$A = P(1+\frac{r}{n})^{r \cdot n}$ $y = ab^x$

HALG2 $A = Pe^{rt}$ $y = ae^{kt}$

8.6 – 8.8 Review

Name: Period: Date:

1. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000? Use $k = 0.614$.

$\frac{2000}{20} = \frac{20e^{.614t}}{20}$ $\ln 100 = .614t$

$t \doteq 7.5$ hrs

2. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant k in the decay formula for the substance. $k \doteq .02166$

$.5a = ae^{k(32)}$

$\frac{\ln .5}{32} = \frac{-32k}{32}$

3. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt 60, is radioactive and has a half-life of 5.7 years. Cobalt 60 is used to trace the path of nonradioactive substances in a system. What is the value of k for cobalt 60?

$.5a = ae^{-5.7k}$

$\frac{\ln .5}{-5.7} = -5.7$

$.1216 = k$

4. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use $k = 0.00012$.

$.0025a = ae^{-.00012t}$

$\ln .0025 = -.00012t$

$t \doteq 49929$ yrs or 50,000 yrs

5. **POPULATION** The population of rabbits in an area is modeled by the growth equation $P(t) = 8e^{0.26t}$, where P is in thousands and t is in years. How long will it take for the population to reach 25,000?

$25 = 8e^{0.26t}$

$\ln 25/8 = .26t$

$t \doteq 4.4$ yrs

6. **RADIOACTIVE DECAY** A radioactive element decays exponentially. The decay model is given by the formula $A = A_0e^{-0.04463t}$. A is the amount present after t days and A_0 is the amount present initially. Assume you are starting with 50g. How much of the element remains after 10 days? 30 days?

10 days: $A = 50e^{-0.04463(10)}$

$A \doteq 32$ grams

30 days: 13.1 grams

7. **POPULATION** A population is growing continuously at a rate of 3%. If the population is now 5 million, what will it be in 17 years' time? $A = 5{,}000{,}000e^{.03(17)} = 8{,}326{,}456$

8. **BACTERIA** A certain bacteria is growing exponentially according to the model $y = 80e^{kt}$. Using $k = 0.071$, find how many hours it will take for the bacteria reach a population of 10,000 cells?

$10{,}000 = 80e^{.071t}$

$\ln 125 = .071t$

68hrs $\doteq t$

$\log_e 1 = x$

$e^0 = 1$

$\log_e 1 = x$

$e^x = 1$

1. PROGRAMMING For reasons having to do with speed, a computer programmer wishes to ~~model~~ population size using a natural base exponential function. However, the programmer is told that the users of the program will be thinking in terms of the annual percentage increase. Let r be the percentage that the population increases each year. Find the value for k in terms of r so that

$$e^{k} = 1 + r.$$

$\log_e (1+r) = k$

$\ln (1+r) = k$

$\ln 1 + \ln r = k$

$0 + \ln r = k$

$\boxed{\ln r = k}$

2. CARBON DATING Archeologists uncover an ancient wooden tool. They analyze the tool and find that it has 22% as much carbon 14 compared to the likely amount that it contained when it was made. Given that the half-life of carbon 14 is about 5730 years, about how old is the artifact? Round your answer to the nearest 100 years.

$.5 = e^{-k(5730)}$

$\ln .5 = -k(5730)$

$1.2097 \times 10^{-4} = k$

$.22 = e^{-(1.2097\times 10^{-4})t}$

$\dfrac{\ln .22}{-1.2097\times 10^{-4}} = t$

$12{,}517 = t$

12,500 yrs

3. POPULATION The doubling time of a population is d years. The population size can be modeled by an exponential equation of the form Pe^{kt}, where P is the initial population size and t is time. What is k in terms of d?

$2P = Pe^{kd}$

$\ln 2 = kd$

$\boxed{\dfrac{\ln 2}{d} = k}$

4. POPULATION Louisa read that the population of her town has increased steadily at a rate of 2% each year. Today, the population of her town has grown to 68,735.

Population	68,735	67,387	66,066	64,770
Year	Today	−1	−2	−3

Based on this information, what was the population of her town 100 years ago?

$68{,}735 = Pe^{.02(100)}$

$68{,}735 = P(7.389056099)$

$9{,}302 = P$

9486

5. CONSUMER AWARENESS Jason wants to buy a brand new high-definition (HD) television. He could buy one now because he has \$7000 to spend, but he thinks that if he waits, the quality of HD televisions will improve. His \$7000 earns 2.5% interest annually compounded continuously. The television he wants to buy costs \$5000 now, but the cost increases each year by 7%.

$7000e^{.025t}$

a. Write a natural base exponential function that gives the value of Jason's account as a function of time t.

$A = Pe^{rt}$

$A = 7000e^{.025t}$

b. Write a natural base exponential function that gives the cost of the television Jason wants as a function of time t.

$5000e$ $\quad \log_e 1.07t$

$C = 5{,}000e^{(\ln 1.07)t}$

same as $C = 5000 \cdot 1.07^t$

$b^{\log_b x} = x$

c. In how many years will the cost of the television exceed the value of the money in Jason's account? In other words, how much time does Jason have to decide whether he wants to buy the television? Round your answer to the nearest tenth of a year.

$7000e^{.025t} < 5000e^{(\ln 1.07)t}$

$\ln 7000 + \ln e^{.025t} < \ln 5000 + \ln e^{(\ln 1.07)t}$

$\ln 7000 - \ln 5000 < -\ln e^{.025t} + \ln e^{(\ln 1.07 t)}$

$\ln 7/5 < -.025t + \ln 1.07t$

$.33647 < -.025t + .0676586485t$

$.33647 < .0426586485t$

$7.887 < t$

about 7.9 yrs

II. Solve each equation or inequality. Round to 4 decimal places.

1. $2^x = 15$

$\log 2^x = \log 15$

$x = \log 15 / \log 2$

3.9069

2. $4^{2a} > 45$

$\log 4^{2a} > \log 45$

$2a > \log 45 / \log 4$

$a > 1.3730$

3. $11^{x+4} > 57$

$x+4 > \frac{\log 57}{\log 11}$

$x > -2.3139$

4. $3^{b^2} = 64$

$b^2 = \frac{\log 64}{\log 3}$

$b = \sqrt{\frac{\log 64}{\log 3}} \doteq \pm 1.9457$

5. $7^{3y-1} > 2^{2y+4}$

$(3y-1)\log 7 > (2y+4)\log 2$

$(3y)\log 7 - \log 7 > (2y)\log 2 + 4\log 2$

$3y(.8451) - .8451 > 2y(.3010) + 1.2041$

$2.5353y - .8451 > .602y + 1.2041$

$1.9333y > 2.0492$

$y > 1.0599$

6. $5^{m^2+1} = 30$

$(m^2+1) = \frac{\log 30}{\log 5}$

$m = \pm 1.0551$

III. Evaluate to 4 decimal places.

1. $\log_4 62$ $\frac{\log 62}{\log 4}$

2.9771

2. $e^{.075}$

1.0779

3. $\ln 0.6$

$-.5108$

4. $\log_3 21$ $\frac{\log 21}{\log 3}$

2.7712

IV. Write an equivalent exponential or logarithmic expression.

1. $e^x = 10$

$\log_e 10 = x$

$\ln 10 = x$

2. $\ln x = 2.3026$

$e^{2.3026} = x$

3. $e^3 = 9x$

$\ln 9x = 3$

4. $\ln .02 = x$

$e^x = .02$

V. Solve each equation or inequality. Round to nearest ten-thousandth.

1. $25e^x = 1000$

$e^x = 40$

$\ln e^x = \ln 40$

$x = \ln 40$

$x \doteq 3.6889$

2. $e^{.075x} > 25$

$\ln e^{.075x} > \ln 25$

$.075x > \ln 25$

$x > \ln 25 / .075$

42.9183

3. $3 \ln 2x > 9$

$\ln 2x > 3 \rightarrow \log_e 2x > 3$

$2x > e^3$

$x > e^3/2$

$x > 10.04$

4. $\ln(x+2) = 4$

$x+2 = e^4$

$x = e^4 - 2$

$x = 52.5982$

5. $5 + 4e^{2x} = 17$

$4e^{2x} = 12$

$e^{2x} = 3$

$\ln e^{2x} = \ln 3$

$2x = \ln 3$

$x = \frac{\ln 3}{2}$

$x = .5493$

6. $\ln(2x+3) > 0$

$2x+3 > e^0$

$2x+3 > 1$

$2x > -2$

$x > -1$