Solving Trigonometric Equations

Solve by using linear algebraic methods, algebra and identities

Solve all for $0° < x < 360°$

Solve using 

1) $\sqrt{\cos^2 x} = \sqrt{1}$

What angle(s) makes the equation true?

$\cos x = \pm 1$

$\cos x = 1$ $\quad x = 0°$

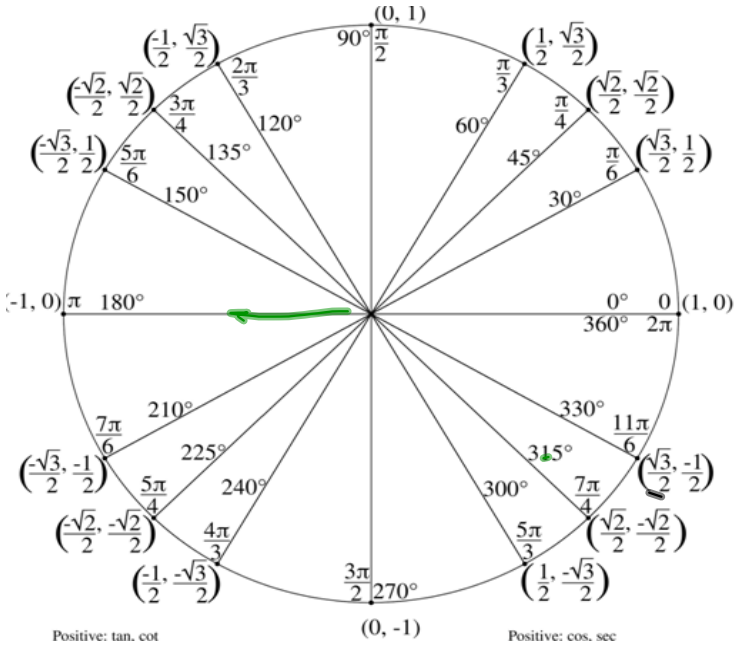
$\cos x = -1$ $\quad x = 180°$

2) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 30°, 150°$


(0, 1)
90°
$\frac{\pi}{2}$
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{2\pi}{3}$
120°
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{3}$
60°
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
135°
$\frac{\pi}{4}$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
45°
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$
150°
$\frac{\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
30°
(-1, 0)
π
180°
0°
360°
0
2π
(1, 0)
210°
$\frac{7\pi}{6}$
$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
225°
$\frac{5\pi}{4}$
$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
240°
$\frac{4\pi}{3}$
$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
330°
$\frac{11\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$
315°
300°
$\frac{7\pi}{4}$
$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\frac{5\pi}{3}$
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\frac{3\pi}{2}$
270°
(0, -1)
Positive: tan. cot
Positive: cos. sec

$\sin x = 1$

$x = 90°$

4) $2\cot x + 1 = -1$

$2\cot x = -2$ use reciprocal

$\cot x = -1 \Rightarrow \tan x = -1$

$X = 135°, 315°$

5) $2\sin x + 3 = 4$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 30, 150$

Solve by Factoring

6) $\sin x \tan x = \sin x$

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0$ $\quad \tan x - 1 = 0$

$\tan x = 1$

$x = 0, 180$ $\quad x = 45, 225$

8) $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0 \qquad \cos x - 1 = 0$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = 120°, 240°, 0°$

Solve by Squaring

use pythagorean identity Now

FOIL 9) $(\tan x + \sqrt{3})^2 = (\sec x)^2$

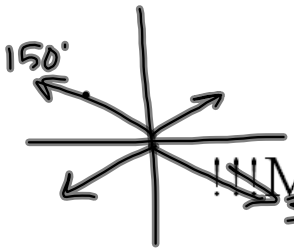
$(\tan x+\sqrt{3})(\tan x+\sqrt{3}) = \sec^2 x$

$\tan^2 x + 2\sqrt{3}\tan x + 3 = 1 + \tan^2 x$

$2\sqrt{3}\tan x + 2 = 0$

$\frac{2\sqrt{3}\tan x}{2\sqrt{3}} = \frac{-2}{2\sqrt{3}}$

$\tan x = -\frac{2}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}$



$x = 150°, 330°$

!!!Must Check Solutions in Original!!!

13. $3\cos^2 x = \sin^2 x$

use Pythagorean Identity to change $\sin^2 x$

$3\cos^2 x - \sin^2 x = 0$

$3\cos^2 x - (1 - \cos^2 x) = 0$

$3\cos^2 x - 1 + \cos^2 x = 0$

$4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$0 < x < 2\pi$ $\cos x = \pm\frac{1}{2}$

$\cos x = \frac{1}{2}$ $\cos x = -\frac{1}{2}$

$x = 60, 300$ $x = 120, 240$

$x = \pi/3, 5\pi/3$ $x = 2\pi/3, 4\pi/3$