the larger integer

$x-5$ = the smaller integer

$$\begin{cases} x-y=5 \\ -y=-x+5 \\ y=x-5 \end{cases}$$

Set-up $\rightarrow 2\left(\frac{1}{x}\right) + \frac{1}{x-5} = \frac{23}{66}$

$$\frac{2 \cdot 66x(x-5)}{x} + \frac{1 \cdot 66x(x-5)}{x-5} = \frac{23 \cdot 66x(x-5)}{66}$$

$$132(x-5) + 66x = 23x(x-5)$$

$$132x - 660 + 66x = 23x^2 - 115x$$

$$198x - 660 = 23x^2 - 115x$$

$$0 = 23x^2 - 313x + 660$$

$$0 = (23x-60)(x-11)$$

$$\begin{array}{l} 23x-60=0 \text{ or } x-11=0 \\ 23x=60 \quad \boxed{x=11} \\ x=60/23 \end{array}$$

The two integers are 11 and 6.

The difference between the reciprocals of two consecutive positive odd integers is $\frac{2}{15}$.

Find the integers.

Let x = the first positive odd integer.

$x+2$ = the next odd integer

Set-up $\rightarrow \frac{1}{x} - \frac{1}{x+2} = \frac{2}{15}$

$$\frac{1 \cdot 15x(x+2)}{x} - \frac{1 \cdot 15x(x+2)}{x+2} = \frac{2 \cdot 15x(x+2)}{15}$$

$$15(x+2) - 15x = 2x(x+2) \rightarrow 0 = 2(x^2 + 2x - 15)$$

$$15x + 30 - 15x = 2x^2 + 4x \quad 0 = 2(x - 3)(x + 5)$$

$$30 = 2x^2 + 4x$$

$$x - 3 = 0 \text{ or } x + 5 = 0$$

$$0 = 2x^2 + 4x - 30 \quad \boxed{x = 3} \quad x = -5 \text{ negative}$$

The two integers are 3 and 5.

We have two equivalent formulas to choose from when solving **work-rate problems**. If two people are working together on a job then their rates add and they can perform the job working together in a shorter amount of time.

If we let x = time it takes person 1 to complete the task then his work rate is $\frac{1}{x}$. In other words, he can complete the 1 job in x number of hours. If we let y = time it takes person 2 to complete the task and t = time it takes with both working together we get the following formulas:

Work – Rate Formulas: $\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$ or alternatively $\frac{t}{x} + \frac{t}{y} = 1$

B. Work-Rate Problems

Bill's garden hose can fill the pool in 12 hours. His neighbor has a hose that can fill the pool in 15 hours. How long will it take to fill the pool using both hoses?

Let t = time it takes to fill the pool with both hoses.

$$\text{Rate of Bill's hose} + \text{Rate of Neighbor's hose} = \text{Rate of Both hoses}$$

$$\frac{1}{12} + \frac{1}{15} = \frac{1}{t}$$



The rates add,
not the times.

$$\frac{1}{12} 60t + \frac{1}{15} 60t = \frac{1}{t} 60t$$

$$5t + 4t = 60$$

$$9t = 60$$

$$\begin{aligned} t &= \frac{60}{9} \\ &= 6\frac{6}{9} \\ &= 6\frac{2}{3} \end{aligned}$$

Both hoses can fill the pool in $6\frac{2}{3}$ hours.

Joe can complete his yard work in 3 hours. If his son helps it will only take 2 hours working together. How long would the yard work take if his son was working alone?

Let y = time it takes Joe's son to do the yard.

$$\text{Set-up} \rightarrow \frac{1}{3} + \frac{1}{y} = \frac{1}{2}$$

$$\frac{1}{3} \cdot 6y + \frac{1}{y} \cdot 6y = \frac{1}{2} \cdot 6y$$

$$2y + 6 = 3y$$

$$6 = y$$

Working alone, it would take Joe's son 6 hours.

Norm and Cliff can paint the office in 5 hours working together. Being a professional painter, Norm can paint twice as fast as Cliff. How long would it take Cliff to paint the office by himself?

Let x = time it takes Norm to paint the office alone.

$2x$ = time it will take Cliff working alone

$$\text{Set-up} \rightarrow \frac{1}{x} + \frac{1}{2x} = \frac{1}{5}$$

$$\frac{1}{x} \cdot 10x + \frac{1}{2x} \cdot 10x = \frac{1}{5} \cdot 10x$$

$$10 + 5 = 2x$$

$$15 = 2x$$

$$\frac{15}{2} = x$$

Back Substitute:

$$2x = 2\left(\frac{15}{2}\right) = 15$$

So it would take Cliff 15 hours to paint the office alone.

Cliff is slower so it will take him twice as long as Norm working alone.

We have set up **uniform motion problems** using the formula $D = rt$. For the following motion problems we will need the equivalent formula $\frac{D}{r} = t$ to set up the equations.

C. Uniform Motion Problems

The first leg of Mary's road trip consisted of 120 miles of traffic. When the traffic cleared she was able to drive twice as fast for 300 miles. If the total trip took 9 hours how long was she stuck in traffic?

Let x = Mary's speed in traffic

$2x$ = speed after traffic

Set-up

$$\frac{120}{x} + \frac{300}{2x} = 9$$

$$\frac{120 \cdot 2x}{x} + \frac{300 \cdot 2x}{2x} = 9 \cdot 2x$$

$$240 + 300 = 18x$$

$$540 = 18x$$

$$\frac{540}{18} = \frac{18x}{18}$$

$$30 = x$$

$$\text{so } 60 = 2x$$

She was in traffic for 4 hours.

	$D = r \cdot t$		
1 st leg	120	x	$\frac{120}{x}$
2 nd leg	300	$2x$	$\frac{300}{2x}$
			9 hrs ← Total

set-up

time in traffic

$$\text{was } \frac{120}{x} = \frac{120}{30}$$

$$= 4 \text{ hrs.}$$

A passenger train can travel 20mph faster than a freight train. If the passenger train can cover 390 miles in the same time it takes the freight train to cover 270 miles, how fast is each train?

let x = speed of freight train.
 $x+20$ = speed of passenger train.

Set-up $\rightarrow \frac{270}{x} = \frac{390}{x+20}$

Set-up

	D	r	t
freight train	270	x	$\frac{270}{x}$
passenger train	390	$x+20$	$\frac{390}{x+20}$

"same time"

$$270(x+20) = 390x$$

$$270x + 5400 = 390x$$

$$5400 = 120x$$

$$\frac{5400}{120} = \frac{120x}{120}$$

$$45 = x$$

$$\text{so } 65 = x+20$$

Freight train speed is 45mph and passenger train's speed is 65mph

Billy rode his skateboard 24 miles to his grandmother's house for the day. It was a rough ride so he borrowed his grandmother's bicycle for the return trip. Going twice as fast on the bicycle the return trip took 2 hours less time. What was his average speed on the bicycle?

let x = speed on skateboard
 $2x$ = speed on bike

Set-up $\frac{24}{x} - \frac{24}{2x} = 2$

Set-up

	D	r	t
skateboard	24	x	$\frac{24}{x}$
Bicycle	24	$2x$	$\frac{24}{2x}$

$$\frac{24}{x} \cdot x - \frac{12}{x} \cdot x = 2 \cdot x$$

$$24 - 12 = 2x$$

$$12 = 2x$$

$$6 = x$$

Billy averaged 6mph on the skateboard and 12mph on the bicycle.

Brett lives on the river 45 miles upstream from town. When the current is 2mph he can row his boat downstream to town for supplies and back in 14 hours. What is his average rowing speed in still water?

let x = Brett's rowing speed

Set-up

	D	r	t
Downstream	45	$x+2$	$\frac{45}{x+2}$
Upstream	45	$x-2$	$\frac{45}{x-2}$
			14 Total time

$D = rt$
 $\frac{D}{r} = t$

$$\frac{45}{x+2} + \frac{45}{x-2} = 14$$

$$\frac{45(x+2)(x-2)}{(x+2)} + \frac{45(x+2)(x-2)}{(x-2)} = 14(x+2)(x-2)$$

$$45(x-2) + 45(x+2) = 14(x+2)(x-2)$$

$$45x - 90 + 45x + 90 = 14(x^2 - 4)$$

$$90x = 14x^2 - 56$$

$$0 = 14x^2 - 90x - 56$$

$$0 = 2(7x^2 - 45x - 28)$$

$$0 = 2(7x+4)(x-7)$$

$$7x+4=0 \text{ or } x-7=0$$

$$7x=-4$$

$$x=-\frac{4}{7}$$

Negative

$x=7$

Brett's rowing speed in still water is 7mph.

Variation problems often set up as rational equations. Given two quantities x and y the following keywords indicate a particular relationship where k is called the variation constant.

<u>Direct Variation:</u>	<u>Inverse Variation:</u>	<u>Joint Variation:</u>
y <i>varies directly as</i> x or y <i>is directly proportional to</i> x	y <i>varies inversely as</i> x or y <i>is inversely proportional to</i> x	y <i>varies jointly as</i> x or y <i>is jointly proportional to</i> x
$y = kx$	$y = \frac{k}{x}$	$y = kxy$

D. Variation Problems

Weight on Earth varies directly with the weight on the Moon. With his equipment, an astronaut weighs 360 pounds on earth but only 60 pounds on the moon. If another astronaut had landed on the moon that weighed 54 pounds with her equipment, how much would she weigh on Earth with equipment?

Let y = weight on Earth
 x = weight on Moon

Set-up $\rightarrow y = kx$

To find k : $360 = k(60)$

$$\frac{360}{60} = \frac{k(60)}{60}$$

$$6 = k$$

Model: $y = 6x$

Now $y = 6(54)$
 $= 324$

On earth she would weigh 324 lbs.

The weight of a body varies inversely as the square of its distance from the center of the Earth. If a person weighs 175 pounds on the surface of the earth ($r \approx 4000$ miles) how much will he weigh at 1000 miles above the Earth's surface?

Let y = weight of the body
 x = Distance from the center of the earth

Set-up $\rightarrow y = \frac{k}{x^2}$

To find k : $175 = \frac{k}{(4000)^2}$

$$175(4000)^2 = K$$

$$2.8 \times 10^9 = K$$

Model: $y = \frac{2.8 \times 10^9}{x^2}$

Find y when $x = 4000 + 1000$

$$y = \frac{2.8 \times 10^9}{(5000)^2} = \frac{2.8 \times 10^9}{2.5 \times 10^7}$$

$$= 1.12 \times 10^2$$

$$= 112$$

He will weigh 112 lbs at 1000 miles up.

The distance d that a free-falling object has fallen is directly proportional to the square of the time that it has fallen. An object falls 120 feet in 4 seconds. Find the constant of proportionality.

Let d = the distance the object has fallen
 t = the time that the object takes to fall.

$$\begin{aligned} \text{set-up } d &= kt^2 \\ 120 &= k(4)^2 \quad (120 \text{ ft in } 4 \text{ sec}) \\ 120 &= k \cdot 16 \\ \frac{120}{16} &= k \\ k &= \frac{15}{2} \quad \checkmark \end{aligned}$$

The area of an ellipse varies jointly as a and b , that is, half the major and minor axes. If the area of an ellipse is 300π square units when $a = 10$ and $b = 30$ then what is the constant of proportionality? Give a formula for the area of an ellipse.

Let A = the area of an ellipse

$$\begin{aligned} \text{Set-up } \rightarrow A &= kab \\ 300\pi &= k(10)(30) \\ 300\pi &= k \cdot 300 \\ \frac{300\pi}{300} &= k \\ k &= \pi \end{aligned}$$

$$\text{Area of an Ellipse } A = \pi ab$$

Chapter 7 *Rational Expressions and Equations*

Sample Exam

Please answer all the questions and show work where appropriate.

1. Evaluate $f(-2)$ given $f(x) = \frac{x^2 - 2x - 15}{4x^2 - 25}$.
2. Simplify and state the restrictions to the domain: $\frac{x^2 - x - 6}{2x^2 - 5x - 3}$
3. Divide: $\frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \div \frac{9x^2 - 1}{3x^2 - 25x + 8}$
4. Add: $\frac{3x}{x+2} + \frac{x}{2-x} + \frac{8}{x^2 - 4}$
5. Subtract: $\frac{x}{x^2 + 2x - 3} - \frac{2}{2x^2 + x - 3}$
6. Simplify: $\frac{\frac{1}{7} - \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{49}}$
7. Solve $F = \frac{9C + 160}{5}$ for C .
8. The sum of the reciprocals of two consecutive odd integers is $\frac{4}{3}$. Set up an algebraic equation and use it to find the two integers.
9. Norm can paint the office by himself in 5 hours. Cliff could do the same job in 7 hours. If they work together how long will it take them to paint the office?
10. An executive went on an 8 hour business trip that required a 165 mile bus ride to the airport then another 2750 miles by airplane. If the airplane speed was 10 times that of the bus, how fast was it moving?

Chapter 7 *Rational Expressions and Equations*
Sample Exam Answers

1. $\frac{7}{9}$
2. $\frac{x+2}{2x+1}$ with restrictions $x \neq 3, -\frac{1}{2}$
3. $\frac{x-8}{x-2}$
4. $\frac{2(x-2)}{x+2}$
5. $\frac{(2x-3)(x+2)}{(x+3)(x-1)(2x+3)}$
6. $-\frac{7x}{7+x}$ or $-\frac{7x}{x+7}$
7. $C = \frac{5F-160}{9}$
8. 1 and 3
9. Approximately 2.92 hours
10. 550 mph

Evaluate $f(-2)$ given $f(x) = \frac{x^2 - 2x - 15}{4x^2 - 25}$.

$$f(x) = \frac{x^2 - 2x - 15}{4x^2 - 25}$$

$$f(-2) = \frac{(-2)^2 - 2(-2) - 15}{4(-2)^2 - 25}$$

$$= \frac{4 + 4 - 15}{16 - 25}$$

$$= \frac{-7}{-9} = \frac{7}{9}$$

Simplify and state the restrictions to the

domain: $\frac{x^2 - x - 6}{2x^2 - 5x - 3}$

$$\frac{x^2 - x - 6}{2x^2 - 5x - 3} = \frac{(x-3)(x+2)}{(2x+1)(x-3)}$$

$$= \frac{x+2}{2x+1} \quad \text{Restrictions } x \neq 3, -\frac{1}{2}$$

Divide: $\frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \div \frac{9x^2 - 1}{3x^2 - 25x + 8}$

$$\frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \div \frac{9x^2 - 1}{3x^2 - 25x + 8}$$

$$= \frac{3x^2 - 5x - 2}{x^2 - 4x + 4} \cdot \frac{3x^2 - 25x + 8}{9x^2 - 1}$$

$$= \frac{(3x+1)(x-2)}{(x-2)(x-2)} \cdot \frac{(3x-1)(x-8)}{(3x+1)(3x-1)}$$

$$= \frac{x-8}{x-2}$$

Add: $\frac{3x}{x+2} + \frac{x}{2-x} + \frac{8}{x^2 - 4}$

$$\frac{3x}{x+2} + \frac{x}{2-x} + \frac{8}{x^2 - 4}$$

$$= \frac{3x(x-2)}{(x+2)(x-2)} + \frac{x(x+2)}{(x-2)(x+2)} + \frac{8}{(x+2)(x-2)}$$

$$= \frac{3x(x-2) - x(x+2) + 8}{(x+2)(x-2)}$$

$$= \frac{3x^2 - 6x - x^2 - 2x + 8}{(x+2)(x-2)}$$

$$= \frac{2x^2 - 8x + 8}{(x+2)(x-2)}$$

$$= \frac{2(x^2 - 4x + 4)}{(x+2)(x-2)}$$

$$= \frac{2(x-2)(x-2)}{(x+2)(x-2)}$$

$$= \frac{2(x-2)}{(x+2)}$$

Subtract: $\frac{x}{x^2 + 2x - 3} - \frac{2}{2x^2 + x - 3}$ $\frac{x}{x^2 + 2x - 3} - \frac{2}{2x^2 + x - 3}$ $= \frac{x(2x+3)}{(x+3)(x-1)(2x+3)} - \frac{2(x+3)}{(2x+3)(x-1)(x+3)}$ $= \frac{x(2x+3) - 2(x+3)}{(x+3)(x-1)(2x+3)}$ $= \frac{2x^2 + 3x - 2x - 6}{(x+3)(x-1)(2x+3)}$ $= \frac{2x^2 + x - 6}{(x+3)(x-1)(2x+3)}$ $= \frac{(2x-3)(x+2)}{(x+3)(x-1)(2x+3)}$	Simplify: $\frac{\frac{1}{7} - \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{49}}$ $\frac{\frac{1}{7} - \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{49}} = \frac{\left(\frac{x-7}{7x}\right)}{\left(\frac{49-x^2}{49x^2}\right)}$ $= \frac{x-7}{7x} \cdot \frac{49x^2}{49-x^2}$ $= \frac{(x-7)}{7x} \cdot \frac{49x^2}{(7+x)(3-x)}$ $= -\frac{7x}{7+x} \text{ or } -\frac{7x}{x+7}$
---	---

Solve $F = \frac{9C+160}{5}$ for C.
$F = \frac{9C+160}{5}$ $5F = 9C + 160$ $5F - 160 = 9C$ $\frac{5F - 160}{9} = C$

The sum of the reciprocals of two consecutive odd integers is $\frac{4}{3}$. Set up an algebraic equation and use it to find the two integers.

Let x = the first odd integer
 $x+2$ = the next odd integer

→ set-up $\frac{1}{x} + \frac{1}{x+2} = \frac{4}{3}$

$$3(x+2) + 3x = 4x(x+2)$$

$$3x + 6 + 3x = 4x^2 + 8x$$

$$6x + 6 = 4x^2 + 8x$$

$$0 = 4x^2 + 2x - 6$$

$$0 = 2(2x^2 + x - 3)$$

$$0 = 2(2x+3)(x-1)$$

$2x+3=0 \text{ or } x-1=0$
 $2x=-3 \quad +1 \quad +1$
 $x=-3/2 \quad x=1$
 not an integer

The two odds are 1 & 3.

Norm can paint the office by himself in 5 hours. Cliff could do the same job in 7 hours. If they work together how long will it take them to paint the office?

Let t = time to paint working together

set-up $\rightarrow \frac{1}{5} + \frac{1}{7} = \frac{1}{t}$

$$7t + 5t = 35$$

$$12t = 35$$

$$t = \frac{35}{12} \approx 2.92 \text{ hrs}$$

Working together it will take approximately 2.92 hrs.

An executive went on an 8 hour business trip that required a 165 mile bus ride to the airport then another 2750 miles by airplane. If the airplane speed was 10 times that of the bus, how fast was it moving?

Let x = speed of the Bus ($D = r \cdot t$ or $\frac{D}{r} = t$)

	$D = r \cdot t$		
Bus	165	x	$\frac{165}{x}$
Plane	2750	$10x$	$\frac{2750}{10x}$
			8 hr

$\rightarrow$ set-up $\frac{165}{x} + \frac{2750}{10x} = 8$

$$1650 + 2750 = 80x$$

$$4400 = 80x$$

$$4400/80 = x$$

Speed of the plane is 550 mph $55 = x$



Chapter 5 - 7_Cumulative Review

Contents

Typically, Chapters 5 – 7 will be the material covered on an Elementary Algebra final exam. At this point we will review and reinforce the techniques we have learned. The best way to do this is to partition 2 hours of your time to take the following sample final exam. When you are finished check your answers and go back and find problems similar to the ones you missed. When you have successfully answered all the questions you will be ready to move on to Intermediate Algebra.

- ✓ *Sample Final Exam*
- ✓ *Sample Final Exam Answers*

Chapters 5 – 7 _ Cumulative Review

Sample Final Exam

Please answer all the questions and show work where appropriate.

1. Simplify: $\frac{-5x^{-2}y^{-3}z}{125x^3y^{-5}z^0}$

2. Simplify: $\left(\frac{2x^{-3}y^4}{6x^{-5}y}\right)^{-2}$

3. Subtract: $(2x^2 - 3x + 5) - (3x^2 + 5x - 10)$

4. Multiply: $(3x - 3)(2x^2 - 5x + 9)$

5. Divide: $(6x^4 - 11x^3 + 6x^2 - 16x + 1) \div (3x - 1)$

6. Factor: $x^4 - 81$

7. Factor: $x^5 - 4x^3 + 8x^2 - 32$

8. Solve: $(2x - 6)(x - 2) = 2(x + 13)$

9. Find a quadratic equation, with integer coefficients, that has solutions $\{-\frac{1}{3}, 5\}$.

10. The base of a triangle is 3 cm less than twice its height. If the area is 10 cm^2 find the length of the base.

11. Find the x and y - intercepts of $f(x) = 3x^2 + 14x - 5$

Chapters 5 – 7 _ Cumulative Review
Sample Final Exam

Page 2

12. Subtract: $\frac{1}{x^2 - 9} - \frac{2}{3 - x}$

13. Subtract: $\frac{5x + 2}{4x - 8} - \frac{2x + 5}{x^2 - x - 2}$

14. Simplify: $\frac{2 + \frac{3}{x} - \frac{5}{x^2}}{1 - \frac{4}{x} + \frac{3}{x^2}}$

15. Divide: $\frac{2x^2 + 9x - 5}{2x^2 + x - 1} \div \frac{x^2 - 25}{x^2 - 10x + 25}$

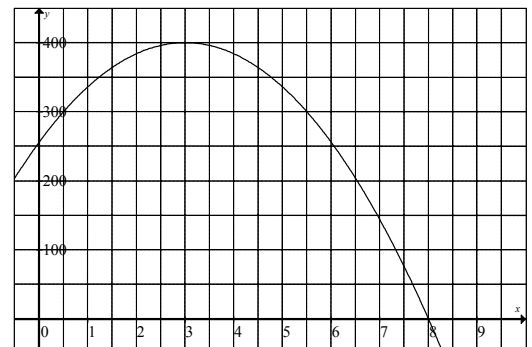
16. Solve: $\frac{3}{2x - 6} - \frac{2x - 3}{2x^2 - 5x - 3} = \frac{1}{2x + 1}$

17. Solve for a : $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

18. An office utilizes two printers an old one and a new one. The older printer takes twice as long to print than the new one. Using them together to print a certain job it takes 10 minutes. How long would it take the old printer working alone to print the same job?

19. One number is 4 less than the other. When the reciprocal of the smaller is subtracted from the reciprocal of the larger the difference is $-\frac{1}{8}$. Find the two numbers.

20. Given the graph of $f(x) = -16x^2 + 96x + 256$:
- At what point in the domain does the graph attain its maximum?
 - What is the maximum?
 - Find the y-intercept.



Chapters 5 – 7 _ Cumulative Review

Sample Final Exam Answers

1. $-\frac{y^2z}{25x^5}$

2. $\frac{9}{x^4y^6}$

3. $-x^2 - 8x + 15$

4. $6x^3 - 21x^2 + 42x - 27$

5. $2x^3 - 3x^2 + x - 5 - \frac{4}{3x-1}$

6. $(x^2 + 9)(x + 3)(x - 3)$

7. $(x + 2)(x - 2)(x + 2)(x^2 - 2x + 4)$

8. $\{-1, 7\}$

9. $3x^2 - 14x - 5 = 0$

10. 5 cm

11 x-intercepts $(-5, 0)$ and $(\frac{1}{3}, 0)$

y-intercept $(0, -5)$

12. $\frac{2x+7}{(x+3)(x-3)}$

13. $\frac{5x+9}{4(x+1)}$

14. $\frac{2x+5}{x-3}$

15. $\frac{x-5}{x+1}$

16. No Solution

17. $a = \frac{-bc}{c-b}$ or $a = \frac{bc}{b-c}$

18. 15 minutes

19. 8 and 4

20. a. $x = 3$

b. $y = 400$

c. $(0, 256)$

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