6.3 Solving Quadratic Equations by FACTORING

Use the Zero Product Property

If $ab = 0$, then either $a = 0$ or $b = 0$

Example: **Two Roots**

Solve by factoring:

$x^2 = -4x$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$ $x+4=0$

$x = -4$

Equation must be set equal to ZERO

Example: **Double Root**

Solve by factoring:

$$x^2 - 6x = -9$$

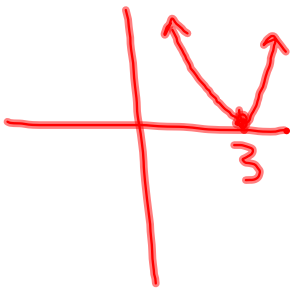
$+9 \quad +9$

$x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$x = 3$

one solution



Equation must be set equal to ZERO

Example: Solve by factoring:

$$4x^2 - 13x - 12 = 0$$

$(x-4)(4x+3)=0$

$x-4=0$ $\quad$ $4x+3=0$

$x=4$ $\quad$ $x=-\frac{3}{4}$

Equation must be set equal to ZERO

Write a quadratic equation whose roots are $-\frac{2}{3}$ and 6

$x = -\frac{2}{3}$ $x = 6$

$(3x + \frac{2}{3} \cdot 3)(x - 6) = 0$

$(3x+2)(x-6)=0$

$3x^2 - 18x + 2x - 12 = 0$

$3x^2 - 16x - 12 = 0$

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6.4 Solving Quadratic Equations by COMPLETING the SQUARE

Use the Square Root Property

If $x^2 = n$, then $x = \pm\sqrt{n}$

The quadratic equation contains a **perfect square trinomial** equal to a constant

Example:

Solve using the Square Root Property:

$$x^2 + 14x + 49 = 64$$

$$(x+7)(x+7) = 64$$

factor to a perfect sq. trinomial

$$\sqrt{(x+7)^2} = \sqrt{64}$$

$$x + 7 = \pm 8$$

$$x = -7 \pm 8$$

$x = -7+8$ $\quad$ $x = -7-8$

$x = 1$ $\quad$ $x = -15$

If the quadratic equation is *not* a perfect square trinomial, you may ***complete the square*** to get a perfect square trinomial.

Steps to completing the square:

For a quadratic equation in $ax^2 + bx + c = 0$

1. Subtract c to the other side
2. Find one half of b
3. Square the the result of step 2
4. Add the result of step 3 to both sides
5. Rewrite the trinomial
6. Use the square root property

Solve

$x^2+2x+6=0$ -6 -6

$x^2+2x+1^2=-6+1^2$ } $\frac{1}{2}\cdot 2=1$

factor $(x+1)^2=-5$

sq. root property $\sqrt{(x+1)^2}=\sqrt{-5}$

$x+1=\pm\sqrt{-1}\sqrt{5}$

$x+1=\pm i\sqrt{5}$

$x=-1\pm i\sqrt{5}$

Example: Solve: $x^2 + 4x - 12 = 0$
completing the square

1. Subtract c $x^2+4 = 12$

2. Find one-half of b $\frac{1}{2}\cdot 4 = 2$

3. Square #2 2^2

4. Add to both sides $x^2+4x+2^2 = 12+2^2$

5. Rewrite the perfect square trinomial
$(x+2)^2 = 16$

6. Solve using the square root property
$\sqrt{(x+2)^2} = \sqrt{16}$

$x+2 = \pm 4$

$x = -2 \pm 4$

$x = -2+4$ $x = -2-4$

$x = 2$ $x = -6$

Example: Solve: $x^2 + 2x + 3 = 0$
completing the square

$\frac{1}{2}\cdot 2 = 1$

$x^2 + 2x = -3$

$x^2 + 2x + 1 = -3 + 1$

$(x+1)^2 = -2$

$x + 1 = \pm\sqrt{-2}$

$x = -1 \pm i\sqrt{2}$

Example: Solve: $x^2 - 8x - 11 = 0$
completing the square

$\frac{1}{2}\cdot 8 = 4$

$x^2 - 8x = 11$

$x^2 - 8x + 4^2 = 11 + 4^2$

$(x-4)^2 = 27$

$x - 4 = \pm\sqrt{27}$

$x = 4 \pm 3\sqrt{3}$

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Completing the square when $a \neq 1$

$$3x^2 - 11x - 4 = 0$$

Step 1. Divide off coefficient from all terms

$$x^2 - \frac{11}{3}x - \frac{4}{3} = 0$$

Step 2 continue steps half of x-term, square, add to both sides

$$\tfrac{1}{2}\cdot\tfrac{11}{3} = \tfrac{11}{6} \quad x^2 - \tfrac{11}{3}x + \left(\tfrac{11}{6}\right)^2 = \tfrac{4}{3} + \left(\tfrac{11}{6}\right)^2$$

Step 3 Perfect squares

$$\left(x - \frac{11}{6}\right)^2 = \frac{169}{36}$$

Step 4 Solve for x

$$x - \frac{11}{6} = \pm\frac{13}{6}$$

$$x = \frac{11}{6} \pm \frac{13}{6}$$

$$x = \frac{11}{6} + \frac{13}{6} \qquad x = \frac{11}{6} - \frac{13}{6}$$

$$x = 4 \qquad x = -\frac{1}{3}$$

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(41) $\frac{3x^2}{3} - \frac{4x}{3} - \frac{2}{3} = 0$

$x^2 - \frac{4}{3}x - \frac{2}{3} = 0$

$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = \frac{2}{3} + \left(\frac{2}{3}\right)^2$

$\left(x - \frac{2}{3}\right)^2 = \frac{2 \cdot 3}{3 3} + \frac{4}{9}$

$\sqrt{\left(x - \frac{2}{3}\right)^2} = \sqrt{\frac{10}{9}}$

$x - \frac{2}{3} = \pm \frac{\sqrt{10}}{3}$ $+\frac{2}{3}$ $+\frac{2}{3}$

$x = \frac{2}{3} \pm \frac{\sqrt{10}}{3}$