

List all possible rational zeros for each polynomial function.

1) $f(x) = x^4 - 3x^3 - 7x^2 - 27x - 18$
 $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

2) $f(x) = 2x^5 - x^4 + x^3 - 3x^2 - x + 5$
 $\pm \frac{5}{2}, \pm 5, \pm \frac{1}{2}, \pm 1$
 p: $\pm 5 \pm 1$
 q: $\pm 2 \pm 1$

Make a Descartes chart detailing the possible number and type(s) of zeros that may exist for this function:

3) $f(x) = 5x^8 - x^6 + 7x^4 - 8x^2 - 3$

TOT	POS	NEG	C
8	3	3	2
8	1	3	4
8	3	1	4
8	1	1	6

4) Find all the zeros for this function:

$f(x) = x^3 - 7x^2 + 16x - 10$

$\frac{p}{q} \mid \begin{array}{ccc|c} & 1 & -7 & 16 & -10 \\ & 1 & -6 & 10 & 0 \\ & & x^2 - 6x + 10 = 0 & & \end{array}$

$\frac{p}{q} : \pm 1, \pm 2, \pm 5, \pm 10$
 $\frac{6 \pm \sqrt{36 - 4(1)(10)}}{2}$
 $\frac{6 \pm \sqrt{-4}}{2}$
 $\frac{6 \pm 2i}{2}$

$1, 3+i, 3-i$

5) Given $x+5$ is a factor of the following function, find the remaining factors:

$f(x) = 3x^3 + 20x^2 + 23x - 10$ $(x+2)(3x-1)$

$-5 \mid \begin{array}{ccc|c} 3 & 20 & 23 & -10 \\ & -15 & -25 & 10 \\ \hline 3 & 5 & -2 & 0 \end{array}$
 $3x^2 + 5x - 2$

6) Apply the Remainder Theorem/Synthetic substitution to find $f(-2)$ if $f(x) = x^3 + 4x^2 - 3x + 2$

$-2 \mid \begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ & -2 & -4 & 14 \\ \hline 1 & 2 & -7 & 16 \end{array}$

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7) Is $(y+2)$ a factor of $6y^3 + 13y^2 - 10y - 24$? Support your position algebraically & explain your Position with a complete sentence.

$-2 \mid \begin{array}{ccc|c} 6 & 13 & -10 & -24 \\ & -12 & -2 & 24 \\ \hline 6 & 1 & -12 & 0 \end{array}$

yes, $y+2$ is a factor because when the polynomial is divided by $y+2$ the remainder is zero.

Solve each & describe the appearance of each graph:

8) $x^2 - 10x + 25 = 0$

$(x-5)^2 = 0$

$x = 5$ (mult. 2)

upward opening parabola touching but not crossing x-axis at $x=5$.

9) $x^3 + 16x = 0$

$x(x^2 + 16) = 0$

$x = 0$ $x^2 = -16$
 $x = \pm 4i$

at end up, cuts thru x-axis @ origin; left end ↓

10) The zeros of a cubic polynomial $P(x)$ are $-3, 2,$ & 4 . Also, $P(0) = 72$. Write $P(x)$ in standard form.

$72 = a(x+3)(x-2)(x-4)$
 $72 = a(0+3)(0-2)(0-4)$

$72 = 24a$

$3 = a$

$P(x) = 3(x+3)(x-2)(x-4)$

$3x^3 - 9x^2 - 30x + 72$

11) Use graphing, synthetic division, factoring, or whatever technique necessary to find all the roots of the function:

$$Q(x) = x^3 + 2x^2 + 4x + 8$$

$$\textcircled{-2, 2i, -2i}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 4 & 8 \\ & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & \end{array}$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

12) Write in standard form the least degree polynomial function with integral coefficients having zeros that include -3, 1, & -3i.

$$P(x) = x^4 + 2x^3 + 6x^2 + 18x - 27$$

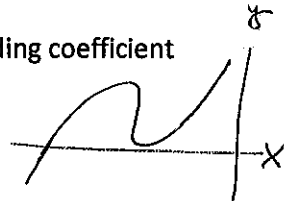
$$(x+3)(x-1)(x+3i)(x-3i)$$

$$(x^2 + 2x - 3)(x^2 + 9)$$

13) Describe the graph of a polynomial function with the given characteristics:

a) 1 real zero w/odd multiplicity, 2 imaginary zeros & a positive leading coefficient

left ↓ right ↑
cuts thru x-axis @ real zero



b) 1 positive real zero w/multiplicity of two & a negative leading coefficient

downward opening parabola
that only touches x-axis @
pos real zero



c) 4 real zeros w/odd multiplicity & positive leading coefficient

"W" shaped graph cutting
thru x-axis @ each of 4
real zeros.

