Name: Jun Class: ______________ Date: ________ ID: A

HALG2 Chapter 8 Test Review

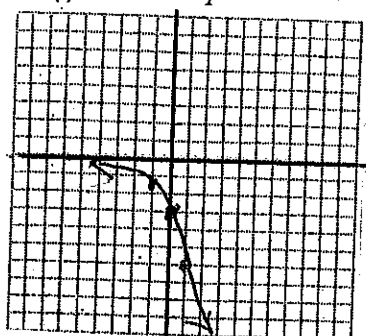
State the domain and range. State if each is exponential growth or exponential decay. Graph #1 & #2.

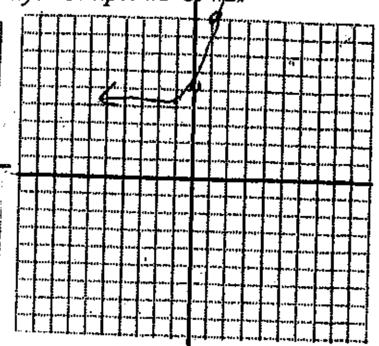
1. $f(x) = -3(2)^x$

x	y
0	-3
-1	-3/2
1	-6

2. $y = 5^x + 4$ ⟶

x	y
0	5
1	9
-1	4 1/5





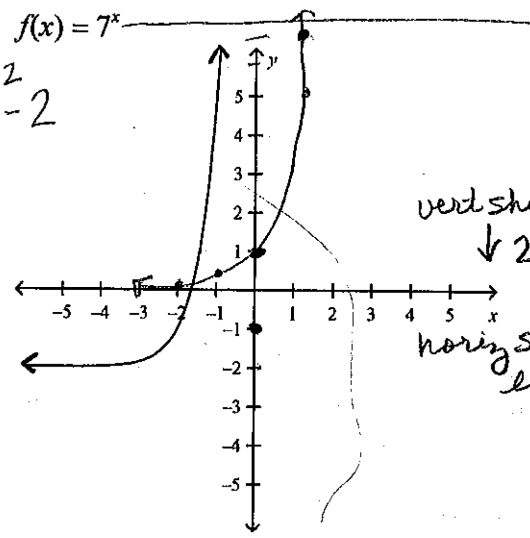
3. $f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} - 7$

For the graph, f(x) is the parent function and g(x) is the transformation of f(x). Use the graph to determine g(x).

4. $f(x) = 7^x$ ⟶

x	y
0	1
1	7
-1	1/7
-2	1/49

$y = 7^{x+2} - 2$



g(x) ⟵

vert shift ↓2

horiz shift left 2

$y = 7^x - 2$

0	-1
1	5
-1	$-2\frac{1}{7}$

The production of a new car model in the year 1995 was 1.25 million units. In the year 2005, it rose to 3.5 million units.

$3.5 = 1.25b^{10}$
$2.8 = b^{10}$
$1.108 = b$

5. Write an exponential function to model the cars y produced after x years.

$y = 1.25(1.108)^x$

6. Assuming that production continued at the same rate of increase, estimate production in 2010.

$y = 1.25(1.108)^{15} \approx 5.82$ million units

Every ten years, the Bureau of the Census counts the number of people living in the United States. In 1830, the population of the U.S. was 12.87 million. By 1840, this number had grown to 17.07 million.

$17.07 = 12.87(b)^{10}$ $1.32634 = b^{10}$

7. Write the equation in terms of x, the number of $\boxed{\text{decades}}$ x since 1830. Estimate the U.S. population in 2020 if the population increases at the same rate.

$y = 12.87(1.33)^x$

8. Find the population based on this information for the year 2000. If the actual population was 281.42 million in 2000, did the population continue to grow at the same rate at which it was growing in the early 1800s?

$2000 \rightarrow y = \underset{\text{mill}}{12.87}(1.33)^{17} = 1640.8$ mill $= 1.640$ billion

NO → slowed considerably

Solve for x. Do as much as you possibly can without a calculator!!!

9. $\log_4 x = 2$ $\quad 4^2 = x \quad 16$

10. $\log_6 \frac{1}{216} = x$ $\quad 6^x = \frac{1}{216} \rightarrow 6^x = 6^{-3} \rightarrow x = -3$

11. $\log_{16} x = \frac{3}{4}$ $\quad 16^{3/4} = x \quad (\sqrt[4]{16})^3 = 2^3 = 8$

12. $\log_4 x \le 6$ $\quad x \le 4^6 \quad 0 < x \le 4096$

13. $\log_2(x^2+5) = \log_2 6x$ $\quad x^2+5 = 6x$ $\quad x^2-6x+5=0$ $\quad (x-1)(x-5)=0$ $\quad x=1;\ x=5$

Solve each equation. Do as much as possible without a calculator!!!!

14. $\log_{10} 5 + \log_{10} x = \log_{10} 25$ $\quad \log_{10} 5x = \log_{10} 25$ $\quad 5x = 25$ $\quad x=5$

15. $\log_5 x + \log_5 (x+23) = \log_5 50$ $\quad \log_5(x^2+23x) = \log_5 50 \rightarrow x^2+23x-50=0$ $\quad (x-2)(x+25)$ $\quad x=2, \cancel{-25}$

16. $2\log_3 x - \log_3 4 = \log_3 16$ $\quad \log_3 \frac{x^2}{4} = \log_3 16$ $\quad \frac{x^2}{4} = 16 \rightarrow x^2 = 64$ $\quad x=8$

17. $\log_2 0.5 + 2\log_2 x = \log_2 5 + \log_2 40$ $\quad \frac{1}{2}x^2 = 200$ $\quad x^2 = 400$ $\quad x = 20$

18. Building codes after an earthquake require structures to be 30 times stronger than they were under the old code. Structures built to the new code can withstand earthquakes of magnitude 8.5. Find what magnitude of earthquake a structure built to the old code could withstand.

Let I = intensity of old code quake $\quad 10^{8.5} = 30I$ $\quad \frac{10^{8.5}}{30} = I \doteq 1 \times 10^7 \quad \therefore M = 7$

19. The 2001 Nisqually earthquake in Washington had a Richter scale magnitude of 6.8. Suppose an architect has designed a building strong enough to withstand an earthquake 30 times as intense as the Nisqually quake. Find the magnitude of the strongest quake this building can withstand. $\quad 30(10^{6.8})$

$M = \log I$

Suppose the increasing number of English-speaking people in the world is modeled by the equation $N = O.e^{r.t}$ where N represents the number of people who can at present speak the language, O represents the people who could speak the language some time ago, r is the increasing rate, and t is the time period.

20. If the number of English speaking people has risen to 2.5 billion from 1.5 billion with an increasing rate of 0.02, then find the time interval in which this number is attained.

$2.5 = 1.5e^{.02t}$ $\quad \ln\frac{2.5}{1.5} = .02t$ $\quad 25.54 = t$ $\quad (\approx 26 \text{ yrs})$

Ben bought a car for \$20,000 in 2003. The car depreciates at a constant rate of 21% per year.

$y = ab^x$ $\quad y = 20{,}000(.79)^7$

21. Find the price of the car in the year 2010. Round to the nearest dollar. $\quad \$3841^{00}$

22. How long will it be until the depreciated cost of the car is worth \$14,000?

$14000 = 20{,}000(.79)^t$

$\log \frac{7}{10} = t \log .79$

$\frac{\log .7}{\log .79} = t$ $\quad$ abt 1.5 years

1950
+171
2121

.1918855 < t(.0012?)
477 < t
170.9

23. John buys a house for $350,000. If housing prices increase by 17% per year, how long will it be until his property is worth $700,000? $y=ab^x$ $700 = 350(1.17)^t$ $\log 2 = \log 1.17^t$ $\frac{\log 2}{\log 1.17} = t$ 4.41 years

24. In a 1950 census, one city's population of males was 3.5 million and that of females was 2.25 million. The population of females increases by 0.68% and that of males by 0.42% per year. If these rates continue, find when will the female population will be higher than the male population. 2121

$3.5(1.0042)^t < 2.25(1.0068)^t$
$\log 3.5 + t\log 1.0042 < \log 2.25 + t \log 1.0068$
$\log 3.5 - \log 2.25 < t\log 1.0068 - t\log 1.0042$

M: $3.5(1.0042)^t$ F: $2.25(1.0068)^t$

25. Assume that a company sold 5.75 million motorcycles and 3.5 million cars in the year 2005. The growth in the sale of $\boxed{\text{two}}$ motorcycles is 16% every year and that of cars is 25% every year. Find when the sale of cars will be more then the sale of motorcycles.

26. Write an exponential function for the graph that passes through (0, 5) & (4, 3125). $3125 = 5b^4$ $625 = b^4$ $5 = b$ $y = 5(5)^x$

27. Without using a calculator, use $\log_5 3 = .6826$ and $\log_5 4 = .8614$ to approximate the value of each expression:

a) $\log_5 12$ $\log_5 3 + \log_5 4$ $.6826 + .8614$ 1.5440

b) $\log_5 (27/16)$ $\log_5 27 - \log_5 16$ $\log_5 3^3 - \log_5 4^2$ $3(\log_5 3) - 2(\log_5 4)$ $3(.6826) - 2(.8614)$ $2.0478 - 1.7228$ 0.325

28. Suppose you deposit $5,000 into an account paying 3% compounded continuously. Find the balance to the nearest dollar after 7 years. $5000e^{.03(7)} =$ $6168

29. Sam's bank pays 2.8% annual interest compounded continuously on savings accounts. Sam put $2,000 into the account. How long will it take for the initial deposit to double in value? Assume no additional deposits or withdrawals and round your answer to the nearest quarter of a year. $4000 = 2000e^{.028t}$ $2 = e^{.028t}$ $\ln 2 = .028t$ $\frac{\ln 2}{.028} = t =$ 24.75 yrs

30. Determine the balance in a retirement account after 20 years if $5,000 was invested at 6.05% interest compounded monthly. $A = 5{,}000\left(1 + \frac{.0605}{12}\right)^{12\cdot 20}$

31. Solve.

$$\left(\frac{1}{2}\right)^{4x+1} = 8^{2x+1}$$

$(2^{-1})^{4x+1} = (2^3)^{2x+1}$
$2^{-4x-1} = 2^{6x+3}$
$-4x-1 = 6x+3$
$-10x = 4$
$x = -4/10 = -2/5$

32. Evaluate without using a calculator:
$\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27$

$\log_3 3^3 + \log_9 (9)^{\frac{3}{2}} + 1 + \log_{81} 81^{\frac{3}{4}} + \log_{243} 243^{\frac{3}{5}}$
$3 + 3/2 + 1 + 3/4 + 3/5 = 4 + \frac{30}{20} + \frac{15}{20} + \frac{12}{20} = 4 + \frac{57}{20} =$ $6\frac{17}{20}$

Key:
(1) D: {all ℝ} R {y | y < 0}; growth (2) D: {all ℝ} R: {y | y > 4}; growth (3) D: {all ℝ} R: {y | y < 7}; decay
(4) $g(x) = 7^{x+2} - 2$ (5) $y = 1.25(1.108)^x$ (6) abt 5.82 million (7) $y = 12.87(1.33)^x$; 2902.42 million
(8) 1.64 billion; No, growth rate slowed considerably (9) 16 (10) −3 (11) 8 (12) 4,096 (13) 1, 5
(14) 5 (15) 2 (16) 8 (17) 20 (18) 7 (19) 8.3 (20) abt 26 yrs (21) $3,841 (22) abt 1.5 yrs
(23) abt 4.4 yrs (24) 2121 (25) 2012 (26) $y = 5(5)^x$ (27) a) 1.5440 b) 0.3250
(28) $6,168 (29) 24.75 yrs (30) $ ~~16,155.63~~ 16,716.53 (31) −2/5 (32) $6\frac{17}{20}$