GLENCOE MATHEMATICS

Algebra 1

Parent and Student Study Guide Workbook

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Algebra 1
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To the Parents of *Glencoe Algebra 1* Students

You teach your children all the time. You taught language to your infants and you read to your son or daughter. You taught them how to count and use basic arithmetic. Here are some ways you can continue to reinforce mathematics learning.

- Encourage a positive attitude toward mathematics.
- Set aside a place and a time for homework.
- Be sure your child understands the importance of mathematics achievement.

The *Glencoe Algebra 1* ***Parent and Student Study Guide Workbook*** is designed to help you support, monitor, and improve your child's math performance. These worksheets are written so that you do not have to be a mathematician to help your child.

The ***Parent and Student Study Guide Workbook*** includes:

- A 1-page *worksheet* for every lesson in the Student Edition (98 in all). Completing a worksheet with your child will reinforce the concepts and skills your child is learning in math class. Upside-down answers are provided right on the page.
- A 1-page *chapter review* (14 in all) for each chapter. These worksheets review the skills and concepts needed for success on tests and quizzes. Answers are located on pages 113–118.

Online Resources
For your convenience, these worksheets are also available in a printable format at **www.algebra1.com/parent_student.**

Algebra 1 Online Study Tools can help your student succeed.

- **www.algebra1.com/extra_examples**
  shows you additional worked-out examples that mimic the ones in the textbook.
- **www.algebra1.com/self_check_quiz**
  provides a self-checking practice quiz for each lesson.
- **www.algebra1.com/vocabulary_review**
  checks your understanding of the terms and definitions used in each chapter.
- **www.algebra1.com/chapter_test**
  allows you to take a self-checking test before the actual test.
- **www.algebra1.com/standardized_test**
  is another way to brush up on your standardized test-taking skills.

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1-1 Variables and Expressions *(Pages 6–9)*

Letters such as *x* and *y* in a mathematical expression are called **variables**. Variables are symbols that are used to represent unspecified numbers. Any letter may be used as a variable. An **algebraic expression** consists of one or more numbers and variables along with one or more arithmetic operations. In multiplication expressions, the quantities being multiplied are called **factors**, and the result is the **product**. An expression such as x^y is called a **power**. The variable *x* is the **base** and *y* is called the **exponent**. The exponent indicates the number of time the base is used as a factor.

Examples

Verbal Expression	Algebraic Expression
2 less than the product of 5 and a number *y*	$5y - 2$
the product of 4 and *a* divided by the product of 3 and *b*	$4a \div 3b$
nine feet shorter than the height of the tree (*T* = tree height)	$T - 9$
one-third as costly as a first-class ticket (*f* = price of first class ticket)	$\frac{f}{3}$

Symbols	Words	Meaning
3^1	3 to the first power	3
3^2	3 to the second power or 3 squared	$3 \cdot 3$
3^5	3 to the fifth power	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$4s^3$	4 times *s* to the third power or 4 times *s* cubed	$4 \cdot s \cdot s \cdot s$

Practice

Write an algebraic expression for each verbal expression.

1. the sum of *g* and 14

2. 10 less than the square of *n*

3. *K* to the fifth power

4. the product of 6 and *r* increased by one third of *q*

5. the product of 12 and *y*

6. 3 years younger than her sister (*s* = sister's age)

Write a verbal expression for each algebraic expression.

7. $x^3 - 5$

8. 6^4

9. $\frac{n^2}{7}$

10. $2(p + 4)$

Write each expression as an expression with exponents.

11. $5 \cdot 5$

12. $9 \cdot 9 \cdot 9 \cdot 9$

13. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

14. $n \cdot n \cdot n$

15. **Standardized Test Practice** Evaluate $2^4 + 5^3$.

A 14 **B** 23 **C** 141 **D** 7^7

Answers: **1.** $g + 14$ **2.** $n^2 - 10$ **3.** K^5 **4.** $6r + \frac{1}{3}q$ **5.** $12y$ **6.** $s - 3$ **7.** 5 less than the cube of *x* **8.** 6 to the fourth power **9.** *n* squared divided by 7 **10.** twice the sum of *p* and 4 **11.** 5^2 **12.** 9^4 **13.** 2^5 **14.** n^3 **15.** C

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1-2 Order of Operations *(Pages 11–15)*

Numerical and algebraic expressions often contain more than one operation. A rule is needed to let you know which operation to perform first. The rule is called the **order of operations**.

Order of Operations	1. Simplify the expressions inside grouping symbols, such as parentheses ( ), brackets [ ], and braces { }, and as indicated by fraction bars. 2. Evaluate all powers. 3. Do all multiplications and divisions from left to right. 4. Do all additions and subtractions from left to right.

Examples

Evaluate each expression.

a. $15 + 3 \cdot 21$

$15 + 3 \cdot 21 = 15 + 63$ Multiply 3 by 21.
$= 78$ Add 15 and 63.

b. $\dfrac{8 + 2^3}{(3 + 1) \cdot 2}$

Since this expression is a fraction, the numerator and denominator should each be treated as a single value. Think of the expression as $(8 + 2^3) \div [(3 + 1) \cdot 2]$.

$(8 + 2^3) \div [(3 + 1) \cdot 2]$
$= (8 + 8) \div [4 \cdot 2]$ Evaluate 2^3; add 3 and 1.
$= 16 \div 8$ Add 8 and 8; multiply 4 and 2.
$= 2$ Divide 16 by 8.

Try These Together

Evaluate each expression.

1. $7 \cdot 2 + 1$ **2.** $2 + 3^2 \cdot 4 - 1$ **3.** $3(8 + 2) \div 5 - 4$

HINT: Refer to the order of operations above to help you remember which operations to perform first.

Practice

Evaluate each expression.

4. $\frac{8}{4} + 3$ **5.** $12 - 6 + 2 \cdot 3$ **6.** $2(3 + 5) - 4$

7. $15(2) - 6$ **8.** $60 - (13 + 5)$ **9.** $6 + 2(3)$

10. $2[2(2 + 2)] + 1$ **11.** $(15)(3)^2 + (4 - 2)$ **12.** $2(1.5 + 2.5) + 7$

13. $\dfrac{3(2^2) + 2(3^2)}{4}$ **14.** $\dfrac{17 + 3^3 - 4(2)}{2}$ **15.** $80 - (20 + 5)$

Evaluate each expression if $x = 5$, $y = 1$, and $z = 3$.

16. $(x + 5)(y + z)$ **17.** $x(xy + z)$ **18.** $2(x + y) + z$

19. **Standardized Test Practice** Evaluate the expression $2 + (3 + 4)2 + 6 - 5(2)$.

A 10 **B** 11 **C** 12 **D** 13

Answers: **1.** 15 **2.** 37 **3.** 2 **4.** 5 **5.** 12 **6.** 12 **7.** 24 **8.** 42 **9.** 12 **10.** 17 **11.** 137 **12.** 15 **13.** $7\frac{1}{2}$ **14.** 18 **15.** 55 **16.** 40 **17.** 40 **18.** 15 **19.** C

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1-3 Open Sentences *(Pages 16–20)*

Mathematical statements with one or more variables are called **open sentences**. An open sentence is neither true nor false until the variable has been replaced by a value. Finding a replacement for the variable that results in a true sentence is called **solving the open sentence**. This replacement is called a **solution** of the open sentence. A sentence that contains an equals sign (=) is called an **equation**. A sentence that has the symbols <, >, ≤, or ≥ is called an **inequality**. A **set** of numbers from which replacements for a variable may be chosen is called a **replacement set**. Each object or number in a set is called an **element**, or member. The **solution set** of an open sentence is the set of all replacements for the variable that make the sentence true.

Examples

a. Is the equation $3a + 12 = 25$ true if $a = 4$?

$3a + 12 = 25$	
$3(4) + 12 = 25$	*Replace a with 4.*
$12 + 12 = 25$	*Multiply 3 by 4.*
$24 \neq 25$	*Since 24 is not equal to 25, the equation is not true for the replacement value of 4.*

b. Find the solution set for the inequality $7b + 2 \geq 37$ if the replacement set is {3, 4, 5, 6}.

Replace b with	$7b + 2 \geq 37$	True or False?
3	$7(3) + 2 \geq 37 \rightarrow 23 \geq 37$	false
4	$7(4) + 2 \geq 37 \rightarrow 30 \geq 37$	false
5	$7(5) + 2 \geq 37 \rightarrow 37 \geq 37$	true
6	$7(6) + 2 \geq 37 \rightarrow 44 \geq 37$	true

Therefore, the solution set is {5, 6}.

Try These Together

1. Is the equation $x + \frac{1}{3} = \frac{1}{4} + \frac{3}{4}$ true if $x = \frac{1}{2}$?

2. Find the solution set for $3g - 2 < 16$ if the replacement set is {2, 4, 6, 8}.

Practice

State whether each equation is true or false for the value of the variable given.

3. $a + \frac{1}{8} = \frac{6}{8} + \frac{1}{4}, a = \frac{7}{8}$

4. $4x^2 + 2(5) = 40, x = 4$

5. $2x^2 + 3(2) = 56, x = 5$

6. $\frac{1}{g^2 + 1} \leq \frac{1}{5}, g = 2$

Find the solution set for each inequality. The replacement set is y = {5, 10, 15, 20}.

7. $y - 3 \leq 13$

8. $y + 2 > 10$

9. $3y - 12 \geq 15$

10. **Standardized Test Practice** Which of the following is the solution set for the inequality $3x^2 + 4(2) \leq 56$ if the replacement set is {2, 3, 4, 5, 6, 7}?

A {5, 6, 7} **B** {2, 3, 4} **C** {4, 5, 6} **D** {3, 4, 5}

Answers: **1.** false **2.** {2, 4} **3.** true **4.** false **5.** true **6.** true **7.** {5, 10, 15} **8.** {10, 15, 20} **9.** {10, 15, 20} **10.** B

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1-4 Identity and Equality Properties *(Pages 21–25)*

You can use the following properties to justify the steps you use when you evaluate an expression.

Additive Identity Property	The sum of any number and 0 is equal to that number. For any number a, $a + 0 = 0 + a = a$.
Multiplicative Identity Property	Since the product of any number and 1 is equal to the number, 1 is called the multiplicative identity. For any number a, $a \cdot 1 = 1 \cdot a = a$.
Multiplicative Property of Zero	For any number a, $a \cdot 0 = 0 \cdot a = 0$.
Multiplicative Inverse Property	Two numbers whose product is 1 are called multiplicative inverses or reciprocals. For every nonzero number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property of Equality	The reflexive property of equality says that any number is equal to itself. For any number a, $a = a$.
Symmetric Property of Equality	The symmetric property of equality says that if one quantity equals a second quantity, then the second quantity also equals the first. For any numbers a and b, if $a = b$, then $b = a$.
Transitive Property of Equality	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any expression.

Practice

Name the multiplicative inverse of each number or variable. Assume that no variable represents zero.

1. 5 **2.** $\frac{3}{5}$ **3.** $\frac{4}{c}$ **4.** $1\frac{1}{3}$

Name the property or properties illustrated by each statement.

5. $x \cdot 1 = x$ **6.** $\frac{15}{3} + 4 = 5 + 4$ **7.** $\frac{2}{3} \cdot \frac{3}{2} = 1$

8. $3 \cdot 0 = 0$ **9.** $11 - 2 = 11 - 2$ **10.** $0 + n = n$

11. If $13 = 4 + 9$, then $4 + 9 = 13$.

12. If $x + 5 = 3$ and $3 = y$, then $x + 5 = y$.

13. **Standardized Test Practice** Name the multiplicative inverse of $\frac{x + 2}{5}$. Assume that $x + 2 \neq 0$.

A $x + \frac{5}{2}$ **B** $\frac{5}{x + 2}$ **C** $\frac{5}{x} + 2$ **D** $\frac{1}{x} + \frac{5}{2}$

Answers: 1. $\frac{1}{5}$ **2.** $\frac{5}{3}$ **3.** $\frac{c}{4}$ **4.** $\frac{3}{4}$ **5.** multiplicative identity **6.** substitution property of equality **7.** multiplicative inverse **8.** multiplicative property of zero **9.** reflexive property of equality **10.** additive identity **11.** symmetric property of equality **12.** transitive property of equality **13.** B

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1-5 The Distributive Property *(Pages 26–31)*

A **term** is a number, a variable, or a product or quotient of numbers and variables. Some examples of terms are x^2 and $3y$. The expression $3a + 5$ has two terms. **Like terms** are terms that contain the same variable, with corresponding variables having the same power. For example, $2x^2$ and $7x^2$ are like terms, but $4b^2$ and $2b$ are not. The expressions $8g + 4g$ and $12g$ are **equivalent expressions** because they denote the same number. An expression is in **simplest form** when it is replaced by an equivalent expression having no like terms and no parentheses. The **coefficient** of a term is the numerical factor. For example, in $8g$, 8 is the coefficient. You can use these facts plus the **Distributive Property** to simplify expressions.

Distributive Property	For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$; $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Examples

a. Rewrite $7(2x + 3)$ without parentheses.

Use the Distributive Property.

$7(2x + 3) = 14x + 21$

The expression $14x + 21$ is in simplest form because it has no parentheses and no like terms.

b. Simplify the expression $3x^2 + 2x + 6x + x^2$.

Group and combine like terms using the Distributive Property.

$3x^2 + 2x + 6x + x^2$

$= 3x^2 + x^2 + 2x + 6x$ — *Rearrange the terms.*

$= (3 + 1)x^2 + (2 + 6)x$ — *Remember, $x^2 = 1x^2$.*

$= 4x^2 + 8x$ — *Simplify.*

Practice

Use the distributive property to rewrite each expression without parentheses.

1. $3(a + 4)$ **2.** $2(x + 3)$ **3.** $(h - 5)6$

4. $-3(b + f)$ **5.** $x(2 + y)$ **6.** $a(b + c)$

Simplify each expression, if possible. If not possible, write *in simplest form*.

7. $4x + 2x$ **8.** $6a + 3b$ **9.** $12xy + 4xy$

10. $11m + 7m^2 + 5m^2$ **11.** $10b + 6b^2 + 4b^3$ **12.** $27x^2 - 18x^2$

13. $15b^3 + 10b + 20b^3$ **14.** $2x^2 + 2x^2$ **15.** $3y^4 - 9y^5 + 15y^4 + 3y^6$

16. Mental Math How would you use the Distributive Property to find the product of 6 and 104 mentally? Show your steps.

17. Standardized Test Practice Use the Distributive Property to rewrite the expression $2(m + 4h + 2a)$ without using parentheses.

A $2m + 4h + 2a$ **B** $2m + 8h + 4a$ **C** $m + 4h^2 + 4a$ **D** $4m + 4h + 4a$

Answers: 1. $3a + 12$ **2.** $2x + 6$ **3.** $6h - 30$ **4.** $-3b - 3f$ **5.** $2x + xy$ **6.** $ab + ac$ **7.** $6x$ **8.** in simplest form **9.** $16xy$ **10.** $11m + 12m^2$ **11.** in simplest form **12.** $9x^2$ **13.** $35b^3 + 10b$ **14.** $4x^2$ **15.** $18y^4 - 9y^5 + 3y^6$ **16.** $6(100 + 4) = 600 + 24 = 624$ **17.** B

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1-6 Commutative and Associative Properties

(Pages 32–36)

You can use the Commutative and Associative Properties with other properties you have studied to evaluate or simplify expressions.

Commutative Property	The Commutative Property says that the order in which you add or multiply two numbers does not change their sum or product. For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Property	The Associative Property says that the way you group three numbers when you add or multiply them does not change their sum or product. For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Examples **Simplify.**

a. $2x^2 + 7x + 5x^2$

$2x^2 + 7x + 5x^2$
$= 2x^2 + 5x^2 + 7x$ — *Commutative (+)*
$= (2 + 5)x^2 + 7x$ — *Distributive Property*
$= 7x^2 + 7x$ — *Simplify.*

b. 642×7

642×7
$= (600 + 40 + 2)7$ — *Substitution (=)*
$= 4200 + 280 + 14$ — *Distributive Property*
$= 4494$ — *Add.*

Practice

Name the property illustrated by each statement.

1. $3 + 4 = 4 + 3$
2. $2 \cdot 9 = 9 \cdot 2$
3. $xy = yx$
4. $g + h + 2 = g + 2 + h$
5. $(2 + 5) + 7 = 2 + (5 + 7)$
6. $(6 \cdot 5)x = 6(5x)$
7. $7 + m = m + 7$
8. $3(4 \cdot 5) = (4 \cdot 5)3$
9. $ab + c = c + ab$

Simplify.

10. $3x + 2y + x$
11. $7a + 3n + 3a$
12. $8d + 2c + 2d + c$
13. $3m^4 + m^2 + 2m^4$
14. $10b^2 + 10b + 10b^2$
15. $\frac{1}{4}d + \frac{2}{3}g + \frac{1}{4}d$
16. $2(4x + y) - 3x$
17. $9 + 3(pq - 2) + pq$
18. $1.8(a + b) + 2.1(1 + a)$

19. Write an algebraic expression for the verbal expression "six times the sum of g and a increased by $3g$." Then simplify, indicating the properties used.

20. **Standardized Test Practice** Name the property or properties illustrated by the statement $s + t = t + s$.

A Associative only
B Commutative only
C Associative and Commutative
D neither Associative nor Commutative

Answers: **1.** commutative (+) **2.** commutative (×) **3.** commutative (×) **4.** commutative (+) **5.** associative (+) **6.** associative (×) **7.** commutative (+) **8.** commutative (×) **9.** commutative (+) **10.** $4x + 2y$ **11.** $10a + 3n$ **12.** $10d + 3c$ **13.** $5m^4 + m^2$ **14.** $20b^2 + 10b$ **15.** $\frac{1}{2}d + \frac{2}{3}g$ **16.** $5x + 2y$ **17.** $4pq + 3$ **18.** $3.9a + 1.8b + 2.1$ **19.** See Answer Key. **20.** B

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1-7 Logical Reasoning *(Pages 37–42)*

The statement *If it is raining outside, then I will wear my raincoat* is called a conditional statement. All **conditional statements** can be written in the form *If A, then B*. Statements of this form are known as **if-then statements**. *A*, the portion of the statement immediately following *if*, is called the **hypothesis**. *B*, the portion of the statement immediately following *then*, is called the **conclusion**.

The process of using definitions, rules, properties, or facts as a means of validating conditional statements is **deductive reasoning**. If a true conditional exists, with a known true hypothesis, then deductive reasoning permits the reader to acknowledge that the conclusion is true for the scenario. A counterexample can be used to show that a conditional is not correct. A **counterexample** is a specific situation in which a statement is false. Only one counterexample is necessary to show that a statement is incorrect.

Examples

a. Identify the hypothesis and the conclusion.

If 3a + 12 = 24, then a = 4.
Hypothesis: 3a + 12 = 24
Conclusion: a = 4

b. Write the conditional in if-then form.

I will attend the school play on Friday.
Hypothesis: It is Friday
Conclusion: I will attend the school play
If it is Friday, then I will attend the school play.

Try These Together

Identify the hypothesis and the conclusion. Write in if-then form.

1. I will earn an A for a score of 90% or higher.

2. Tom will play inside when the weather is bad.

Practice

Use deductive reasoning to verify whether each conditional is *true* or *false*. If it is false, provide a counterexample.

3. If there is a rainbow, then it must have rained while the Sun was shining.

4. If the flowers are wet, then it rained.

5. **Standardized Test Practice** Which numbers are counterexamples for the conditional statement.
If $x \cdot y = 60$, then x and y are positive numbers.

A $x = 10, y = 6$ **B** $x = 3, y = 20$ **C** $x = -2, y = -30$ **D** $x = 1, y = 60$

Answers: **1.** H: score of 90% or higher; C: earn an A; If I score 90% or higher, then I will earn an A. **2.** H: weather is bad; C: Tom will play inside; If the weather is bad, then Tom will play inside. **3.** True **4.** False, an irrigation system could also cause flowers to be wet **5.** C

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1-8 Graphs and Functions *(Pages 43–48)*

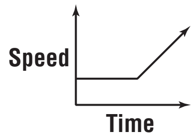
A **function** is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input. For example, vegetables are often sold by the pound. So, the weight of a vegetable would be the input and the total price would be the output. In this example, the price you pay depends on the weight of the vegetables. The weight of the vegetables that you purchase is the **independent variable** or **quantity**. The price you pay for the vegetables is the **dependent variable** or **quantity**. On a graph, the independent variable is usually graphed on the **horizontal axis**, and the dependent variable is graphed on the **vertical axis**. **Ordered pairs** are used to locate points on the graph. The ordered pair (0, 0) corresponds to the **origin**. A **relation** is a set of ordered pairs. The set of first numbers in the ordered pair is the **domain** of the relation, while the set of second numbers is the **range**.

Example

Marco rides his bike to school every morning. For a certain time, he rides at a steady rate. When he gets near the school, he must ride down a steep hill that causes him to pick up speed. What are the independent and dependent quantities? What would a graph of this situation look like?

Time is the independent quantity. Marco's speed is the dependent quantity because it depends on the time.

This graph shows that Marco's speed remains constant for most of the time when he rides to school, but increases near the end of his ride when he goes down the hill.

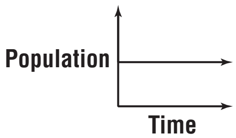


Practice

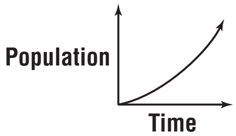
Identify the graph that matches the statement. Explain your answer.

1. The population of humans on Earth is increasing faster and faster each year.

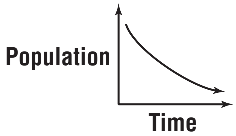
a.



b.

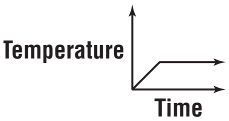


c.

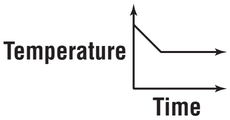


2. **Standardized Test Practice** On a summer day, when the temperature in Marjorie's apartment rises to 80°F, the air conditioner comes on and cools the apartment to 76°F. The air conditioner then switches off and stays off until the temperature rises to 80°F again. Then the cycle repeats. Which graph represents this situation?

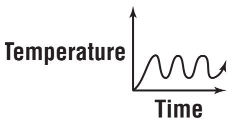
A



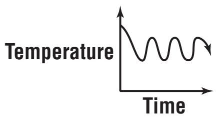
B



C



D



Answers: 1. b **2.** C

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1-9 Statistics: Analyzing Data by Using Tables and Graphs *(Pages 50–55)*

Numerical information, or **data**, can be analyzed using a variety of means. In basic statistics, the most common forms of data representation are tables, bar graphs, circle graphs, and line graphs. A **table** displays individual pieces of data in row and column form. **Bar graphs** are picture representations of data that consist of a series of rectangles, or bars, that compare different categories of data. Bar graphs can also display multiple sets or types of data simultaneously. A **circle graph** represents data as a piece, or percentage, of a whole set. The total pieces, or percentages, in a circle graph should have a sum of 100%. **Line graphs** consist of a series of ordered pairs that are connected to form a line. A line graph is particularly useful when displaying change. Also, a line graph can be beneficial when making predictions of future change or future trends.

Example

Malik collected the following data from his classmates. The data are a representation of the month in which the birthday of each of Malik's 25 classmates occurs.

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Boys	1	0	2	1	0	2	2	0	1	1	1	1
Girls	2	1	1	3	0	0	2	2	0	1	0	1
Total	3	1	3	4	0	2	4	2	1	2	1	2

Practice

Use the table above to answer each question.

1. How many total students in Malik's class have a birthday in either January or February?

a. 3 **b.** 4 **c.** 5

2. How many more students have a birthday in July than in June?

a. 0 **b.** 1 **c.** 2

3. **Standardized Test Practice** Malik would like to display the data he collected in a different form. He would like to make a graph that would compare the number of boys' birthdays to the number of girls' birthdays for each month. Which type of graph should he construct to show the comparison of the two different types of data?

A bar graph **B** circle graph **C** line graph

Answers: 1. b **2.** c **3.** A

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1 Chapter Review

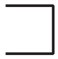
Crack the Code

Use the secret code in the box at the right to crack these problems.

Secret Code

1	4	7
2	5	8
3	6	9

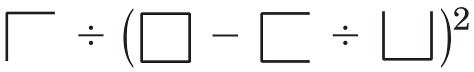
 = 1

 = 2

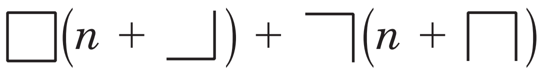
and so on

1. 

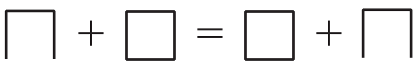
2. 

3. 

4. 

5. 

6. What property is illustrated below?



Answers are located in the Answer Key.

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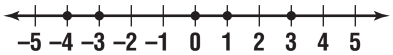
2-1 Rational Numbers on the Number Line *(Pages 68–72)*

A **number line** is a visual representation of the numbers from **negative infinity** to **positive infinity**, which means it extends indefinitely in two directions. The number line consists of **negative numbers** on its left, zero in the middle, and **positive numbers** on its right. You can **graph** a number on the number line by drawing a point on the place on the number line that corresponds to the given number. For example, to graph -5 on the number line, you would place a point on the tick mark that is five places to the left of zero. -5 is called the **coordinate** of this point. The **absolute value**, or distance from zero on the number line, of -5 is 5 because -5 is 5 units away from zero, $|-5| = 5$.

The numbers on the number line can be grouped into different categories. The **natural numbers** are the numbers in the set {1, 2, 3, 4, 5, ...}. The three dots in the set signify that the set continues in this pattern indefinitely. The **whole numbers** are the numbers {0, 1, 2, 3, 4, ...}. **Integers** are the whole numbers and their opposites {..., -2, -1, 0, 1, 2, ...}. A **rational number** is any number that can be expressed as a fraction whose denominator is not equal to zero. For example, $-\frac{2}{3}$, $\frac{4}{5}$, $\frac{30}{10}$, and $\frac{9}{2}$ are all rational numbers. The rational numbers can also be expressed in decimal form. More specifically, the decimal equivalent of any rational number will terminate or will repeat. If the decimal repeats it should be written with bar notation. Notice that $-\frac{2}{3} = 0.\overline{6}$, $\frac{4}{5} = 0.8$, $\frac{30}{10} = 3$, and $\frac{9}{2} = 4.5$.

Examples

a. Name the set of numbers graphed.



The graph shows the set: {−4, −3, 0, 1, 3}.

b. Find the absolute value.

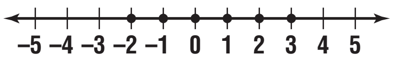
|10|

10 is ten units from zero in the positive direction. Therefore, |10| = 10.

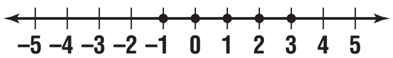
Practice

Name the set of numbers graphed.

1.



2.



3.

4.

Graph each set of numbers on a number line.

5. {integers from -2 to 6, inclusive}

6. {-4, -3, -2, -1}

7. {integers less than 1 but greater than -4}

8. {integers greater than 2}

9. {integers less than or equal to 3}

10. {integers less than $-4 + (-1)$}

11. **Standardized Test Practice** Which number shows the absolute value of -30?

A $|-30| = -30$ **B** $|-30| = 30$ **C** $|-30| = \frac{1}{30}$ **D** $|-30| = -\frac{1}{30}$

Answers: 1. {3, 2, 1, 0, −1, −2} **2.** {−1, 0, 1, 2, 3} **3.** {3, 4, 5,...} **4.** {−2, −3, −4, −5,...} **5–10.** See Answer Key. **11.** B

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2-2 Adding and Subtracting Rational Numbers *(Pages 73–78)*

The **absolute value** of a number is its distance from zero on a number line and is denoted by bars around a quantity. These absolute value bars can serve as grouping symbols. For example, $|-3 + 1| = 2$ since $-3 + 1 = -2$ and $|-2| = 2$.

Adding Integers	• To add integers with the same sign, add their absolute values. The sum has the same sign as the integers. • To add integers with different signs, subtract the lesser absolute value from the greater absolute value and give the result the same sign as the integer with the greater absolute value.
Additive Inverse Property	For every number a, $a + (-a) = 0$.
Subtracting Integers	To subtract a number, add its **additive inverse** or **opposite**. For any numbers a and b, $a - b = a + (-b)$.

Examples

a. Find −9 + 16.

The addends have different signs, so find the difference of their absolute values.

$|16| - |-9| = 16 - 9$ *or* 7

Use the sign of 16 because it has the greater absolute value.

$-9 + 16 = 7$

b. Find −3 − 4.

Rewrite this problem as an addition problem.

$-3 - 4 = -3 + (-4)$ *To subtract 4, add −4.*

The addends have the same sign, so add and keep the same sign.

$-3 - 4 = -7$

Practice

1. State the additive inverse and absolute value of −111.

Find each sum or difference.

2. $-100 + 82$ **3.** $-8 + 17$ **4.** $4 - (-12)$ **5.** $-10 - (-24)$

6. $|-23 - (-8)|$ **7.** $|-111 - (-56)|$ **8.** $-15 + (-3)$ **9.** $13 - (-2)$

Simplify each expression.

10. $6t + (-14t)$ **11.** $-7s + (-15s)$ **12.** $-8n - (-13n)$ **13.** $-16p - 4p$

Evaluate each expression if $x = -3$, $y = 4$, and $z = -6$.

14. $x + 12$ **15.** $y + z$ **16.** $|z| - y$ **17.** $-|z - 8|$

18. **Standardized Test Practice** Simplify $-12 - (-14)$.

A −2 **B** −16 **C** 16 **D** 2

Answers: 1. 111; 111 **2.** −18 **3.** 9 **4.** 16 **5.** 14 **6.** 15 **7.** 55 **8.** −18 **9.** 15 **10.** $-8t$ **11.** $-22s$ **12.** $5n$ **13.** $-20p$ **14.** 9 **15.** −2 **16.** 2 **17.** −14 **18.** D

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2-3 Multiplying Rational Numbers *(Pages 79–83)*

The product of two numbers having the *same sign* is positive. The product of two numbers having *different signs* is negative. It is also useful to note that multiplying a number or expression by -1 results in the opposite of the number or expression. This is called the **multiplicative property of -1**.

Examples

a. Evaluate $-3x^2$ for $x = -\frac{2}{3}$.

$-3x^2 = -3\left(-\frac{2}{3}\right)^2$	*Replace x with $-\frac{2}{3}$.*
$= -3\left(\frac{4}{9}\right)$	$\left(-\frac{2}{3}\right)^2 = -\frac{2}{3} \cdot \left(-\frac{2}{3}\right)$ or $\frac{4}{9}$
$= -\frac{\overset{1}{\cancel{3}}}{1}\left(\frac{4}{\underset{3}{\cancel{9}}}\right)$	*Divide out common factors.*
$= -\frac{4}{3}$ or $-1\frac{1}{3}$	*Multiply. The signs are different, so the product is negative.*

b. Simplify $(-1)(2x)(-3y) + (4x)(-5y)$

$(-1)(2x)(-3y) + (4x)(-5y)$

$= 2x(-1)(-3y) + (4x)(-5y)$	*Commutative Property*
$= 2x(3y) + (-20xy)$	*Multiply.*
$= 6xy + (-20xy)$	*Multiply.*
$= -14xy$	*Combine like terms.*

Practice

Find each product.

1. $(-2)(3)(-5)$

2. $5.26(-0.011)$

3. $-10.01(-10.11)$

4. $2\left(\frac{3}{5}\right)\left(-\frac{5}{7}\right)$

5. $\left(-\frac{8}{11}\right)\left(\frac{9}{10}\right)$

6. $\left(-\frac{7}{10}\right)\left(-\frac{13}{21}\right)$

7. $\left(-\frac{8}{13}\right)(0)\left(-\frac{4}{5}\right)$

8. $3\left(\frac{4}{9}\right)(-4)\left(\frac{6}{7}\right)$

9. $\left(-\frac{2}{5}\right)(-4)\left(-\frac{3}{8}\right)$

10. $5\left(\frac{3}{4}\right)(-4)(-2)$

11. $8(-0.25)(-3)$

12. $\frac{2}{7}(-21)(13)\left(\frac{1}{14}\right)$

Evaluate each expression if $r = -\frac{1}{8}$, $s = \frac{4}{5}$, $t = -2\frac{9}{10}$, and $w = -1\frac{2}{9}$.

13. $4rs$

14. $2tw$

15. $rt - s$

16. $s^2\left(-\frac{1}{8}\right)$

Simplify.

17. $2m\left(-\frac{1}{3}n\right) + 3m(-2n)$

18. $1.2(3x + y) - 0.8(22x - 2y)$

19. **Standardized Test Practice** The velocity of an object t seconds after the object is dropped from the top of a tall building is about $-9.8t$ meters per second (m/s). What is its velocity 2.5 seconds after it is dropped?

A -24.5 m/s **B** -7.3 m/s **C** 7.3 m/s **D** 18.4 m/s

Answers: **1.** 30 **2.** -0.05786 **3.** 101.2011 **4.** $-\frac{6}{7}$ **5.** $-\frac{36}{55}$ **6.** $\frac{13}{30}$ **7.** 0 **8.** $-4\frac{4}{7}$ **9.** $-\frac{3}{5}$ **10.** 30 **11.** 6 **12.** $-5\frac{4}{7}$ **13.** $-\frac{2}{5}$ **14.** $7\frac{4}{45}$ **15.** $-\frac{7}{16}$ **16.** $-\frac{2}{25}$ **17.** $-6\frac{2}{3}mn$ **18.** $-14x + 2.8y$ **19.** A

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2-4 Dividing Rational Numbers *(Pages 84–87)*

You can use the same rules of signs when dividing rational numbers that you used for multiplying.

Dividing Two Rational Numbers	The quotient of two numbers having the *same sign* is positive. The quotient of two numbers having *different signs* is negative.

If a fraction has one or more fractions in the numerator or denominator, it is a **complex fraction**. To simplify a complex fraction, rewrite it as a division expression.

Examples

a. Simplify $\frac{\frac{4}{7}}{-8}$.

Rewrite the complex fraction as $\frac{4}{7} \div (-8)$.

$\frac{4}{7} \div (-8) = \frac{4}{7} \cdot \left(-\frac{1}{8}\right)$ *Multiply by* $-\frac{1}{8}$*, the reciprocal of −8.*

$= -\frac{4}{56}$ *or* $-\frac{1}{14}$ *The signs are different, so the product is negative.*

b. Simplify $\frac{-2x + 10y}{5}$.

$\frac{-2x + 10y}{5} = \frac{-2x}{5} + \frac{10y}{5}$ *Divide each term by 5.*

$= -\frac{2}{5}x + 2y$ *Simplify.*

Practice

Simplify.

1. $22 \div \left(\frac{11}{13}\right)$
2. $24 \div \left(-\frac{1}{8}\right)$
3. $\frac{-14}{-2}$
4. $\frac{-\frac{15}{64}}{3}$
5. $\frac{-\frac{30}{7}}{-10}$
6. $\frac{8}{-\frac{4}{9}}$
7. $\frac{-32m}{8}$
8. $-18t \div \frac{8}{9}$
9. $\frac{2a + 8}{4}$
10. $\frac{8x + 42y}{6}$
11. $\frac{-12h + (-18g)}{3}$
12. $\frac{54s + 3w}{-6}$

Evaluate each expression if $x = 4$, $y = -5$, and $z = -1.5$.

13. $\frac{y}{z}$
14. $\frac{xy}{xz}$
15. $\frac{x + z}{3}$

16. **Standardized Test Practice** How many boxes of peanuts can you get from 52 pounds of peanuts if each box holds $1\frac{5}{8}$ pounds of peanuts?

A 84 **B** 32 **C** 26 **D** 50

Answers: **1.** 26 **2.** −192 **3.** 7 **4.** $-\frac{5}{64}$ **5.** $\frac{3}{7}$ **6.** −18 **7.** $-4m$ **8.** $-20\frac{1}{4}t$ **9.** $\frac{1}{2}a + 2$ **10.** $1\frac{1}{3}x + 7y$ **11.** $-4h - 6g$ **12.** $-9s - \frac{1}{2}w$ **13.** $3\frac{1}{3}$ **14.** $3\frac{1}{3}$ **15.** $\frac{5}{6}$ **16.** B

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2-5 Statistics: Displaying and Analyzing Data *(Pages 88–94)*

Two common methods of displaying data are line plots and stem-and-leaf plots. Data are analyzed most often by frequency and central tendency. The **frequency** is the number of times an individual element occurs within the data. Central tendency consists of the mean (the average of the elements in the data), median (the middle most number when the data are arranged numerically), and mode (the number that occurs the most).

You can display numerical data on a number line with a **line plot**.

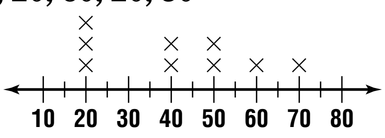
Drawing a Line Plot	• Draw and label a number line. • Choose a scale that includes the range of values in the data from least to greatest. • Choose an interval and divide the number line into these intervals. • Draw the line plot making a mark (such as an X) above the number line to show each item in the data.

Example

Draw a line plot for this data: 20, 40, 70, 50, 40, 20, 60, 20, 50

These values range from 20 to 70, so the scale on the number line must include these values.

An interval of 10 fits this data.



In a **stem-and-leaf plot**, the greatest common place-value of the data is used to form the **stems**. The numbers in the next greatest place-value position are then used to form the **leaves**.

Example

Organize the following test scores into a stem-and-leaf plot.
72, 69, 98, 77, 92, 85, 79, 86, 90, 98, 83, 100, 77, 98, 91

Stem	Leaf
6	9
7	2 7 7 9
8	3 5 6
9	0 1 2 8 8 8
10	0

8|3 = 83

a. Which grade occurred most frequently? 98 (three times)

b. What were the highest and lowest grades? 69 and 100

c. How many people scored 80 or above? 10 people

Practice

Use the stem-and-leaf plot above to answer the following questions.

1. What is the frequency of 77?

2. What is the mean of the data rounded to the hundreths place?

3. What is the median of the data?

4. What is the mode of the data?

5. **Standardized Test Practice** Use the above line plot to answer the following question. What is the mode of the data?

A 3 **B** 20 **C** 40 **D** 41.1

Answers: 1. 2 **2.** 86.3 **3.** 86 **4.** 98 **5.** B

2-6 Probability: Simple Probability and Odds

(Pages 96–101)

You can calculate the chance, or **probability**, that a particular event will happen by finding the ratio of the number of ways the event can occur to the number of possible outcomes. The probability of an event may be written as a fraction, decimal, or percent. When outcomes have an equal chance of occurring, they are **equally likely**. When an outcome is chosen without any preference, the outcome occurs at **random**.

Definition of Probability	**probability of an event** or $P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$
Definition of Odds	**odds** of an event = number of ways the event can occur : number of ways the event cannot occur = successes : failures

Examples

a. Find the probability of randomly choosing the letter *p* in the word "apple."

There are 2 p's and 5 letters in all.

P(choosing a p) $= \frac{2}{5}$

The probability is $\frac{2}{5}$*, 0.4, or 40%.*

b. Find the odds of randomly selecting the letter *p* in the word "Mississippi."

There are 11 letters in the word. Two letters are p's and 11 − 2 or 9 letters are not p's.

Odds of selecting a p
= number of p's : number not p's
= 2:9   2:9 is read "2 to 9."

Try These Together

1. What is the probability of rolling a 1 or a 2 using a 6-sided number cube?
HINT: The number of favorable outcomes is 2.

2. From a group of 125 boys and 150 girls, what are the odds of randomly selecting a girl?
HINT: Remember to simplify your ratio.

Practice

Determine the probability of each event.

3. You toss a coin and get heads.

4. A person was born on a weekday.

Find the probability of each outcome if a computer randomly chooses a letter in the word "mathematical."

5. the letter *t* **6.** the letter *a* or *c* **7.** the letter *d* **8.** not an *m*

Find the odds of each outcome if a computer randomly chooses a letter in the word "Alabama."

9. the letter *a* **10.** the letter *b* **11.** a consonant **12.** not a *g*

13. **Standardized Test Practice** What are the odds of randomly selecting a dime from a dish containing 11 pennies, 6 nickels, 5 dimes, and 3 quarters?

A 5:1 **B** 1:5 **C** 1:4 **D** 4:1

Answers: 1. $\frac{1}{3}$ **2.** 6:5 **3.** $\frac{1}{2}$ **4.** $\frac{5}{7}$ **5.** $\frac{1}{6}$ **6.** $\frac{1}{3}$ **7.** 0 **8.** $\frac{5}{6}$ **9.** 4:3 **10.** 1:6 **11.** 3:4 **12.** 7:0 **13.** C

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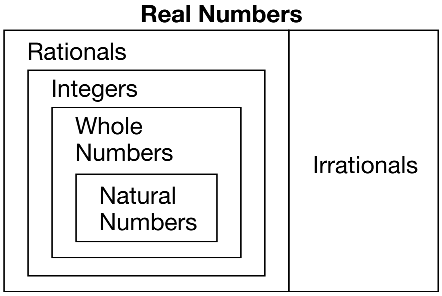
2-7 Square Roots and Real Numbers

(Pages 103–109)

If $x^2 = y$, then x is a **square root** of y. A rational number, like 81, whose square root, 9, is a rational number, is called a **perfect square**. The number 81 has two square roots, 9 and −9. The **radical sign** $\sqrt{\ }$ is used to indicate a nonnegative or **principal square root**. For example, $\sqrt{81} = 9$.

A square root of a positive rational number that is not a perfect square is an **irrational number**. An irrational number is a number that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

The set of rational numbers and the set of irrational numbers together form the set of **real numbers**. The graph of the set of all real numbers is the entire number line.



Examples

a. Find $\sqrt{0.09}$.

$\sqrt{0.09} = 0.3$ since $(0.3) \cdot (0.3) = 0.09$

b. Find $-\sqrt{0.4}$ to the nearest hundredth using a calculator.

$\sqrt{0.4} \approx 0.63$, so $-\sqrt{0.4} \approx -0.63$

Practice

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if necessary.

1. $\sqrt{\frac{9}{16}}$
2. $\sqrt{441}$
3. $-\sqrt{\frac{121}{196}}$
4. $-\sqrt{961}$
5. $\sqrt{6.4}$

Evaluate each expression. Use a calculator if necessary. Round to the nearest hundredth if necessary.

6. $\sqrt{a}$, if $a = 729$
7. $-\sqrt{cd}$, if $c = 36$ and $d = 81$
8. $\sqrt{q + r}$, if $q = 42$ and $r = 30$

Name the set or sets of numbers to which each real number belongs. Use N for natural numbers, W for whole numbers, Z for integers, Q for rational numbers, and I for irrational numbers.

9. $\sqrt{64}$
10. $\frac{-20}{2}$
11. $\sqrt{50}$
12. $-\sqrt{100}$

13. **Standardized Test Practice** A rectangular field has a length of ℓ feet and a width of w feet. The distance from any corner of the field to the diagonally-opposite corner is $\sqrt{\ell^2 + w^2}$. What is the diagonal distance across a field that is 96 feet long and 28 feet wide?

A 144 ft **B** 100 ft **C** 124 ft **D** 114 ft

Answers: **1.** $\frac{3}{4}$ **2.** 21 **3.** $-\frac{11}{14}$ **4.** −31 **5.** 2.53 **6.** 27 **7.** −54 **8.** 8.49 **9.** N, W, Z, Q **10.** Z, Q **11.** I **12.** Z, Q **13.** B

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2 Chapter Review

Vacation Getaway

1. You have won a free two-week vacation to anywhere around the world. Simplify each expression to find the average temperature in December in degrees Celsius for each city.

a. Berlin, Germany $-9 \cdot \left(-\frac{1}{9}\right)$ ________°C

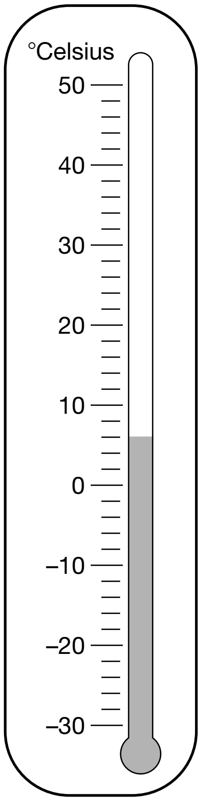
b. London, England $-5 + 10$ ________°C

c. Montreal, Quebec $-6 + (-1)$ ________°C

d. Paris, France $\frac{3}{8} \div \frac{2}{16}$ ________°C

e. Beijing, China $-10 \cdot \frac{1}{5}$ ________°C

f. Sao Paulo, Brazil $-63 \div (-3)$ ________°C



2. The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Estimate the temperatures in Fahrenheit for each city by using $F \approx 2C + 32$.

3. Why do you think the average temperature in Sao Paulo, Brazil, is so high compared to the other cities?

4. Which city would you choose for your free vacation if you go in December? Why?

Answers are located in the Answer Key.

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3-1 Writing Equations *(Pages 120–126)*

Chapter 3

You can use a four-step plan to solve problems.

Problem-Solving Plan	1. Explore the problem. 2. Plan the solution. 3. Solve the problem. 4. Examine the solution.
Writing an Equation	Many verbal sentences that express numerical relationships can be written as equations. **Define a variable** to represent one of the unspecified numbers or measures referred to in the sentence or problem. Some words that suggest the equals sign are • is • is equal to • is as much as • equals • is the same as • is identical to

Examples **Translate each verbal sentence into an equation or inequality.**

a. Juan has 3 more books than Maria, and together they have 15 books.

Let m = the number of books Maria has.
$(m + 3) + m = 15$

b. Twice the sum of the square of a number and 14 is greater than 32.

Let x = the number.
$2(x^2 + 14) > 32$

Practice

1. A farmer has a rectangular field that is 200 feet longer than it is wide. The perimeter of the field is 4000 feet.

a. If w represents the width of the field, what expression represents the length of the field?

b. What expression represents the perimeter of the field?

c. What equation expresses the fact that the perimeter is 4000 feet?

Translate each sentence into an equation, inequality, or formula.

2. The product of x and the cube of y is 30.

3. The area of a circle is the product of π and the square of the radius.

4. Two-thirds of the sum of a, the square of b, and c is the same as 45.

5. The sum of m and n is at least twice as large as the difference of m and n.

6. A Kodiak bear begins having 3 cubs every 3 years starting at age 6. If the average lifespan of a Kodiak bear is 29 years, how many cubs does a mother bear average in a lifetime?

7. **Standardized Test Practice** What is the width of a rectangular field that has a perimeter of 4000 feet if the length of the field is 200 feet greater than the width?

A 1800 ft **B** 1100 ft **C** 900 ft **D** 800 ft

Answers: 1a. $w + 200$ **1b.** $w + (w + 200) + w + (w + 200)$ or $4w + 400$ **1c.** $4w + 400 = 4000$ **2.** $xy^3 = 30$ **3.** $A = \pi r^2$ **4.** $\frac{2}{3}(a + b^2 + c) = 45$ **5.** $m + n \geq 2(m - n)$ **6.** 24 cubs **7.** C

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3-2 Solving Equations by Using Addition and Subtraction *(Pages 128–134)*

You can add or subtract the same number on each side of an equation and the result is an **equivalent equation**. Equivalent equations have the same solution.

Addition Property of Equality	For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Solving Equations	To **solve an equation** means to get the variable (with a coefficient of 1) by itself on one side of the equation. You can do this by undoing what has been done to the variable, using the properties of equality.

Examples

a. Solve $x - \frac{2}{3} = \frac{1}{3}$.

The number $\frac{2}{3}$ has been subtracted from x. The opposite of subtracting $\frac{2}{3}$ is adding $\frac{2}{3}$. Add $\frac{2}{3}$ to each side of the equation. $x - \frac{2}{3} + \frac{2}{3} = \frac{1}{3} + \frac{2}{3}$ is an equivalent equation. Simplify to obtain $x = 1$.

Check: Is $1 - \frac{2}{3} = \frac{1}{3}$? Yes.

The solution is 1.

b. Solve $9 + y = 13$.

Write an equivalent equation by subtracting 9 from each side of the original equation.

$9 + y - 9 = 13 - 9$, so $y = 4$.

Check: Does $9 + 4 = 13$? Yes.

The solution is 4.

Try These Together

1. Solve $a + (-8) = 17$.
HINT: Add 8 to each side.

2. Solve $b - (-18) = 4$.
HINT: This equation is equivalent to $b + 18 = 4$.

Practice

Solve each equation. Check your solution.

3. $11 - c = -16$
4. $5.4 = d + 6.2$
5. $e - (-23) = 31$
6. $4.8 + f = 9.6$
7. $g - (-20) = 11$
8. $14 = h - 21$
9. $-2.8 = j + (-5.1)$
10. $-12 + k = -19$
11. $m + (-8) = \frac{1}{2}$

12. Age Minya is 30 years younger than her mom, and the sum of their ages is 58. How old is Minya?

13. **Standardized Test Practice** If the low temperature for the day is −14°F and the high is 22°F, by how much did the temperature increase?

A 8°F **B** 18°F **C** 28°F **D** 36°F

Answers: **1.** 25 **2.** −14 **3.** 27 **4.** −0.8 **5.** 8 **6.** 4.8 **7.** −9 **8.** 35 **9.** 2.3 **10.** −7 **11.** $8\frac{1}{2}$ **12.** 14 **13.** D

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3-3 Solving Equations by Using Multiplication and Division *(Pages 135–140)*

You can solve a multiplication or division equation by using the Multiplication and Division Properties of Equality.

Multiplication Property of Equality	For any numbers a, b, and c, if $a = b$, then $ac = bc$.
Division Property of Equality	For any numbers a, b, and c, with $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Examples

a. Solve $\left(2\frac{1}{2}\right)x = 1\frac{3}{4}$.

Rewrite the mixed numbers as improper fractions.

$\frac{5}{2}x = \frac{7}{4}$ *Multiply each side by $\frac{2}{5}$, the reciprocal of the number that is multiplied by x.*

$\left(\frac{2}{5}\right)\left(\frac{5}{2}\right)x = \left(\frac{7}{4}\right)\left(\frac{2}{5}\right)$ so $x = \frac{14}{20}$ or $\frac{7}{10}$.

b. Solve $7y = -63$.

Since y has been multiplied by 7, divide each side by 7 to isolate the variable.

$\frac{7y}{7} = \frac{-63}{7}$, so $y = -9$.

Try These Together

1. Solve $-5a = 55$.

HINT: Divide each side by −5 or multiply by $\frac{1}{-5}$.

2. Solve $\frac{x}{-5} = 4$.

HINT: Multiply each side by −5.

Practice

Solve each equation. Check your solution.

3. $6y = 54$ **4.** $-7d = -84$ **5.** $22b = 176$ **6.** $2.4f = 21.6$

7. $0.36g = 1.8$ **8.** $\frac{1}{6}k = 8$ **9.** $-\frac{4}{5}m = 2$ **10.** $\frac{n}{8} = -4$

11. $\frac{p}{-6} = \frac{7}{12}$ **12.** $\left(-2\frac{1}{3}\right)q = 21$ **13.** $5x = \frac{10}{13}$ **14.** $\frac{-r}{8} = -18$

Define a variable, write an equation and solve the problem.

15. Two-thirds of a number is $9\frac{3}{5}$.

16. Negative fourteen times a number is 84.

Complete.

17. If $6a = 36$, then $3a =$ __?__.

18. If $2d = 7$, then $10d =$ __?__.

19. **Standardized Test Practice** There are nine boys in a class. If the boys make up three-eighths of the entire class, how many students are in the class?

A 72 **B** 24 **C** 20 **D** 10

Answers: **1.** −11 **2.** −20 **3.** 9 **4.** 12 **5.** 8 **6.** 9 **7.** 5 **8.** 48 **9.** $-2\frac{1}{2}$ **10.** −32 **11.** $-3\frac{1}{2}$ **12.** −9 **13.** $\frac{2}{13}$ **14.** 144 **15.** $14\frac{2}{5}$ **16.** −6 **17.** 18 **18.** 35 **19.** B

3-4 Solving Multi-Step Equations *(Pages 142–148)*

Solving Multi-Step Equations	• Work backward to isolate the variable and solve the equation. • Use subtraction to undo addition, and use addition to undo subtraction. • Use multiplication to undo division, and use division to undo multiplication.

Consecutive integers are integers in counting order, such as −3, −2, and −1.

Examples

a. Solve $\frac{2x - 3}{4} = 9$.

Multiply each side by 4 to eliminate the fraction.

$4\left(\frac{2x - 3}{4}\right) = 9(4)$

$2x - 3 = 36$

Next, undo the subtraction by adding 3 to each side.

$2x - 3 + 3 = 36 + 3$

$2x = 39$

Last, undo the multiplication by dividing each side by 2.

$\frac{2x}{2} = \frac{39}{2}$

$x = 19\frac{1}{2}$

b. Find 3 consecutive odd integers whose sum is −3.

Let n = the least odd integer. Then n + 2 = the next greater odd integer, and n + 4 = the greatest of the three odd integers.

$n + (n + 2) + (n + 4) = -3$

$3n + 6 = -3$ *Add like items.*

$3n + 6 - 6 = -3 - 6$ *Subtract 6 from each side.*

$3n = -9$ *Simplify.*

$\frac{3n}{3} = \frac{-9}{3}$ *Divide each side by 3.*

$n = -3$ *Simplify.*

n + 2 = −3 + 2 or −1 and n + 4 = −3 + 4 or 1, so the consecutive odd integers are −3, −1, and 1.

Practice

Solve each equation. Check your solution.

1. $10 - 7p = -18$ **2.** $-1.9r + 9.3 = 15$ **3.** $6 = \frac{s}{3}$ **4.** $\frac{-4m - 3}{-6} = -9$

5. $-6 = \frac{-2n - 3}{4}$ **6.** $\frac{t}{5} - 4 = -10$ **7.** $11 = -7 - \frac{g}{3}$ **8.** $\frac{5}{6}b + 8 = -11$

9. $13 = -8 - 3t$ **10.** $-\frac{3 + n}{7} = -5$ **11.** $\frac{s + 4}{-2} = -16$ **12.** $3 - 9t = 21$

Define a variable, write an equation, and solve each problem.

13. Find two consecutive odd integers whose sum is 128.

14. Find three consecutive even integers whose sum is 90.

15. **Standardized Test Practice** Sally is eight years older than John. John is fourteen years older than Kareem. If the sum of all three ages is 90, how old is Kareem?

A 8 **B** 18 **C** 28 **D** 40

Answers: 1. 4 **2.** −3 **3.** 18 **4.** $-14\frac{1}{4}$ **5.** $10\frac{1}{2}$ **6.** −30 **7.** −54 **8.** $-22\frac{4}{5}$ **9.** −7 **10.** 32 **11.** 28 **12.** −2 **13.** 63, 65 **14.** 28, 30, 32 **15.** B

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3-5 Solving Equations with the Variable on Each Side *(Pages 149–154)*

To solve an equation that has the variable on both sides, use the properties of equality to write an equivalent equation that has the variable on only one side. Then solve. When you solve equations that contain grouping symbols, you may need to use the distributive property to remove the grouping symbols. Some equations may have no solution because there is no value of the variable that will result in a true equation. For example, $x + 1 = x + 2$ has no solution; it cannot be true. An equation that is true for every value of the variable is called an **identity**. For example, $x + x = 2x$ is true for every value of x.

Examples

a. Solve $3(x - 2) = 4x + 5$.

First use the distributive property to remove the parentheses.

$3x - 6 = 4x + 5$

Next, collect all the terms with x on one side of the equal sign by subtracting 3x from each side.

$3x - 6 - 3x = 4x + 5 - 3x$

$-6 = x + 5$ *Add like terms.*

$-6 - 5 = x + 5 - 5$ *Subtract 5 from each side.*

$-11 = x$ *Simplify.*

b. Solve $\frac{1}{2}y = \frac{1}{3}y + 2$.

First, multiply each side by 6, the LCD, to clear the fractions from the problem.

$6 \cdot \frac{1}{2}y = 6\left(\frac{1}{3}y + 2\right)$

$6 \cdot \frac{1}{2}y = 6 \cdot \frac{1}{3}y + 6 \cdot 2$

$3y = 2y + 12$

Next, collect all the terms with y on one side of the equal sign by subtracting 2y from each side.

$3y - 2y = 2y - 2y + 12$

$y = 12$

Try These Together

1. Solve $4x + 3 = 5x + 7$.

HINT: Subtract 4x from each side.

2. Solve $7 + 3t = \frac{6 - t}{2}$.

HINT: Multiply each side by 2.

Practice

Solve each equation. Then check your solution.

3. $18 + 2n = 4n - 9$

4. $10 - 2.7y = y + 9$

5. $\frac{2}{3}n + 6 = \frac{1}{4}n - 3$

6. $11.1c - 2.4 = -8.3c + 6.4$

7. $3 - 4x = 8x + 8$

8. $\frac{3}{5}d + 5 = \frac{1}{3}d - 3$

9. $3(2x - 1) = 9(x + 3)$

10. $2(2x - 5) = 6x + 4$

11. $-6(4x + 1) = 5 - 11x$

12. $\frac{5}{6}(12p + 4) = -13p + 4$

13. $-8\left(\frac{1}{4}n - 3\right) = n + 2$

14. $\frac{2 + t}{3} = 4 - \frac{6}{7}t$

15. **Standardized Test Practice** Nine less than half n is equal to one plus the product of $-\frac{1}{8}$ and n. Find the value of n.

A 24   **B** −21   **C** 8   **D** 16

Answers: 1. −4 **2.** $-\frac{8}{7}$ **3.** 13.5 **4.** $\frac{10}{37}$ **5.** $-21\frac{3}{5}$ **6.** $\frac{44}{97}$ **7.** $-\frac{5}{12}$ **8.** −30 **9.** −10 **10.** −7 **11.** $-\frac{11}{13}$ **12.** $\frac{2}{69}$ **13.** $7\frac{1}{3}$ **14.** 2.8 **15.** D

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3-6 Ratios and Proportions *(Pages 155–159)*

A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y, $x:y$, or $\frac{x}{y}$. An equation stating that two ratios are equal is called a **proportion**. In $\frac{a}{b} = \frac{c}{d}$, the numbers a and d are the **extremes** and the numbers b and c are the **means**.

Means-Extremes Property of Proportions	In a proportion, the product of the extremes is equal to the product of the means. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. The cross products, ad and bc, in a proportion are equal.

You can write proportions that involve a variable and then use cross products to solve the proportion.

Examples

a. Do the ratios $\frac{3}{4}$ and $\frac{4}{3}$ form a proportion?

$\frac{3}{4} \stackrel{?}{=} \frac{4}{3}$ *Check cross products.*

$3 \cdot 3 \stackrel{?}{=} 4 \cdot 4$

$9 = 16$ *False*

Since $9 \neq 16$, $\frac{3}{4}$ and $\frac{4}{3}$ do not form a proportion.

b. Solve the proportion $\frac{3}{4} = \frac{x + 2}{x}$.

Set the cross products equal to each other.

$3(x) = 4(x + 2)$

$3x = 4x + 8$ *Distribute.*

$3x - 4x = 4x + 8 - 4x$ *Subtract 4x from each side.*

$-1x = 8$ *Simplify.*

$\frac{-1x}{-1} = \frac{8}{-1}$ *Divide each side by −1.*

$x = -8$ *Simplify.*

The solution is −8.

Practice

Use cross products to determine whether each pair of ratios forms a proportion.

1. $\frac{7}{8}, \frac{28}{34}$ **2.** $\frac{15}{20}, \frac{5}{7}$ **3.** $\frac{100}{240}, \frac{5}{12}$ **4.** $\frac{7.5}{10}, \frac{21}{28}$

Solve each proportion.

5. $\frac{8}{5} = \frac{x}{35}$ **6.** $\frac{a}{12} = \frac{6}{18}$ **7.** $\frac{2.2}{6} = \frac{11}{y}$ **8.** $\frac{5}{p} = \frac{7}{8.4}$

9. $\frac{20}{35} = \frac{2x}{7}$ **10.** $\frac{1}{2} = \frac{22}{c - 5}$ **11.** $\frac{12}{v + 3} = \frac{1}{4}$ **12.** $\frac{8}{9} = \frac{d - 1}{18}$

13. Medicine Your doctor has prescribed two teaspoons of medicine to be taken every six hours. How much medicine will you have taken in 4 days? (*Hint:* Convert 4 days into hours.)

14. Standardized Test Practice Two out of every seven people at a particular high school play in the band. If the school has 742 students, how many of them are in the band?

A 106 students **B** 212 students **C** 371 students **D** 1484 students

Answers: 1. no **2.** no **3.** yes **4.** yes **5.** 56 **6.** 4 **7.** 30 **8.** 6 **9.** 2 **10.** 49 **11.** 45 **12.** 17 **13.** 32 teaspoons **14.** B

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3-7 Percent of Change *(Pages 160–164)*

Finding Percent of Change	percent of change $= \dfrac{\text{amount of change}}{\text{original amount}}$ amount of change = original amount − new amount percent of decrease ⇒ new amount is less than original amount percent of increase ⇒ new amount is more than original amount

Examples

a. Find the percent of change if the original price of an item is $56 and the new price $32. Is this change a percent of increase or decrease?

amount of change: 56 − 32 or 24

$\frac{\text{amount of change}}{\text{original amount}} = \frac{24}{56}$ *or about 0.43*

The percent of change is 43%.

Since the new amount is less than the original amount, 32 < 56, this is a percent of decrease.

b. A book with an original price of $15 is on sale at a discount of 25%. If the sales tax is 10%, what is the final price of the book?

Discount = 25% of original price
= 0.25 ·15 or $3.75
Sale price = $15 − $3.75 or $11.25
Tax = 10% of sale price
= 0.10 · $11.25 or $1.13
Final = $11.25 + $1.13
= $12.38

Try This Together

1. original: 500 tons
new: 640 tons
Is this change a percent of increase or decrease? Find the percent of change.
HINT: Subtract to find the amount of change.

Practice

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of increase or decrease. Round to the nearest whole percent.

2. original: 12 cm
new: 30 cm

3. original: 40 mph
new: 70 mph

4. original: $14.99
new: $8.99

5. original: 100 lb
new: 120 lb

6. original: 50¢
new: 69¢

7. original: 16 oz
new: 20 oz

Find the final price of each item.

8. printer: $101.98
discount: 15%

9. notebook: $1.49
sales tax: 7.5%

10. gum: $0.45
sales tax: 8%

11. **Standardized Test Practice** All shirts at a store are reduced by 40%. If sales tax is 8.5%, find the final price of a shirt that normally costs $18.

A $7.20 **B** $10.80 **C** $11.72 **D** $19.53

Answers: 1. increase; 28% **2.** increase; 150% **3.** increase; 75% **4.** decrease; 40% **5.** increase; 20% **6.** increase; 38% **7.** increase; 25% **8.** $86.68 **9.** $1.60 **10.** $0.49 **11.** C

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3-8 Solving Equations and Formulas

(Pages 166–170)

Some equations contain more than one variable. To solve an equation or formula for a specific variable, you need to get that variable by itself on one side of the equation. When you divide by a variable in an equation, remember that division by 0 is undefined.

When you use a formula, you may need to use **dimensional analysis**, which is the process of carrying units throughout a computation.

Examples

a. Solve the formula $d = rt$ for t.

The variable t has been multiplied by r, so divide each side by r to isolate t.

$\frac{d}{r} = \frac{rt}{t}$ or $\frac{d}{r} = t$

Thus $t = \frac{d}{r}$, *where* $r \neq 0$.

b. Find the time it takes to drive 75 miles at an average rate of 35 miles per hour.

Use the formula you found for t in Example A.

$t = \frac{d}{r}$

$t = \frac{75 \text{ mi}}{35 \frac{\text{mi}}{\text{h}}}$ *Use dimensional analysis.* $\frac{\text{mi}}{\frac{\text{mi}}{\text{h}}} = \frac{\text{mi}}{1} \cdot \frac{\text{h}}{\text{mi}} = \text{h}$

$t = 2\frac{1}{7}$ *hours*

Try These Together

1. Solve $4a + b = 3a$ for a.

HINT: Begin by subtracting 3a from each side.

2. Solve $\frac{c + d}{3} = 2c$ for c.

HINT: Begin by multiplying each side by 3.

Practice

Solve each equation for the variable specified.

3. $f = epd$, for e

4. $12g + 31h = -8g$, for h

5. $y = mx + b$, for b

6. $v = r + at$, for r

7. $\frac{3x + y}{c} = 4$, for c

8. $\frac{5xy + n}{2} = -6$, for y

9. $m + n + 2p = 3$, for m

10. $6y + z = bc - 2y$, for y

11. $3x - 4y = 7$, for y

12. $s = \frac{n}{2}(a + t)$, for n

13. $v = \frac{4}{3}r$, for r

14. $W = mgh$, for g

15. $PV = nRT$, for V

16. $G = F - D$, for D

17. $6t + 62s = \frac{1}{2}(3t - 42s)$, for t

18. $3c + 5d = 7d - 6c$, for d

19. **Standardized Test Practice** Four ninths of a number c increased by 4 is 18 less than one eighth times another number d. Solve for c.

A $c = \frac{9}{32}d + 31\frac{1}{2}$ **B** $c = \frac{4}{72}d + \frac{4}{72}$ **C** $c = \frac{9}{32}d - 49\frac{1}{2}$ **D** $c = \frac{4}{72}d - 31\frac{1}{2}$

Answers: **1.** $a = -b$ **2.** $c = \frac{d}{5}$ **3.** $e = \frac{f}{pd}$ **4.** $h = \frac{-20g}{31}$ **5.** $b = y - mx$ **6.** $r = v - at$ **7.** $c = \frac{3x + y}{4}$ **8.** $y = \frac{-n - 12}{5x}$ **9.** $m = 3 - n - 2p$ **10.** $y = \frac{bc - z}{8}$ **11.** $y = \frac{3x - 7}{4}$ **12.** $n = \frac{2s}{a + t}$ **13.** $r = \frac{3}{4}v$ **14.** $g = \frac{W}{mh}$ **15.** $V = \frac{nRT}{P}$ **16.** $D = F - G$ **17.** $t = -\frac{166s}{9}$ **18.** $d = \frac{9c}{2}$ **19.** C

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3-9 Weighted Averages *(Pages 171–177)*

Sometimes the numbers that go into an average do not all have the same weight or importance. In such cases, you may want to use a **weighted average**. Two applications of weighted averages are mixture problems and problems involving **uniform motion**, or motion at a constant rate or speed. The formula *distance = rate · time*, or $d = rt$ is used to solve uniform motion problems.

Example

How much pure juice and 20% juice should you mix to make 4 quarts of 50% juice?

Let p = the amount of pure juice to be added. Then, make a table of the information.

	Quarts	Amount of Juice
Pure juice (100%)	p	100% of $p = 1 \cdot p$ or p
20% juice	$4 - p$	20% of $4 - p = 0.2(4 - p)$
50% juice	4	50% of $4 = 0.5 \cdot 4$ or 2

Next, write an equation with the expression for each amount of juice.

pure juice + 20% juice = 50% juice

$$p + 0.2(4 - p) = 2$$
$$p + 0.8 - 0.2p = 2$$
$$(1 - 0.2)p + 0.8 = 2$$
$$0.8p + 0.8 = 2$$
$$0.8p = 1.2$$
$$p = 1.5$$

You should mix 1.5 quarts of pure juice with 4 − 1.5 or 2.5 quarts of 20% juice to obtain a 4 quart mixture that is 50% juice.

Practice

1. **Entertainment** Symphony tickets cost $16 for adults and $8 for students. A total of 634 tickets worth $8432 were sold. Use the table to find how many adult and student tickets were sold.

	Number Sold	Price Per Ticket	Total Price
Adult Tickets	x		
Student Tickets	$634 - x$		

2. **Transportation** A truck and a jeep leave Melbourne, the truck heading east and the jeep heading west. The jeep is traveling 5 mph slower than the truck. In 3 hours, the vehicles are 465 miles apart. Draw a diagram of the situation and then use the table to find the speed of each vehicle. (*Hint:* eastbound distance + westbound distance = total distance apart.)

	Rate (mph)	Time (hours)	Distance (miles)
Truck	x	3	
Jeep		3	

3. **Standardized Test Practice** A group of twenty people bought popcorn at a movie. A regular popcorn cost $2 and a large popcorn cost $3. If the total bill for popcorn was $49, how many bags of each size did they buy?

A 5 regular, 15 large **B** 12 regular, 8 large
C 11 regular, 9 large **D** 7 regular, 13 large

Answers: 1. 420 adult; 214 student **2.** See Answer Key for diagram; truck: 80 mph, jeep: 75 mph **3.** C

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3 Chapter Review

Phrase Find

Solve the eleven equations. The numbers in the puzzle below are solutions to the equations. For example, if the solution to an equation is $c = 1$, then look at the puzzle for the number 1. When you find the number 1, write “c” in the blank. Repeat this procedure for each equation.

Solve.

1. $n + (-3) = -6$

2. $a - (-4) = 9$

3. $8u = -96$

4. $-\frac{2}{3}g = -4$

5. $3e - 5 = 16$

6. $\frac{i}{6} + 7 = 2$

7. $2 - s = 9$

8. $\frac{\ell + 1}{8} = 4$

9. $4(2b - 1) = -76$

10. $5r + 7 = 2r + 19$

11. $2(3f + 3) = 9(f - 2)$

___	___	___	___	___	___	___		___	___		___	___	___
5	31	6	7	−9	4	5		−30	−7		8	−12	−3

Answers are located in the Answer Key.

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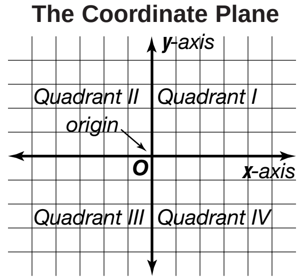
4-1 The Coordinate Plane *(Pages 192–196)*

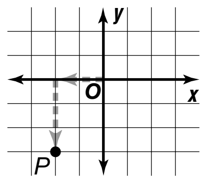
You can graph points on the **coordinate plane**, shown below.



You name points in the coordinate plane with *ordered pairs* of the form (x, y). The first number is the ***x*-coordinate** and corresponds to numbers on the horizontal or ***x*-axis**. The second is the ***y*-coordinate** and corresponds to numbers on the vertical or ***y*-axis**. These two axes divide the plane into four **quadrants**. The quadrants are numbered in a counterclockwise direction, starting at the upper right corner of the plane. The axes intersect at their zero points, a point called the **origin**, which has an ordered pair of (0, 0).

Example

Graph the point $P(-2, -3)$. Name the quadrant in which the point is located.



Move from the origin 2 units to the left, since the x-coordinate is negative.

Then move 3 units down, since the y-coordinate is negative.

This point is in Quadrant III.

Try This Together

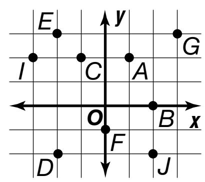
Use the graph in PRACTICE below.

1. Write the ordered pair that names point A. Name the quadrant in which the point is located.
   HINT: Write your ordered pair in the form (x, y).

Chapter 4

Practice

Write the ordered pair for each point. Name the quadrant in which the point is located.



2. C **3.** D **4.** E **5.** F

6. G **7.** B **8.** I **9.** J

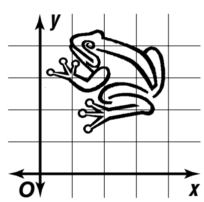
Graph each point.

10. $M(3, 1)$ **11.** $P(2, -3)$ **12.** $T(-4, 2)$ **13.** $N(0, 4)$ **14.** $G(-5, -3)$

15. $K(-3, 3)$ **16.** $Q(5, -2)$ **17.** $Y(3, 0)$ **18.** $V(0, -1)$ **19.** $W(4, 4)$

20. **Standardized Test Practice** Which of the following gives the coordinates of the point where the frog's eye is located?

A (1, 4) **B** (1.5, 4)
C (4, 1) **D** (4, 1.5)



Answers: 1. (1, 2), Quadrant I **2.** (−1, 2), Quadrant II **3.** (−2, −2), Quadrant III **4.** (−2, 3), Quadrant II
5. (0, −1), border of Quadrants III and IV **6.** (3, 3), Quadrant I **7.** (2, 0), border of Quadrant 1 and IV **8.** (−3, 2), Quadrant II
9. (2, −2), Quadrant IV **10–19.** See Answer Key. **20.** B

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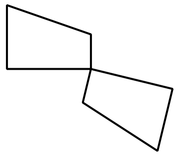
4-2 Transformations on the Coordinate Plane *(Pages 197–203)*

The movement of a geometric figure is called a **transformation**. Before a figure is transformed it is known as a **preimage**. After the transformation, the figure is referred to as an **image**. Transformations can be categorized as a reflection, translation, dilation, or rotation. In a **reflection**, the figure is flipped over a line. A **translation** is when a figure is slid horizontally, vertically, or both. In **dilations**, the figure is enlarged or reduced. A rotation is when a figure is turned around a point.

Example

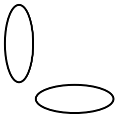
Which type of transformation does this picture show?

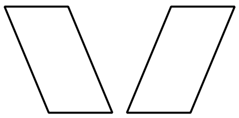
The figure has been rotated around the point that is the lower right corner of the original figure. This is a rotation.

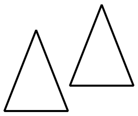


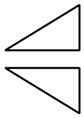
Practice

Tell whether each geometric transformation is a translation, reflection, dilation, or rotation.

1. 

2. 

3. 

4. 

Find the coordinates of the vertices of the image.

5. Preimage is $\triangle ABC$ with vertices $A(1, 4)$, $B(5, 1)$, and $C(1, 1)$. The figure is translated 2 units right and 4 units up.

6. Preimage is $\triangle ABC$ with vertices $A(1, 4)$, $B(5, 1)$, and $C(1, 1)$. The figure is reflected about the y-axis.

7. Preimage is $\triangle ABC$ with vertices $A(1, 4)$, $B(5, 1)$, and $C(1, 1)$. The figure is rotated 90° about point B.

8. Preimage is $\triangle ABC$ with vertices $A(1, 4)$, $B(5, 1)$, and $C(1, 1)$. The figure is dilated by a factor of 2.

9. **Standardized Test Practice** Find the coordinates of the vertices of the image when the quadrilateral $\square WXYZ$ is translated 5 units left and 4 units down. The preimage vertices are $W(1, 0)$, $X(2, 3)$, $Y(4, 1)$, and $Z(3, -3)$.

A $W'(4, 4), X'(3, 1), Y'(1, 3), Z'(2, 7)$

B $W'(6, -4), X'(8, -1), Y'(9, -3), Z'(8, -7)$

C $W'(-4, 4), X'(-3, 1), Y'(-1, 3), Z'(-2, 7)$

D $W'(-4, -4), X'(-3, -1), Y'(-1, -3), Z'(-2, -7)$

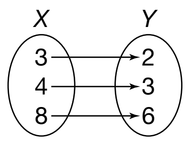
Answers: **1.** rotation **2.** reflection **3.** translation **4.** reflection **5.** $A'(3, 8)$, $B'(7, 5)$, $C'(3, 5)$ **6.** $A'(-1, 4)$, $B'(5, 1)$, $C'(-1, 1)$ **7.** $A'(9, -2)$, $B'(5, 1)$, $C'(9, 1)$ **8.** $A'(2, 8)$, $B'(10, 2)$, $C'(2, 2)$ **9.** D

4-3 Relations *(Pages 205–211)*

A *relation* is a set of ordered pairs. A relation can be represented by a mapping. A **mapping** shows a pairing of each x element in the *domain* with a y element in the *range*. Arrows go from the x element to the y element. You can find the **inverse** of a relation by switching the coordinates in each ordered pair.

Example

Express the relation shown in the mapping below as a set of ordered pairs. Then state the domain, range, and inverse of the relation.



set of ordered pairs: *{(3, 2), (4, 3), (8, 6)}*

domain: *{3, 4, 8}* **range:** *{2, 3, 6}.*

To write the inverse, exchange the x- and y-coordinates.

inverse: *{(2, 3), (3, 4), (6, 8)}*

Try These Together

1. State the domain, range, and inverse of {(3, 7), (2, 8), (1, 9)}.
2. State the domain, range, and inverse of {(−1, 4), (2, 4), (3, 5)}.

HINT: Recall that the domain contains the first, or x-coordinates.

Practice

State the domain and range of each relation.

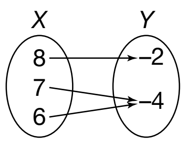
3. {(6, 3), (9, 2), (6, 4)}
4. {(10, −8), (9, −5)}

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then state the domain, range, and inverse of the relation.

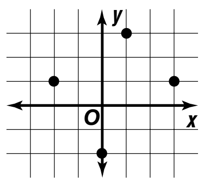
5.

x	y
20	15
22	18
25	19
31	20

6.



7.



8. **School** Emelina has noticed a ratio of 6 boys to 5 girls in her classes. She modeled this using the equation $b = 1.2g$, where b is the number of boys, g is the number of girls, and 1.2 is the ratio $\frac{6}{5}$. Explain why in this situation the solutions to this equation cannot be decimals. Use trial and error to make a table of three whole number values for g that have corresponding whole number values for b.

9. **Standardized Test Practice** What is the domain of the relation, {(2, 7), (3, 5), (2, 8)}?

A {2, 3, 5, 7, 8} **B** {5, 7, 8} **C** {2, 3, 8} **D** {2, 3}

Answers: **1.** D = {1, 2, 3}, R = {7, 8, 9}, Inv = {(7, 3), (8, 2), (9, 1)} **2.** D = {−1, 2, 3}, R = {4, 5} Inv = {(4, −1), (4, 2), (5, 3)} **3.** D = {6, 9}, R = {2, 3, 4} **4.** D = {9, 10}, R = {−8, −5} **5.** {(20, 15), (22, 18), (25, 19), (31, 20)}, D = {20, 22, 25, 31}, R = {15, 18, 19, 20}, Inv ={(15, 20), (18, 22), (19, 25), (20, 31)} **6.** {(8, −2), (7, −4), (6, −4)}, D = {6, 7, 8}, R = {−4, −2}, Inv = {(−4, 6), (−4, 7), (−2, 8)} **7.** {(−2, 1), (0, −2), (1, 3), (3,1)}, D = {−2, 0, 1, 3}, R = {−2, 1, 3}, Inv = {(1, −2), (−2, 0), (3, 1), (1, 3)} **8.** You can't have a fraction of a person. Some possible points in the table: {(5, 6), (10, 12), (15, 18), (20, 24)} **9.** D

4-4 Equations as Relations *(Pages 212–217)*

An **equation in two variables** has solutions that are ordered pairs in the form (x, y). In an equation involving x and y, the set of x values is the domain of the relation.

Solutions of an Equation in Two Variables	If a true statement results when the numbers in an ordered pair are substituted into an equation in two variables, then the ordered pair is a solution of the equation.

Examples

a. Solve $y = 2x - 1$ if the domain is $\{1, 0, -1\}$.

Make a table and substitute each value of x into the equation to determine the corresponding value of y.

domain x	$2x - 1$	range y	ordered pair (x, y)
1	$2(1) - 1$	1	$(1, 1)$
0	$2(0) - 1$	-1	$(0, -1)$
-1	$2(-1) - 1$	-3	$(-1, -3)$

solution set: *$\{(1, 1), (0, -1), (-1, -3)\}$.*

b. Which of the ordered pairs, (3, 5), (0, 1), or (−1, 1), is a solution of $y = 2x - 1$?

Substitute the values for x and y into the equation to see if they make a true statement.

Does $5 = 2(3) - 1$? Yes, $5 = 6 - 1$.

Does $1 = 2(0) - 1$? No, $1 \neq 0 - 1$.

Does $1 = 2(-1) - 1$? No, $1 \neq -2 - 1$.

(3, 5) is a solution of $y = 2x - 1$.

Practice

Which ordered pairs are solutions of the equation?

1. $y = 2x - 7$ **a.** $(4, 1)$ **b.** $(8, 9)$ **c.** $(-1, -5)$ **d.** $(0, 7)$
2. $y = 9x$ **a.** $(2, 11)$ **b.** $(-1, 9)$ **c.** $(-1, -9)$ **d.** $(3, 12)$
3. $2x + y = 18$ **a.** $(1, 15)$ **b.** $(0, 18)$ **c.** $(-2, 14)$ **d.** $(-1, 20)$
4. $y - 3x = 10$ **a.** $(7, 31)$ **b.** $(0, 0)$ **c.** $(0, 10)$ **d.** $(-2, 16)$
5. $5x + 3y = 24$ **a.** $(-1, 5)$ **b.** $(4, 2)$ **c.** $(3, -1)$ **d.** $(0, 8)$

Solve each equation if the domain is $\{-1, 0, 4, 5\}$.

6. $y = 5x + 1$
7. $y = -2x + 3$
8. $x + y = 10$
9. $4x + y = 7$
10. $3x - y = 16$
11. $-6x + 2y = -8$

12. **Anatomy** Alicia believes she's found an equation to describe her height at different ages in her life. The equation is $h = 5a$, where a is age and h is height in inches. Solve for the domain $a = \{5, 10, 12, 20, 25\}$. For which of these ages are the heights unrealistic?

13. **Standardized Test Practice** Which of the following is a solution of the equation $2x - y = 10$?

A $(-2, -6)$ **B** $(-2, 6)$ **C** $(2, 6)$ **D** $(2, -6)$

Answers: 1. a, b **2.** c **3.** b, d **4.** a, c **5.** d **6.** $\{(-1, -4), (0, 1), (4, 21), (5, 26)\}$ **7.** $\{(-1, 5), (0, 3), (4, -5), (5, -7)\}$ **8.** $\{(-1, 11), (0, 10), (4, 6), (5, 5)\}$ **9.** $\{(-1, 11), (0, 7), (4, -9), (5, -13)\}$ **10.** $\{(-1, -19), (0, -16), (4, -4), (5, -1)\}$ **11.** $\{(-1, -7), (0, -4), (4, 8), (5, 11)\}$ **12.** $\{(5, 25), (10, 50), (12, 60), (20, 100), (25, 125)\}$; ages 5, 20, and 25 **13.** D

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4-5 Graphing Linear Equations *(Pages 218–223)*

A **linear equation** may contain one or two variables with no variable having an exponent other than 1. A linear equation can be written in the form $Ax + By = C$, where A, B, and C are any real numbers, and A and B are not both zero. To graph a linear equation, find at least two solutions of the equation. Then, plot the points and draw a straight line through them.

Examples

a. Determine whether the equation $y = 2x - 1$ is a linear equation. If it is, rewrite the equation in the form $Ax + By = C$.

This is a linear equation, since the equation contains only two variables and the power on each variable is 1. First, rewrite the equation so that both variables are on the same side of the equation.

$y = 2x - 1$

$-2x + y = -1$ *Subtract 2x from each side.*

The equation is now in the form Ax + By = C, where A = −2, B = 1, and C = −1.

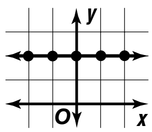
b. Graph the equation $y = 2$.

Select five values for the domain and make a table.

x	y	(x, y)
−2	2	(−2, 2)
−1	2	(−1, 2)
0	2	(0, 2)
1	2	(1, 2)
2	2	(2, 2)

Note that because the equation does not contain the variable x, x can be any value and the y value will still be 2.

Then graph the ordered pairs and connect them to draw the line. Note that the graph of y = 2 is a horizontal line through (0, 2).



Try These Together

1. Rewrite the equation $x = 3$ in the form $Ax + By = C$.

HINT: Since there is no variable y in this equation, use the placeholder 0y.

2. Graph the equation $3x - y = 5$.

HINT: To find values for y more easily, solve the equation for y. Subtract 3x from each side and then divide each side by −1.

Practice

Determine whether each equation is a linear equation. If an equation is linear, rewrite it in the form $Ax + By = C$.

3. $y = 2x^2 - 3$ **4.** $x = 2y + 8$ **5.** $y = -1$

6. $y = -4x + 1$ **7.** $3x = 5y + 7$ **8.** $8 - y = x$

Graph each equation.

9. $y = x + 4$ **10.** $y = 3x - 1$ **11.** $y = 3 - 2x$

12. $y - 3 = 0$ **13.** $y + 5 = 0$ **14.** $x - 2 = 0$

15. $x - y = 6$ **16.** $x + y = 15$ **17.** $2x + y = 4$

18. **Standardized Test Practice** Write the equation $y = 2x - 8$ in the standard form $Ax + By = C$.

A $y + 2x = -8$ **B** $y - 2x = -8$ **C** $-2x + y = -8$ **D** $2x + y = -8$

Answers: 1. $1x + 0y = 3$ **2.** See Answer Key. **3.** no **4.** yes; $x - 2y = 8$ **5.** yes; $0x + y = -1$ **6.** yes; $4x + y = 1$ **7.** yes; $3x - 5y = 7$ **8.** yes; $x + y = 8$ **9–17.** See Answer Key. **18.** C

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4-6 Functions *(Pages 226–231)*

A **function** is a relation in which each element of the domain is paired with *exactly* one element of the range. Equations that are functions can be written in a form called **functional notation**, $f(x)$ (read "f of x"). In a function, x is an element of the domain and $f(x)$ is the corresponding element in the range.

Vertical Line Test	If each vertical line passes through no more than one point of the graph of a relation, then the relation is a function.

Examples

a. Is {(1, 2), (1, 3)} a function? Is {(1, 4), (3, 2), (5, 4)} a function?

1st relation: not a function
This relation has 1 paired with both 2 and 3.

2nd relation: a function
In this relation, each x-value is paired with no more than one y-value. A function can have a y-value paired with more than one x-value.

b. If $f(x) = 3x - 1$ and $g(x) = 2x$, find $f(1)$ and $g(3)$.

$f(x) = 3x - 1$
$f(1) = 3(1) - 1$ or 2 — *Replace x with 1.*
$g(x) = 2x$
$g(3) = 2(3)$ or 6 — *Replace x with 3.*

Practice

Determine whether each relation is a function.

1.

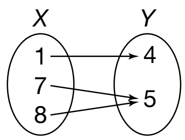
x	y
−1	10
−2	13
−3	16

2.

x	y
2	0
2	−1
3	−4

3.

x	y
33	10
35	8
36	10

4.


5. {(7, 4), (6, 3), (5, 2)}

6. {(15, 0), (15, −2)}

7. {(0, 1), (2, 1), (0, 3)}

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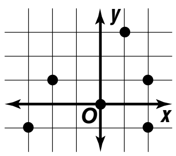
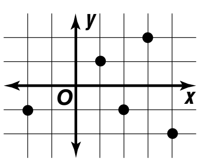
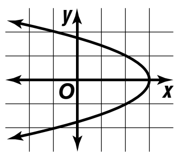


9.


10.


Given $f(x) = -3x$ and $g(x) = x - 5$, find each value.

11. $f(7)$ **12.** $g(7)$ **13.** $g(-8)$ **14.** $f(-1)$

15. $f(a)$ **16.** $g(m)$ **17.** $2[g(9)]$ **18.** $3[f(2)]$

19. **Standardized Test Practice** Martha pays a flat \$50 a month for the use of her cell phone. She also pays \$0.30 for each minute that she talks over 6 hours. The cost of her phone bill can be represented by $f(x) = 50 + 0.30x$, where x is the number of minutes past 6 hours that she uses the phone. Evaluate $f(60)$ to find the amount of her phone bill if she uses the phone for 7 hours.

A \$68.30 **B** \$68.00 **C** \$50.30 **D** \$18.00

Answers: **1.** yes **2.** no **3.** yes **4.** yes **5.** yes **6.** no **7.** no **8.** no **9.** yes **10.** no **11.** −21 **12.** 2 **13.** −13 **14.** 3 **15.** −3a **16.** m − 5 **17.** 8 **18.** −18 **19.** B

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4-7 Arithmetic Sequences *(Pages 233–238)*

An **arithmetic sequence** is a set of numbers in a specific order whose difference between successive terms is constant. Any number in the set is a **term**. To move from one term to the next term a constant number must be added to the previous term. For example, 3, 6, 9, 12,... is an arithmetic sequence because to progress from one term to the next, like 6 to 9, you must add a constant number, 3, to the previous term. In this example, 3 is called the **common difference**. Therefore, an arithmetic sequence can be found with $a_1, a_1 + d, a_2 + d, a_3 + d,...$ where a_1 is the first term of the sequence and d is the common difference. To calculate the nth term of an arithmetic sequence, you can use the formula $a_n = a_1 + (n - 1)d$.

Examples

a. Find the next three terms of the arithmetic sequence 0, 9, 18, 27,...

$9 - 0 = 9$	*Find the common*
$18 - 9 = 9$	*difference by subtracting*
$27 - 18 = 9$	*successive terms.*
$27 + 9 = 36$	*Add the common*
$36 + 9 = 45$	*difference to the next*
$45 + 9 = 54$	*three terms.*

The next three terms are 36, 45, and 54.

b. Find the 7th term of the arithmetic sequence 10, 23, 36,...

$23 - 10 = 13$	*Find the common*
$36 - 23 = 13$	*difference. $d = 13$*
$a_n = a_1 + (n - 1)d$	*Use the formula.*
$a_7 = 10 + (7 - 1)13$	*Substitute.*
$a_7 = 10 + 6 \cdot 13$	*Evaluate by the*
$a_7 = 10 + 78$	*order of operations.*
$a_7 = 88$	

Practice

Find the next three terms of each arithmetic sequence.

1. $1, \frac{1}{2}, 0, \frac{-1}{2},...$

2. 13, 30, 47, 64,...

3. 102, 94, 86, 78,...

4. 4, 8, 12, 16,...

5. $7, \frac{25}{4}, \frac{11}{2}, \frac{19}{4},...$

6. 13, 11, 9, 7,...

7. $-1, -7, -13, -19,...$

8. $-1, 2, 5, 8,...$

9. **Standardized Test Practice** Which of the following is the 24th term of the arithmetic sequence $3, -2, -7, -12,...$?

A -62 **B** -92 **C** -112 **D** -162

Answers: 1. $-1, \frac{-3}{2}, -2$ **2.** 81, 98, 115 **3.** 70, 62, 54 **4.** 20, 24, 28 **5.** $4, \frac{13}{4}, \frac{5}{2}$ **6.** 5, 3, 1 **7.** $-25, -31, -37$ **8.** 11, 14, 17 **9.** C

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4-8 Writing Equations from Patterns

(Pages 240–245)

Points that lie in a linear pattern can be described by an equation.

Writing Equations	First make a table of several ordered pairs from the graph of the relation. Next, find the common differences of the domain and range. Then, write an equation using the ratio of the differences. Check to see if you need to adjust your equation by adding or subtracting a quantity.

Examples

a. Write an equation for the function.

x	6	4	2
y	3	2	1

Find the differences in domain and range values.
domain: *$4 - 6 = -2$ and $2 - 4 = -2$*
range: *$2 - 3 = -1$ and $1 - 2 = -1$*

$\frac{\text{range differences}}{\text{domain differences}} = \frac{-1}{-2}$ *or* $\frac{1}{2}$

This suggest $y = \frac{1}{2}x$ may describe the relation.

Check: *If $x = 6$, then $y = \frac{1}{2}(6)$ or 3 ✓*

If $x = 4$, then $y = \frac{1}{2}(4)$ or 2 ✓

Thus, $y = \frac{1}{2}x$ describes this relation.

b. Write an equation for the function.

x	2	1	0
y	5	2	−1

domain: *$1 - 2 = -1$ and $0 - 1 = -1$*
range: *$2 - 5 = -3$ and $-1 - 2 = -3$*

$\frac{\text{range differences}}{\text{domain differences}} = \frac{-3}{-1}$ *or 3*

This suggests $y = 3x$ may describe the relation.

Check: *If $x = 2$, then $y = 3(2)$ or 6 $6 \neq 5$*
This suggests that 1 should be subtracted from 3x to describe the relation correctly.
You can check to verify that the equation $y = 3x - 1$ describes the relation.

Practice

Write an equation for each function.

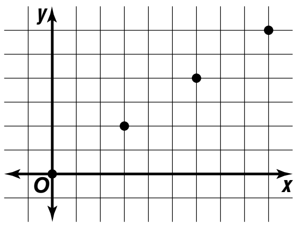
1.

x	4	8	12	16	20
$h(x)$	1	2	3	4	5

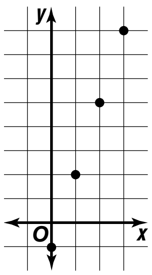
2.

x	2	4	6	8	10	12
$f(x)$	−1	−2	−3	−4	−5	−6

3.



4.



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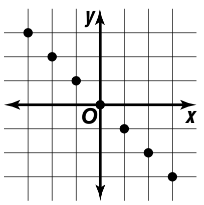

6. Standardized Test Practice The table shows the number of hours worked versus amount of pay. Write an equation in functional notation for the relation.

Hours	20	25	30	35
Pay ($)	160	200	240	280

A $f(h) = 8h$ **B** $f(h) = \frac{1}{8}h$ **C** $f(h) = 5h$ **D** $f(h) = \frac{1}{5}h$

Answers: 1. $h(x) = \frac{1}{4}x$ **2.** $f(x) = -\frac{1}{2}x$ **3.** $y = \frac{3}{2}x$ **4.** $y = 3x - 1$ **5.** $y = -x$ **6.** A

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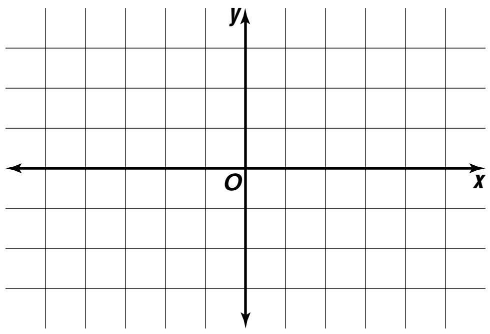
4 Chapter Review

Make a Map

See if you and a parent can find Captain Graphsalot's fleet of ships. Use each clue to graph points that show the locations of his ships. Three or more points in a row indicate the location of a single ship.



Clue 1: Graph {(0, 1), (0, 3), (5, 1)}. State the domain and range of this relation.

Clue 2: Graph {(5, 0), (4, −2), (3, −2)}. State the inverse of the relation.

Clue 3: Solve $y - x = 1$ for the domain {−2, −1, 2}. Plot the points in your graph.

Clue 4: Determine whether each of the following relations is a function. If the relation is a function, graph the given points. If it is not a function, do not graph it.

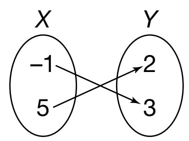
a.

x	y
0	1
0	−1
4	2

b.

x	y
−3	−2
2	−2
1	3

c.



Clue 5: Given $g(x) = x^2 - 5$, find $g(-3)$. This is the number of ships that you should have found in the fleet.

Answers are located in the Answer Key.

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5-1 Slope *(Pages 256–262)*

Definition of Slope	The steepness of a line in the coordinate plane is called its **slope**. It is defined as the ratio of the **rise**, or vertical change in y, to the **run**, or horizontal change in x, as you move from one point to the other.
Determining Slope Given Two Points	Given the coordinates of two points, (x_1, y_1) and (x_2, y_2), on a line, the slope m of the line can be found as follows. $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

Examples

a. What is the slope of the line that passes through (4, −6) and (−2, 3)?

Let $x_1 = 4$, $y_1 = -6$, $x_2 = -2$, *and* $y_2 = 3$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Slope formula*

$m = \frac{3 - (-6)}{-2 - 4}$ *Substitute.*

$m = \frac{9}{-6}$ or $-\frac{3}{2}$ *Simplify.*

b. Find the value of r so that the line through $(r, 4)$ and (0, 5) has a slope of −2.

$-2 = \frac{5 - 4}{0 - r}$ *Slope formula with* $m = -2$, $(x_1, y_1) = (r, 4)$, *and* $(x_2, y_2) = (0, 5)$

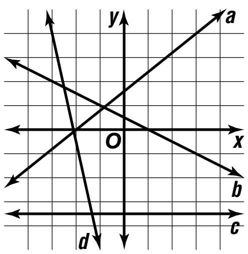
$\frac{-2}{1} = \frac{1}{-r}$

$2r = 1$ *Find the cross products.*

$r = \frac{1}{2}$ *Solve for r.*

Practice

Determine the slope of each line using the graph at the right.



1. line a
2. line b
3. line c
4. line d

Determine the slope of the line that passes through each pair of points.

5. (9, 3), (7, 6)
6. (−3, −2), (9, −5)
7. $\left(\frac{1}{3}, -1\frac{1}{3}\right), \left(2\frac{1}{3}, \frac{1}{3}\right)$

Determine the value of r so the line that passes through each pair of points has the given slope.

8. $(3, r), (5, -9), m = \frac{9}{2}$
9. $(0, -8), (r, 0), m = -\frac{2}{5}$
10. $(5, -4), (6, r), m = 2$

11. **Construction** Ann is building a wheelchair ramp with a 7% incline from her entryway into her sunken living room. The height of the ramp needs to be 21 cm. What will be the length of the ramp?

12. **Standardized Test Practice** What is the slope of the line that passes through (1, −3) and (−2, 6)?

A −3 **B** −1 **C** 1 **D** 3

Answers: **1.** $\frac{4}{5}$ **2.** $-\frac{1}{2}$ **3.** 0 **4.** $-\frac{9}{2}$ **5.** $-\frac{3}{2}$ **6.** $-\frac{1}{4}$ **7.** $\frac{6}{5}$ **8.** −18 **9.** −20 **10.** −2 **11.** 300 cm or 3 m **12.** A

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5-2 Slope and Direct Variation *(Pages 264–270)*

An equation in the form of $y = kx$, where $k \neq 0$, is called **direct variation**. In direct variation we say that *y varies directly with x* or *y varies directly as x*. In the direct variation equation, $y = kx$, k is the **constant of variation**. The constant of variation in a direct variation equation has the same value as the slope of the graph. For example, $y = 5x$ is a direct variation because it is in the form of $y = kx$. The constant of variation of $y = 5x$ is 5. The slope of the linear graph of $y = 5x$ is 5. All direct variation graphs pass through the origin.

Examples

a. For the equation $y = 2x$, which passes through points (2, 4) and (5, 10), show that the slope and the constant of the variation are equal.

2 is the constant of the variation;

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{5 - 2} = \frac{6}{3} = \frac{2}{1} = 2$

b. Write and solve an equation if y varies directly with x and $y = 40$ when $x = 5$.

$y = kx$	*Direct variation form*
$40 = k \cdot 5$	*Substitute values.*
$8 = k$	*Divide each side by 5.*

Therefore, $y = 8x$.

Practice

Name the constant of variation for each equation. Then determine the slope of the line that passes through the given pair of points.

1. $y = \frac{1}{3}x$; (6, 2), (−9, −3)

2. $y = \frac{-5}{2}x$; (−10, 25), (−2, 5)

3. $y = 13x$; (2, 26), (9, 117)

Write a direct variation equation that relates x and y. Assume that y varies directly with x. Then solve.

4. If $y = -32$ when $x = 4$, find x when $y = 24$.

5. If $y = 15$ when $x = 6$, find x when $y = -25$.

6. **Standardized Test Practice** Which equation is *not* an example of a direct variation?

A $y = \frac{-7}{3}x + 1$ **B** $y = \frac{5}{16}x$ **C** $y = 14x$ **D** $y = -9x$

Chapter 5

Answers: **1.** $k = \frac{1}{3}$, $m = \frac{1}{3}$ **2.** $k = \frac{-5}{2}$, $m = \frac{-5}{2}$ **3.** $k = 13$, $m = 13$ **4.** $y = -8x$, $x = -3$ **5.** $y = 2.5x$, $x = -10$ **6.** A

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5-3 Slope-Intercept Form *(Pages 272–277)*

The coordinates at which a graph intersects the axes are known as the ***x*-intercept** and the ***y*-intercept**.

Finding Intercepts	To find the x-intercept, substitute 0 for y in the equation and solve for x. To find the y-intercept, substitute 0 for x in the equation and solve for y.
Slope-Intercept Form of a Linear Equation	If a line has a slope of m and a y-intercept of b, then the slope-intercept form of an equation of the line is $y = mx + b$.

Example

Find the x- and y-intercepts of the graph of $2x + 3y = 5$. Then, write the equation in slope-intercept form.

$2x + 3(0) = 5$ *Let $y = 0$.*
$2x = 5$ *Simplify.*
$x = \frac{5}{2}$ *The x-intercept is $\frac{5}{2}$.*

$2(0) + 3y = 5$ *Let $x = 0$.*
$3y = 5$ *Simplify.*
$y = \frac{5}{3}$ *The y-intercept is $\frac{5}{3}$.*

Slope-Intercept Form: $2x + 3y = 5$
$3y = -2x + 5$ *Subtract 2x from each side.*
$y = -\frac{2}{3}x + \frac{5}{3}$ *Divide each side by 3.*

Note that in this form we can see that the slope m of the line is $-\frac{2}{3}$, and the y-intercept b is $\frac{5}{3}$.

Practice

Find the x- and y-intercepts of the graph of each equation.

1. $6x + 2y = 10$
2. $6x - y = -7$
3. $8y - 5 = 3x$

Write an equation in slope-intercept form of a line with the given slope and y-intercept. Then write the equation in standard form.

4. $m = 5, b = 5$
5. $m = 2, b = -7$
6. $m = -3, b = 0$

Find the slope and y-intercept of the graph of each equation.

7. $7y = x - 10$
8. $8x - \frac{1}{2}y = -2$
9. $4(x - 5y) = 9(x + 1)$

10. Chemistry The graph of an equation to convert degrees Celsius, x, to degrees Fahrenheit, y, has a y-intercept of 32°. Given that water boils at 212°F and at 100°C, write the conversion equation.

11. **Standardized Test Practice** What is the slope-intercept form of an equation for the line that passes through (0, 1) and (3, 37)?

A $y = 12x - 1$ **B** $y = 12x + 1$ **C** $y = -12x - 1$ **D** $y = -12x + 1$

Answers: 1. $\frac{5}{3}$, 5 **2.** $-\frac{7}{6}$, 7 **3.** $-\frac{5}{3}$, $\frac{5}{8}$ **4.** $y = 5x + 5$, $5x - y = -5$ **5.** $y = 2x - 7$, $2x - y = 7$ **6.** $y = -3x$, $3x + y = 0$ **7.** $\frac{1}{7}$, $-\frac{10}{7}$ **8.** 16, 4 **9.** $-\frac{1}{4}$, $-\frac{9}{20}$ **10.** $y = \frac{9}{5}x + 32$ **11.** B

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5-4 Writing Equations in Slope-Intercept Form *(Pages 280–285)*

You now know how to write an equation for any line with a given slope and y-intercept. It is also possible to write an equation for any line with a given slope and any point on the line. In addition, since you know the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, you can also write an equation of any line given two points.

To write an equation given the slope and one point.	Use $y = mx + b$ for the equation. Replace m with the given slope and the coordinates of the given point for x and y. Solve the equation for the y-intercept, b. Rewrite the equation with the slope for m and the y-intercept for b.
To write an equation given two points.	Use the slope formula to calculate m. Chose any of the two given points to use in place of x and y in $y = mx + b$. Replace m with the slope you just calculated. Solve for b. Rewrite the equation with the slope for m and the y-intercept for b.

Examples

Write an equation in slope-intercept form from the given information.

a. The slope is 3 and the line passes through the point (5, 16).

$y = mx + b$	*Use slope-intercept form.*
$y = 3x + b$	*Replace m with the slope.*
$16 = 3 \cdot 5 + b$	*Replace x and y.*
$1 = b$	*Solve for b.*
$y = 3x + 1$	*Rewrite the equation.*

b. The line passes through the points (10, −4) and (−7, 13).

$m = \frac{y_2 - y_1}{x_2 - x_1}$	*Use the slope formula.*
$m = \frac{13 - (-4)}{-7 - 10}$	*Substitute.*
$m = -1$	*Solve for m.*
$y = mx + b$	
$-4 = (-1)10 + b$	*Substitute m, x, and y.*
$6 = b$	*Solve for b.*
$y = -x + 6$	*Rewrite the equation.*

Practice

Write an equation in slope-intercept form from the given information.

1. $m = 3, (0, 4)$

2. $m = -\frac{3}{2}, (0, 6)$

3. $m = \frac{1}{2}, (5, 6.5)$

4. $m = 1, (-5, -7)$

5. $(3, -4), (-6, -1)$

6. $(-10, 47), (5, -13)$

7. $(0, -1), (3, 8)$

8. $(5, 8), (-3, 8)$

9. **Standardized Test Practice** Which is the correct slope-intercept equation for a line that passes through the points $(-15, -47)$ and $(-19, -59)$?

A $y = -3x + 2$ **B** $y = 3x + 2$ **C** $y = -3x - 2$ **D** $y = 3x - 2$

Answers: **1.** $y = 3x + 4$ **2.** $y = -\frac{3}{2}x + 6$ **3.** $y = \frac{1}{2}x + 4$ **4.** $y = x - 2$ **5.** $y = \frac{-1}{3}x - 3$ **6.** $y = -4x + 7$ **7.** $y = 3x - 1$ **8.** $y = 8$ **9.** D

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5-5 Writing Equations in Point-Slope Form

(Pages 286–291)

Point-Slope Form of a Linear Equation	For a given point (x_1, y_1) on a nonvertical line having slope of m, the **point-slope form** of a linear equation is as follows: $$y - y_1 = m(x - x_1).$$ The linear equation of a vertical line, which has an undefined slope, through a point (x_1, y_1) is $x = x_1$.
Standard Form	The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are integers, $A \geq 0$, and A and B are not both zero.

Examples

a. Write the equation, first in point-slope form and then in standard form, of the line that passes through (2, 3) and has a slope of 5.

Point-Slope Form $y - y_1 = m(x - x_1)$

$y - 3 = 5(x - 2)$

$y - 3 = 5x - 10$ *Distribute.*

$5x - 10 = y - 3$ *Reflexive Property (=)*

$5x - y = 7$ *Add 10 and subtract y from each side.*

Standard Form $5x - y = 7$, *where* $A = 5$, $B = -1$, *and* $C = 7$.

b. Write the point-slope form of an equation of the line that passes through (0, 3) and (4, 0).

slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 3}{4 - 0}$ *or* $-\frac{3}{4}$

Point-Slope Form

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{3}{4}(x - 0)$ *Let* $(x_1, y_1) = (0, 3)$

$y - 3 = -\frac{3}{4}x$

Practice

1. Write the point-slope form of an equation of the line that passes through the point $(-1, -4)$ and has a slope of $\frac{2}{5}$.

Write the standard form of an equation of the line that passes through the given point and has the given slope.

2. $(3, -6)$, $m = 3$ **3.** $(9, 7)$, $m = -\frac{1}{4}$ **4.** $(6, -3)$, $m =$ undefined

Write the point-slope form of an equation of the line that passes through each pair of points.

5. $(-6, 1)$, $(5, 9)$ **6.** $(4, 9)$, $(1, 4)$ **7.** $(5, 0)$, $(-6, 4)$

8. $(-7, -8)$, $(2, -7)$ **9.** $(5, -8)$, $(2, -5)$ **10.** $(-6, -8)$, $(5, -8)$

11. **Standardized Test Practice** What is the standard form of an equation of the line that passes through $(3, -3)$ and $(-1, 1)$?

A $x - y = 0$ **B** $x + y = 0$ **C** $y = -(x + 1)$ **D** $2x + 2y = 3$

Answers: **1.** $y + 4 = \frac{2}{5}(x + 1)$ **2.** $3x - y = 15$ **3.** $x + 4y = 37$ **4.** $x + 0y = 6$ **5.** $y - 9 = \frac{8}{11}(x - 5)$ or $y - 1 = \frac{8}{11}(x + 6)$ **6.** $y - 4 = \frac{5}{3}(x - 1)$ or $y - 9 = \frac{5}{3}(x - 4)$ **7.** $y = -\frac{4}{11}(x - 5)$ or $y - 4 = -\frac{4}{11}(x + 6)$ **8.** $y + 8 = \frac{1}{9}(x + 7)$ or $y + 7 = \frac{1}{9}(x - 2)$ **9.** $y + 5 = -1(x - 2)$ or $y + 8 = -1(x - 5)$ **10.** $y + 8 = 0(x - 5)$ or $y + 8 = 0(x + 6)$ **11.** B

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5-6 Geometry: Parallel and Perpendicular Lines *(Pages 292–297)*

Parallel Lines	Lines in the same plane that never intersect are called **parallel lines**. If two nonvertical lines have the same slope, then they are parallel. All vertical lines are parallel.
Perpendicular Lines	Lines that intersect at right angles are called **perpendicular lines**. If the product of the slopes of two lines is −1, then the lines are perpendicular. The slopes of two perpendicular lines are negative reciprocals of each other. In a plane, vertical lines and horizontal lines are perpendicular.

Examples

a. Determine whether the graphs of $2y = -3x + 4$ and $3y = 2x - 9$ are *parallel*, *perpendicular*, or *neither*.

Rewrite each line in slope-intercept form to identify its slope.

$2y = -3x + 4$ $\quad$ $3y = 2x - 9$

$y = -\frac{3}{2}x + 2$ $\quad$ $y = \frac{2}{3}x - 3$

$m = -\frac{3}{2}$ $\quad$ $m = \frac{2}{3}$

Since $-\frac{3}{2} \cdot \frac{2}{3} = -1$, these lines are perpendicular.

b. Write an equation in slope-intercept form of the line that is parallel to the graph of $x + 6y = -12$ and has an x-intercept of 9.

Find the slope of the line given.

$6y = -x - 12 \Rightarrow y = -\frac{1}{6}x - 2$

A line parallel to this line will have the same slope, or $-\frac{1}{6}$. An x-intercept of 9 means the new line passes through (9, 0).

$y - y_1 = m(x - x_1)$ *Point-slope form*

$y - 0 = -\frac{1}{6}(x - 9)$ $\quad$ $m = -\frac{1}{6}, (x_1, y_1) = (9, 0)$

$y = -\frac{1}{6}x + \frac{3}{2}$ *Slope-intercept form*

Practice

Determine whether the graphs of each pair of equations are *parallel, perpendicular*, or *neither*.

1. $x = 4y + 12$
$4y = x + 8$

2. $y = -x + 8$
$x + 2y = 8$

3. $2y = 5x + 6$
$2x + 5y = 5$

Write an equation in slope-intercept form of the line having the following properties.

4. is perpendicular to the graph of $y = \frac{1}{2}x + 6$ and passes through (6, 8)

5. is parallel to the graph of $y = \frac{1}{6}x - 2$ and passes through the origin

6. passes through (1, 0) and is parallel to the graph of $3x - 3y = 5$

7. passes through (0, −7) and is perpendicular to the graph of $x - 2y = 7$

8. is parallel to the x-axis and passes through (4, 5)

9. is perpendicular to the graph of $x - 3y = 6$ and passes through (7, −5)

10. **Standardized Test Practice** What is the slope of a line perpendicular to $y + 3x = 2$?

A −3 $\quad$ **B** $-\frac{1}{3}$ $\quad$ **C** $\frac{1}{3}$ $\quad$ **D** 3

Answers: 1. parallel **2.** neither **3.** perpendicular **4.** $y = -2x + 20$ **5.** $y = \frac{1}{6}x$ **6.** $y = x - 1$ **7.** $y = -2x - 7$ **8.** $y = 5$ **9.** $y = -3x + 16$ **10.** C

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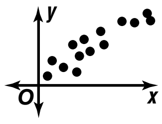
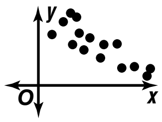
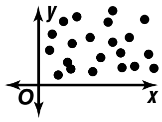
5-7 Statistics: Scatter Plots and Lines of Fit *(Pages 298–305)*

To determine if there is a relationship between a set of data, we can display the data points in a graph called a scatter plot. In a **scatter plot**, the two sets of data are plotted as ordered pairs in the coordinate plane.

Types of Correlations	 In this graph, *x* and *y* have a **positive correlation**. As *x* increases, *y* also increases.	 In this graph, *x* and *y* have a **negative correlation**. As *x* increases, *y* decreases.	 In this graph, *x* and *y* have *no correlation*. In this case; *x* and *y* are not related and are said to be *independent*.

You can sometimes draw a line, called a **line of fit**, that passes close to most of the data points.

Try These Together

Explain whether a scatter plot for each pair of variables would probably show a **positive, negative,** ***or*** **no** ***correlation between the variables.***

1. the number of cars on a freeway and the amount of time for a commute

2. a person's weight and the number of siblings they have

HINT: As one variable increases, does the other also increase?

Practice

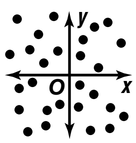
Explain whether a scatter plot for each pair of variables would probably show a positive, negative, or no correlation between the variables.

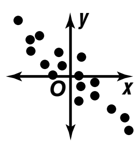
3. the number of extra-curricular activities and the amount of free-time

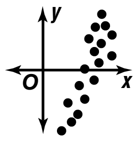
4. the time a student's homework will take and the weight of their backpack

5. the amount of time concert tickets are on sale and the number of tickets left

Determine whether a line of fit should be drawn for each set of data graphed below.

6. 

7. 

8. 

9. **Standardized Test Practice** What type of correlation is there between the number of hours spent talking long distance on the telephone and the amount of the telephone bill?

A positive correlation **B** no correlation

C negative correlation **D** need more information

Answers: 1. positive **2.** no correlation **3.** negative **4.** positive **5.** negative **6.** No, *x* and *y* do not seem to be related. **7.** Yes, *x* and *y* have a negative correlation. **8.** Yes, *x* and *y* have a positive correlation **9.** A

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5 Chapter Review

Quick Draw

On a sheet of graph paper, create a coordinate grid by drawing and labeling the x- and y-axes. Then use the clues below to graph a group of segments and one line. The segments will not be connected in order, but when you finish they will form a recognizable figure.

CLUE 1

Plot (4, 5) and (5, 3). Connect them with a line segment. What is the slope of this segment? ________

CLUE 2

Plot (2, 5) and connect it to (4, 5). What is the slope of this segment?

____

CLUE 3

Plot (5, 1) and connect it to (5, 3). What is the slope of this segment?

____

CLUE 4

Plot (−2, 5) and (−4, 5). Connect them with a line segment. Write an equation for the line that contains this segment.

____

CLUE 5

Plot (−5, 1) and (−5, 3). Connect them with a line segment. Write an equation for the line that contains this segment.

____

CLUE 6

Start at (−5, 3). Use the slope $m = 2$ to rise and run once. Connect the two points with a line segment. Write an equation in point-slope form for the line that contains this segment.

____

CLUE 7

Use $y = \frac{7}{5}x - 6$ to graph the next line segment. Plot the point indicated by the y-intercept. Use the slope to rise and run once. Connect the two points.

CLUE 8

Connect (−5, 1) to (0, −6). What is the slope of this segment?

____

CLUE 9

Use $y = x + 3$ to graph the next line segment. Plot the point indicated by the y-intercept. Use the slope to rise and run twice. Connect the two points.

CLUE 10

Connect (−2, 5) and (0, 3) with a line segment. Write an equation in slope-intercept form for the line that contains this segment.

____

CLUE 11

Graph $-2x + 3y = 3$.

Answers are located in the Answer Key.

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6-1 Solving Inequalities by Addition and Subtraction *(Pages 318–323)*

Addition and Subtraction Properties of Inequalities	For all numbers a, b, and c, the following are true. 1. If $a > b$, then $a + c > b + c$ and $a - c > b - c$. (Also true for $\geq$) 2. If $a < b$, then $a + c < b + c$ and $a - c < b - c$. (Also true for $\leq$)

The solutions of an inequality can be graphed on a number line or written using **set-builder notation**.

Example

Solve $3m - 7 > 4m + 1$. Check your solution, and graph it on a number line.

$3m - 7 > 4m + 1$
$3m - 7 - 3m > 4m + 1 - 3m$
$-7 > m + 1$
$-7 - 1 > m + 1 - 1$
$-8 > m$ or $m < -8$

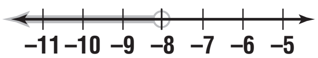
In set builder notation, the solution set is $\{m | m < -8\}$, which is read "the set of all numbers m such that m is less than −8."

Only numbers less than −8 substituted into the original inequality should yield a true statement.

$3(0) - 7 \overset{?}{>} 4(0) + 1$	Let $m = 0$.
$-7 > 1$	False
$3(-9) - 7 \overset{?}{>} 4(-9) + 1$	Let $m = -9$.
$-34 > -35$	True

Since only the number less than −8 yields a true statement, the solution checks.

Graph the point −8 using an open circle, since −8 is not part of the solution. Then draw a heavy arrow to the left to indicate numbers less than −8.



Try These Together

1. Solve and graph $z - 16 < 5$.

2. Solve and graph $j + \frac{1}{2} > 9$.

Practice

Solve each inequality. Then check your solution, and graph it on a number line.

3. $-6 + m > 6$ **4.** $3y \leq 2y + 4$ **5.** $x - 1 < -14$ **6.** $-0.05 \leq v - (-0.06)$

Solve each inequality. Then check your solution.

7. $x + \frac{1}{3} < \frac{1}{6}$ **8.** $-0.8x - 0.7 < 0.3 - 1.8x$ **9.** $5x + 7 \geq 4x + 8$

10. $2h - 5 \leq h + 4$ **11.** $u - 45 \geq 38$ **12.** $2x + \frac{1}{3} \leq 3x + \frac{2}{3}$

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. A number decreased by −3 is at least 10.

14. Twice a number is more than the difference of that number and 4.

15. **Standardized Test Practice** Which number is a solution of $2x \leq x + 8$?

A 12 **B** 11 **C** 9 **D** 6

Answers: 1–6. For graphs, see Answer Key. **1.** $\{z | z < 21\}$ **2.** $\{j | j > 8\frac{1}{2}\}$ **3.** $\{m | m > 12\}$ **4.** $\{y | y \leq 4\}$ **5.** $\{x | x < -13\}$ **6.** $\{v | v \geq -0.11\}$ **7.** $\{x | x < -\frac{1}{6}\}$ **8.** $\{x | x < 1\}$ **9.** $\{x | x \geq 1\}$ **10.** $\{h | h \leq 9\}$ **11.** $\{u | u \geq 83\}$ **12.** $\{x | x \geq -\frac{1}{3}\}$ **13.** $x - (-3) \geq 10$; $\{x | x \geq 7\}$ **14.** $2x > x - 4$; $\{x | x > -4\}$ **15.** D

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6-2 Solving Inequalities by Multiplication and Division *(Pages 325–331)*

When you multiply or divide each side of an inequality by a negative number, you must reverse the direction of the inequality symbol.

Multiplication and Division Properties for Inequalities	For all numbers a, b, and c, the following are true. 1. If c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$, and if c is positive and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. 2. If c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$, and if c is negative and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. These properties also hold true for inequalities involving $\leq$ and $\geq$.

Example

Solve $-5y \leq 12$ and check your solution.

$-5y \leq 12$

$\frac{-5y}{-5} \geq \frac{12}{-5}$ *Divide each side by −5 and change the ≤ to ≥.*

$y \geq -2.4$

In set builder notation, the solution set is $\{y \mid y \geq -2.4\}$.

Check: *Let y be −2.4 and any number greater than −2.4, such as 0.*

$-5(-2.4) \leq 12$ $-5(0) \leq 12$

$12 \leq 12$ ✓ $0 \leq 12$ ✓

Try These Together

1. Solve $3a \leq -27$ and check.

2. Solve $-\frac{5}{7}s < -\frac{5}{14}$ and check.

Practice

Solve each inequality. Then check your solution.

3. $\frac{r}{2} < 68$

4. $-d \leq 59$

5. $-\frac{1}{5}u > 20$

6. $-14c < -49$

7. $\frac{n}{-8} \leq 9$

8. $13b > -91$

9. $\frac{75k}{-4} > \frac{5}{16}$

10. $8 \geq 0.5g$

11. $5 < -t$

12. $\frac{f}{8} \geq \frac{1}{10}$

Define a variable, write an inequality, and solve each problem. Then check your solution.

13. 5 times a number is at most 45.

14. 34 is at least one half of a number.

15. One fifth of a number is at most −10.

16. 60 percent of a number is less than 78.

17. **Standardized Test Practice** Solve $-\frac{1}{2}x \geq \frac{1}{2}$.

A $\{x \mid x \leq -1\}$ **B** $\{x \mid x \geq -1\}$ **C** $\left\{x \mid x \leq -\frac{1}{4}\right\}$ **D** $\left\{x \mid x \geq -\frac{1}{4}\right\}$

Answers: 1. $\{a \mid a \leq -9\}$ **2.** $\left\{s \mid s > \frac{1}{2}\right\}$ **3.** $\{r \mid r < 136\}$ **4.** $\{d \mid d \geq -59\}$ **5.** $\{u \mid u < -100\}$ **6.** $\{c \mid c > 3.5\}$ **7.** $\{n \mid n \geq -72\}$ **8.** $\{b \mid b > -7\}$ **9.** $\left\{k \mid k < -\frac{1}{60}\right\}$ **10.** $\{g \mid g \leq 16\}$ **11.** $\{t \mid t < -5\}$ **12.** $\{f \mid f \geq 0.8\}$ **13.** $5x \leq 45$; $\{x \mid x \leq 9\}$ **14.** $34 \geq \frac{1}{2}x$; $\{x \mid x \leq 68\}$ **15.** $\frac{1}{5}x \leq -10$; $\{x \mid x \leq -50\}$ **16.** $\frac{60}{100}x < 78$; $\{x \mid x < 130\}$ **17.** A

Chapter 6

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6-3 Solving Multi-Step Inequalities *(Pages 332–337)*

Inequalities involving more than one operation can be solved by undoing the operations in reverse order in the same way you would solve an equation with more than one operation. The important exception is that multiplying or dividing an inequality by a negative number reverses the sign of the inequality.

Example

Solve $-3f - 7 \geq -f + 9$.

$-3f - 7 \geq -f + 9$	
$-3f - 7 + f \geq -f + 9 + f$	*Add f to each side.*
$-2f - 7 \geq 9$	*Combine like terms.*
$-2f - 7 + 7 \geq 9 + 7$	*Add 7 to each side.*
$-2f \geq 16$	*Combine like terms.*
$\frac{-2f}{-2} \leq \frac{16}{-2}$	*Divide each side by −2 and change ≥ to ≤.*
$f \leq -8$	*Simplify.*

The solution set is $\{f \mid f \leq -8\}$.

Try These Together

Solve each inequality. Then check your solution.

1. $2a - 18 \leq 5a + 3$

HINT: Begin by collecting all the terms with a on one side of the equality sign.

2. $x - 2 < \frac{x + 4}{4}$

HINT: Begin by multiplying each side by 4.

Practice

Solve each inequality. Then check your solution.

3. $\frac{1}{4}z - 1 \geq 3$

4. $-7x - 8 > 1 - 2x$

5. $2m + 3 > 11$

6. $2w - 3 \geq 8w + 69$

7. $-4 - 2p > 8$

8. $\frac{3h + 1}{4} > -2$

9. $5q - 4 \geq 12 - 3q$

10. $8 + v \geq 2v - 1$

11. $\frac{4(x - 1)}{3} \leq 12$

12. Money Matters Sarah does not want to spend more than \$20 for a backpack. At a certain store all backpacks are on sale for 30% off. If she pays 5% sales tax after the discount, what is the regular price of the most expensive backpack she can buy? Define a variable, write an inequality, and then solve.

13. **Standardized Test Practice** Solve $-\frac{1}{3}x + 3 \geq 0$.

A $\{x \mid x \leq -9\}$ **B** $\{x \mid x \geq -9\}$ **C** $\{x \mid x \leq 9\}$ **D** $\{x \mid x \geq 9\}$

Answers: 1. $\{a \mid a \geq -7\}$ **2.** $\{x \mid x < 4\}$ **3.** $\{z \mid z \geq 16\}$ **4.** $\{x \mid x < -1.8\}$ **5.** $\{m \mid m > 4\}$ **6.** $\{w \mid w \leq -12\}$ **7.** $\{p \mid p < -6\}$
8. $\{h \mid h > -3\}$ **9.** $\{q \mid q \geq 2\}$ **10.** $\{v \mid v \leq 9\}$ **11.** $\{x \mid x \leq 10\}$ **12.** x = cost of backpack; $x - 0.30x + 0.05(x - 0.30x) \leq 20$; \$27.21
13. C

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6-4 Solving Compound Inequalities

(Pages 339–344)

Two inequalities considered together form a **compound inequality**.

AND Compound Inequalities	Compound inequalities that contain the word *and* are true only if both inequalities are true. The graph of a compound inequality containing *and* is the **intersection** of the graphs of the two inequalities that make up the compound inequality. To find the intersection, determine where the two graphs overlap.
OR Compound Inequalities	Compound inequalities that contain the word *or* are true if one or more of the inequalities is true. The graph is the **union** of the graphs of the two inequalities that make up the compound inequality.

Examples

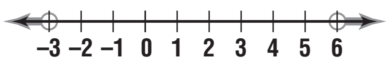
Solve each compound inequality. Then graph the solution set.

a. $2k - 5 > 7$ or $-3k - 1 > 8$

$2k - 5 > 7$ *or* $-3k - 1 > 8$

$2k > 12$ $\quad$ $-3k > 9$

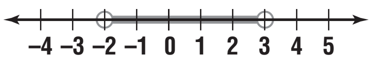
$k > 6$ $\quad$ $k < -3$



b. $4 < n + 6 < 9$

$n + 6 > 4$ *and* $n + 6 < 9$

$n > -2$ $\quad$ $n < 3$



Try These Together

1. Graph the solution set of $a \geq -9$ and $a < 9$.
   HINT: One circle is closed and the other is open.

2. Graph the solution set of $d < -6$ or $d > 4$.
   HINT: Combine the graphs of d < -6 and d > 4

Practice

3. Graph the solution set of $n < 7$ and $n \geq 4$.

Solve each compound inequality. Then graph the solution set.

4. $6g - 8 > 4$ or $6g + 2 < -4$
5. $k + 8 > -4$ or $k - 8 < 8$
6. $1 < 2c - 7 < 7$
7. $5r + 3 \geq -2$ and $r \neq 0$

Define a variable, write a compound inequality, and solve each problem. Then check your solution.

8. The sum of three times a number and two lies between 8 and 11.
9. Eight less than 4 times a number is at most 24 and at least -12.
10. **Standardized Test Practice** If the replacement set is all integers, find the solution set for $1 < x - 1 < 3$.

A {3} **B** {2, 3, 4} **C** all integers **D** no solution

Answers: 1–3. See Answer Key. **4–7.** For graphs, see Answer Key. **4.** $\{g \mid g > 2$ or $g < -1\}$ **5.** $\{k \mid k > -12$ or $k < 16\}$ **6.** $\{c \mid 4 < c < 7\}$ **7.** $\{r \mid r \geq -1$ and $r \neq 0\}$ **8.** $8 < 3x + 2 < 11$; $\{x \mid 2 < x < 3\}$ **9.** $24 \geq 4x - 8 \geq -12$; $\{x \mid 8 \geq x \geq -1\}$ **10.** A

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6-5 Solving Open Sentences Involving Absolute Value *(Pages 345–351)*

An open sentence involving absolute value can be solved by first rewriting it as a compound sentence.

Rewriting Absolute Value Equations Inequalities	• If $|x| = n$, then $x = -n$ or $x = n$. • If $|x| < n$, then $x > -n$ and $x < n$. (Also true for $|x| \leq n$) • If $|x| > n$, then $x < -n$ or $x > n$. (Also true for $|x| \geq n$)

Examples **Solve each open sentence. Then graph the solution set.**

a. $|2 + 4y| < 6$

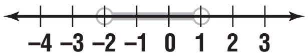
Rewrite as a compound inequality. Then solve.

$2 + 4y > -6$ and $2 + 4y < 6$

$4y > -8$ $\quad$ $4y < 4$

$y > -2$ $\quad$ $y < 1$

The solution set is $\{y|-2 < y < 1\}$.

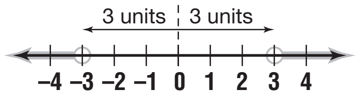


b. $|p| > 3$

Rewrite as a compound inequality. Then solve.

$p < -3$ or $p > 3$

The solution set is $\{p|p < -3$ or $p > 3\}$.



Try These Together

1. Solve $|a - 4| = 7$ and graph the solution set.
HINT: The solution will be two points.

2. Solve $|6s - 4| < 8$ and graph the solution set.
HINT: The solution will be a line segment.

Practice

Solve each open sentence. Then graph the solution set.

3. $|5d + 1| = 9$ **4.** $|2 - 2y| > 8$ **5.** $|3 - n| \leq 4$

6. $|-w + 8| \geq 11$ **7.** $|2g - 6| < 1$ **8.** $|1.1z - 3.3| = 7.7$

Express each statement in terms of an inequality involving absolute value.

9. The weight w in a bicycle trailer is allowed to vary from 60 pounds by no more than 40 pounds.

10. The height h of a person allowed on a roller coaster can vary from 65 inches by no more than 13 inches.

11. **Standardized Test Practice** Solve $|x - 5| \leq 7$.

A $\{x|x \leq 12$ or $x \geq -2\}$ **B** $\{x|-2 \leq x \leq 12\}$

C $\{x|x \leq 12\}$ **D** $\{x|x \geq -2\}$

Answers: 1–8. For graphs, see Answer Key. **1.** $\{-3, 11\}$ **2.** $\{s|-\frac{2}{3} < s < 2\}$ **3.** $\{-2, \frac{8}{5}\}$ **4.** $\{y|y < -3$ or $y > 5\}$ **5.** $\{n|-1 \leq n \leq 7\}$ **6.** $\{w|w \leq -3$ or $w \geq 19\}$ **7.** $\{g|2.5 < g < 3.5\}$ **8.** $\{-4, 10\}$ **9.** $|w - 60| \leq 40$ **10.** $|h - 65| \leq 13$ **11.** B

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6-6 Graphing Inequalities in Two Variables

(Pages 352–357)

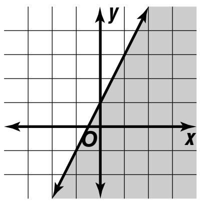
The solution set for an inequality in two variables contains ordered pairs whose graphs fill an area on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge of the half-plane.

Graphing Inequalities in Two Variables	1. Find the boundary by graphing the equation related to the inequality. If the inequality symbol is $<$ or $>$, draw the boundary as a *dashed* line. If the inequality symbol is $\le$ or $\ge$, draw the boundary as a *solid* line to show that the points on the boundary are included in the solution set. 2. Determine which of the two half-planes contains the solutions by choosing a point in each half-plane and testing its coordinates in the inequality. If the coordinates make the inequality true, shade that half-plane.

Example

Graph $y - 2x \le 1$.

Solve the equality for y: $y \le 2x + 1$. Then, graph the related equation $y = 2x + 1$. Draw the line as a solid line since the inequality symbol is less than or equal to. Select a point in each of the half-planes and test it in the inequality.



Test (0, 0)	**Test (−1, 1)**
$y - 2x \le 1$	$y - 2x \le 1$
$0 - 2(0) \le 1$	$1 - 2(-1) \le 1$
$0 \le 1$ True	$3 \le 1$ False

Therefore, the half-plane that contains the point (0, 0) should be shaded.

Practice

Find which ordered pairs from the given set are part of the solution set for each inequality.

1. $y > 2x$, {(−3, −7), (0, 0), (1, 3), (2, 5)}
2. $3y + 2x \le 8$, {(−1, 5), (3, −1), (5, −1), (9, 2)}

Graph each inequality.

3. $x > 4$
4. $x + y \le 2$
5. $3x - 2y \le -5$
6. $2x + 10 < 0$
7. $x - y \ge -4$
8. $y > -3$

9. **Jobs** It takes a librarian 1 minute to renew an old library card and 3 minutes to make a new card. Together, she can spend no more than 30 minutes renewing and making cards. Write an inequality to represent this situation, where x is the number of old cards she renews and y is the number of new cards she makes.

10. **Standardized Test Practice** Which ordered pair is a solution of $x + 2y \le -7$?

A (0, 0) **B** (8, −8) **C** (−5, 3) **D** (−1, 0)

Answers: 1. (1, 3), (2, 5) **2.** (3, −1), (5, −1) **3–8.** See Answer Key. **9.** $x + 3y \le 30$ **10.** B

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6 Chapter Review

Grand Prize

A baseball team is selling calendars, T-shirts, and candy bars to raise money. The student who raises the most money wins the grand prize. To find out what the grand prize is, match each inequality to its graph on the right. Use the letter to the right of the graph to fill in the blanks that correspond with the number of the problem.

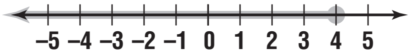
___	___	___	___	___	___	___	___	___
5	9	9	1	5	7	6	6	3

___	___	___	___	___	___	___
8	2	4	7	6	8	3

1. $x - (-2) \leq 6$

K

2. $3t + 8 > 20$

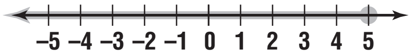


A

3. $\frac{n}{-2} - 7 > -9$

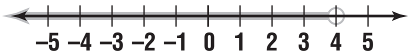
C

4. $d \leq 5$ and $d \geq -3$



N

5. $y \leq 5$ or $y \leq -3$



S

6. $a \geq 5$ and $a \leq -3$

Y

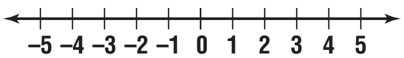
7. $|w - 1| < 4$

I

8. $m < 5$ or $m > -3$

T

9. $|p - 1| \geq 4$



E

Answers are located in the Answer Key.

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7-1 Graphing Systems of Equations

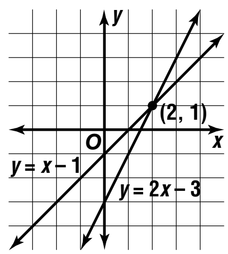
(Pages 369–374)

A set of equations with the same variables forms a **system of equations**. A solution to a system of two equations with two variables is an ordered pair of numbers that satisfies both equations. One way to solve a system of equations is to carefully graph the equations on the same coordinate plane. The coordinates of the point at which the graphs intersect is the solution to the system. If the graphs of the two equations coincide, meaning they are the same line, then there are *infinitely many* solutions to the system. A system of equations with at least one ordered pair that satisfies both equations is **consistent**. It is possible for the graphs of the two equations to be parallel. In this case, the system is inconsistent because there are *no solutions* that satisfy the two equations.

Example

Graph the system of equations to find the solution.
$y = 2x - 3$ and $y = x - 1$

The graphs appear to intersect at the point with coordinates (2, 1). Check this estimate by replacing x with 2 and y with 1 in each equation.



Check: *$y = 2x - 3$ / $y = x - 1$*
$1 = 2(2) - 3$ / $1 = 2 - 1$
$1 = 1$ ✓ / $1 = 1$ ✓

The solution is (2, 1).

Try These Together

Graph each system of equations. Then determine whether the system has* one *solution,* no *solution, or* infinitely many *solutions. If the system has one solution, name it.

1. $y = x + 2$; $y = 2x - 1$

2. $y = x + 2$; $y = x - 1$

3. $y = 2x - 1$; $y = 3(x - 1)$

HINT: Be sure to check your solution by substituting the x- and y-values back into the two equations.

Practice

Graph each system of equations. Then determine whether the system has *one* solution, *no* solution, or *infinitely many* solutions. If the system has one solution, name it.

4. $y = 10 - x$; $y = x + 1$

5. $2x + y = -5$; $3x + 3y = 9$

6. $y = 8 - x$; $y = 4 - \frac{1}{3}x$

7. $y = -3$; $4x + y = 1$

8. **Standardized Test Practice** A canoe can be paddled 10 miles upstream, against the river current, in 5 hours. Paddling downstream the same distance takes 1 hour. Write and then graph a system of equations to solve for the speed c of the canoe in still water and the speed r of the river current. Express the solution to the system as an ordered pair (c, r).

A (3, 7) **B** (7, 3) **C** (4, 6) **D** (6, 4)

Answers: **1–7.** See Answer Key for graphs. **1.** one; (3, 5) **2.** no solution **3.** one; (2, 3) **4.** one; (4.5, 5.5) **5.** one; (−8, 11) **6.** one; (6, 2) **7.** one; (1, −3) **8.** D

7-2 Substitution *(Pages 376–381)*

To solve a system of equations without graphing, you can use the **substitution method** shown in the example below. In general, if you solve a system of equations and the result is a *true* statement, such as $-5 = -5$, the system has *infinitely many* solutions; if the result is a *false* statement, such as $-5 = 7$, the system has *no solution*.

Example

Use substitution to solve the system of equations $x + y = 1$ and $2x + y = -1$.

Step 1: *Solve one of the equations for x or y.*

$x + y = 1$	*Solve the first equation for x since the*
$x = 1 - y$	*coefficient of x is 1.*

Step 2: *Substitute this value into the other equation.*

$2x + y = -1$	*Use the second equation.*
$2(1 - y) + y = -1$	*Substitute 1 − y for x.*
$2 - 2y + y = -1$	*Distribute.*

Step 3: *Solve this equation.*

$2 - 2y + y = -1$	*Solve for y.*
$-y = -3$ or $y = 3$	

Step 4: *Find the value of the other variable using substitution into either equation.*

$x + y = 1$	*Use the first equation.*
$x + 3 = 1$	*Substitute 3 for y.*
$x = -2$	*Solve for x.*

The solution to the system is (−2, 3).

Check: *Substitute −2 for x and 3 for y in each of the original equations and check for true statements.*

Try These Together

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has **no solution** ***or*** **infinitely many** ***solutions.***

1. $3x + y = 19$, $x - 2y = -10$
2. $2x - y = 7$, $8x + y = 3$
3. $y = 2x - 4$, $y = 2x + 2$
4. $y = -5x + 3$, $y = 3x - 3$

HINT: If possible, choose to first solve an equation for a variable that has a coefficient of 1.

Practice

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

5. $5x + 4 = y$, $y - 3x = 7$
6. $3y + x = -1$, $2x + 6 = -3y$
7. $6x - y = 0$, $3x + 4y = 18$
8. $3y - 4x = 2$, $8x = 6y - 4$
9. $2x - y = -4$, $-x + y = -9$
10. $5x - 2y = -6$, $2x + 3y = 9$
11. $3x + y = 28$, $x + 3y = -12$
12. $5x - y = 98$, $-2x + 3y = 5$

13. **Standardized Test Practice** All CDs in the budget bin are priced the same. Packs of AA batteries are on sale. Keisha's total bill (before tax) for 3 CDs and 1 pack of AA batteries was \$39. Eduardo's total for 2 CDs and 3 packs of batteries was \$33. What was the price of a single CD?

A \$3 **B** \$10 **C** \$12 **D** \$13

Answers: **1.** (4, 7) **2.** (1, −5) **3.** no solution **4.** $\left(\frac{3}{4}, -\frac{3}{4}\right)$ **5.** $\left(1\frac{1}{2}, 11\frac{1}{2}\right)$ **6.** $\left(-5, \frac{4}{3}\right)$ **7.** $\left(\frac{2}{3}, 4\right)$ **8.** infinitely many **9.** (−13, −22) **10.** (0, 3) **11.** (12, −8) **12.** (23, 17) **13.** C

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7-3 Elimination Using Addition and Subtraction *(Pages 382–386)*

In systems of equations where the coefficients of terms containing the same variable are *opposites*, the **elimination** method can be applied by adding the equations. If the coefficients of those terms are the *same*, the elimination method can be applied by subtracting the equations.

Examples **Solve each system of equations using elimination.**

a. $x - 2y = 13$ **and** $3x + 2y = 15$

Add the two equations, since the coefficients of the y-terms, −2 and 2, are opposites.

$$\begin{array}{rl} x - 2y = 13 & \\ (+)\ 3x + 2y = 15 & \\ \hline 4x \quad\quad = 28 & \text{Solve for } x. \\ x = 7 & \text{Divide each side by 4.} \end{array}$$

$x - 2y = 13$ — *Use the first equation.*

$7 - 2y = 13$ — *Substitute 7 for x.*

$-2y = 6 \Rightarrow y = -3$

The solution of the system is (7, −3).

b. $3x + 4y = 5$ **and** $3x - y = -5$

Subtract the two equations, since the coefficients of the x-terms are the same.

$$\begin{array}{rl} 3x + 4y = 5 & \\ (-)\ 3x - y = -5 & \\ \hline 5y = 10 & \text{Solve for } y. \\ y = 2 & \text{Divide each side by 5.} \end{array}$$

$3x - y = -5$ — *Use the second equation.*

$3x - 2 = -5$ — *Substitute 2 for y.*

$3x = -3 \Rightarrow x = -1$

The solution of the system is (−1, 2).

Try These Together

State whether addition, subtraction, or substitution would be most convenient to solve each system of equations. Then solve the system.

1. $x - y = 3$; $3x + y = 1$

2. $3x + 4y = 2$; $2x + 4y = 8$

3. $2x + 4y = 8$; $y - 3 = x$

Practice

State whether addition, subtraction, or substitution would be most convenient to solve each system of equations. Then solve the system.

4. $x + 2y = 3$; $-x + y = 6$

5. $x + y = -2$; $x - y = 8$

6. $2y - 3x = 12$; $-2y + 6x = -5$

7. $2x + y = -5$; $x + 3y = 25$

8. $x - 4y = 16$; $2x - 4y = 18$

9. $2x + 4y = 6$; $3x - 4y = 2$

10. $8x + y = 1$; $-8x - 4y = 3$

11. $2x - 5y = -6$; $2x + 3y = -9$

12. Shopping A can of juice and a can of beef stew together cost $2.05. Two cans of juice and a can of beef stew cost $2.70. How much does a single can of juice cost?

13. **Standardized Test Practice** Solve the system. $2y - 5x = 1$; $3y + 5x = 14$

A (3, 1) **B** (1, 3) **C** (−1, 3) **D** (3, −1)

Answers: 1. addition; (1, −2) **2.** subtraction; (−6, 5) **3.** substitution; $\left(-\frac{2}{3}, 2\frac{1}{3}\right)$ **4.** addition; (−3, 3) **5.** addition or subtraction; (3, −5) **6.** addition; $\left(2\frac{1}{3}, 9\frac{1}{2}\right)$ **7.** substitution; (−8, 11) **8.** subtraction; $\left(2, -3\frac{1}{2}\right)$ **9.** addition; $\left(1\frac{3}{5}, \frac{7}{10}\right)$ **10.** addition; $\left(\frac{7}{24}, -1\frac{1}{3}\right)$ **11.** subtraction; $\left(-3\frac{15}{16}, -\frac{3}{8}\right)$ **12.** $0.65 **13.** B

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7-4 Elimination Using Multiplication

(Pages 387–392)

An extension of the elimination method is to multiply one or both of the equations in a system by some number so that adding or subtracting eliminates a variable.

Examples **Solve each system of equations using elimination.**

a. $x - y = 5$ **and** $3x + 2y = 15$

Multiply the first equation by 2 so that the coefficient of the y-terms in the system will be opposites. Then, add the equations and solve for x.

$2(x - y) = 2(5)$ → $2x - 2y = 10$
$3x + 2y = 15$ → $(+)\ 3x + 2y = 15$

$5x = 25$
$x = 5$

$x - y = 5$ — *Use the first equation.*
$5 - y = 5$ — *Substitute 5 for x.*
$-y = 0 \Rightarrow y = 0$

The solution to this system is (5, 0).

b. $2x + 9y = 43$ **and** $5x - 2y = -15$

Multiply the first equation by 5 and the second equation by −2 so that the coefficients of the x-terms in the system will be opposites. Then, add the equations and solve for y.

$5(2x + 9y) = 5(43)$ → $10x + 45y = 215$
$-2(5x - 2y) = -2(-15)$ → $(+)\ -10x + 4y = 30$

$49y = 245$
$y = 5$

$2x + 9y = 43$ — *Use the first equation.*
$2x + 45 = 43$ — *Substitute 5 for y.*
$2x = -2 \Rightarrow x = -1$

The solution to the system is (−1, 5).

Try These Together

Use elimination to solve each system of equations.

1. $2x + y = 4$
$3x - 2y = 6$

2. $-5x + 2y = 5$
$x - y = 2$

3. $4x + 7y = 6$
$6x + 5y = 20$

4. $\frac{x - y}{4} = 1$
$\frac{2x - y}{3} = 4$

Practice

Use elimination to solve each system of equations.

5. $18x + 24y = 288$
$-16x - 12y = -172$

6. $3x + 8y = 11$
$2x + 5y = 18$

7. $y = 4x + 11$
$3x - 2y = -7$

8. $2x - 2y = 16$
$3x + y = 4$

9. $2x + 3y = 0$
$3x + y = 7$

10. $2x + \frac{1}{3}y = -1$
$x - \frac{1}{4}y = -8$

11. $0.4x + 0.2y = 0.4$
$0.2x - 0.3y = 0.4$

12. Algebra Solve using elimination: $\frac{1}{2x - 4} - \frac{2}{y + 1} = 0$ and $\frac{1}{x - 3} - \frac{1}{y + 4} = 0$.

13. **Standardized Test Practice** By which number could you multiply the first equation of the following system to solve the system by elimination?
$-4x - 11y = -32$ and $12x + 10y = 55$

A 3 or −3 **B** 10 or −10 **C** 11 or −11 **D** 12 or −12

Answers: **1.** (2, 0) **2.** (−3, −5) **3.** (5, −2) **4.** (8, 4) **5.** (4, 9) **6.** (89, −32) **7.** (−3, −1) **8.** (3, −5) **9.** (3, −2) **10.** $\left(-3\frac{1}{2}, 18\right)$ **11.** $\left(1\frac{1}{4}, -\frac{1}{2}\right)$ **12.** $\left(\frac{2}{3}, -6\frac{1}{3}\right)$ **13.** A

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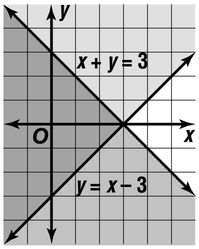
7-5 Graphing Systems of Inequalities

(Pages 394–398)

You can solve **systems of inequalities** by graphing. Recall that the graph of an inequality is a *half-plane*. The intersection of the two half-planes graphed in a system of inequalities represents the solution to the system.

Example

Graph the system of inequalities to find the solution.
$x + y \leq 3$ and $y + 3 \geq x$



Begin by solving each inequality for y. Then, graph each inequality.

$x + y \leq 3$ and $y + 3 \geq x$
$y \leq -x + 3$ $\quad$ $y \geq x - 3$

The solution to the system includes the ordered pairs in the intersection of the graphs of each inequality. This region is shaded dark gray.

Notice that the boundary lines $y = -x + 3$ and $y = x - 3$ are included in the solution, since the inequalities contained $\leq$ and $\geq$ symbols.

Try These Together

Solve each system of inequalities by graphing.

1. $x > 3$, $y \leq 5$

2. $x \leq 4$, $y > -1$

3. $y - 3 > x$, $y + x < 3$

4. $2y + x < 6$, $3x - y > 4$

HINT: Remember to graph inequalities with < or > with dashed lines because these lines are not included in the solution.

Practice

Solve each system of inequalities by graphing.

5. $x < 1$, $y > -4$

6. $2x + y \leq 4$, $3x - y \geq 6$

7. $y + 2 \leq x$, $2y + 2 > 2x$

8. $x + 4 \leq y$, $y > 2$

9. Algebra Solve by graphing.
$x - 4y > 11$
$3x + y \leq 6$
$x \geq 0$

10. **Standardized Test Practice** A dieter limits a snack to 90 Calories. Which is a possible snack combination of 20-Calorie apricots and 3-Calorie celery stalks?

A 4 apricots, 3 celery stalks
B 3 apricots, 10 celery stalks
C 2 apricots, 8 celery stalks
D all of these

Answers: 1–9. See Answer Key. **10.** D

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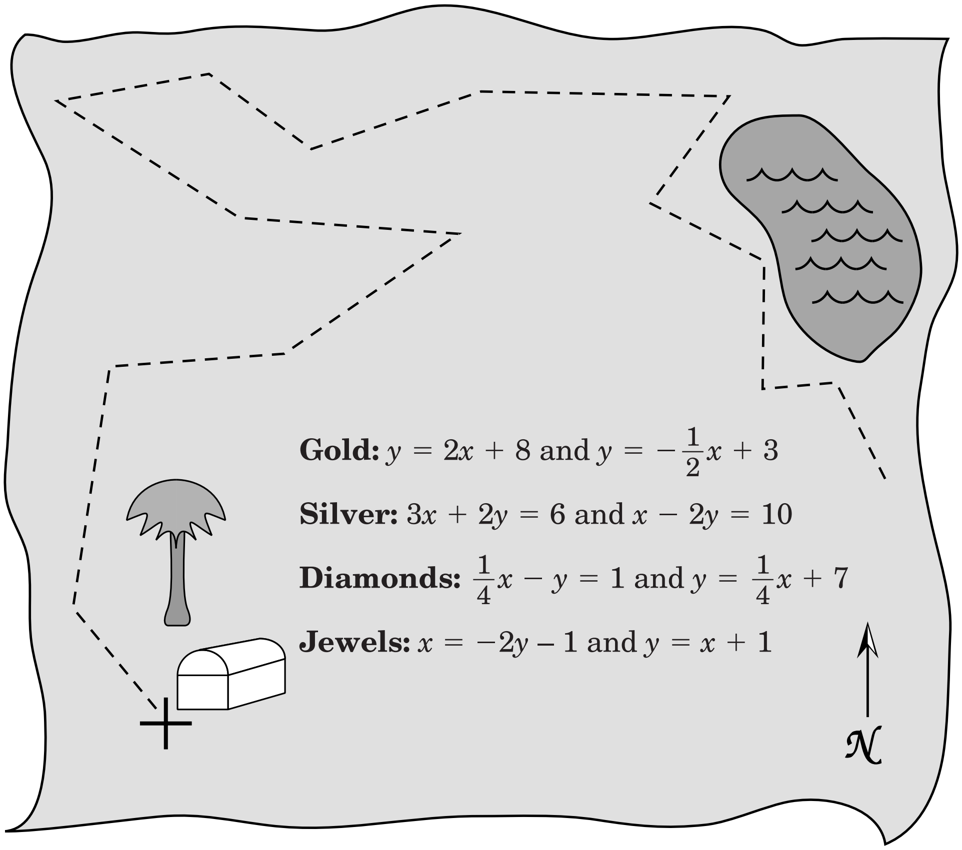
7 Chapter Review

Treasure Hunt

Imagine that you and your parent are on a treasure hunt. The treasure hunt is taking place on a giant coordinate grid that is laid out on the floor of your school gym. You are competing with other parents and students for a grand prize. However, every parent/student team is looking for different treasures.

The treasures for which you are searching are numbered stickers on the floor of this giant coordinate grid. Specifically, you are given a list of four items and a starting point. To locate the treasures, you must plot the intersection of the two graphs listed for each treasure.

Your starting point is the intersection of the graphs of $y = \frac{2}{3}x + 5$ and $2y = x + 10$. Find the coordinates of your starting point by graphing the two equations. Determine the coordinates for the location of each treasure by graphing each pair of equations given in the figure below. Then determine which treasure is closest to your starting point. The winner of the treasure hunt is the first parent/student team who turns in the treasure sticker that was closest to their starting point.



Answers are located in the Answer Key.

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8-1 Multiplying Monomials *(Pages 410–415)*

An expression like $5x^2$ is called a **monomial**. A monomial is a number, a variable, or a product of a number and one or more variables. Monomials that are real numbers are called **constants**. To simplify a product involving monomials, write an equivalent expression in which: (1) there are no powers of powers, (2) each base appears exactly once, and (3) all fractions are in simplest form.

Product of Powers	You can multiply powers with the same base by adding exponents. For any number a, and all integers m and n, $a^m \cdot a^n = a^{m+n}$.
Power of a Power	You can find a power of a power by multiplying exponents. For any number a, and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	A power of a product is the product of the powers. For all numbers a and b, and any integer m, $(ab)^m = a^m b^m$.
Power of a Monomial	The power of a power property and the power of a product property can be combined into the power of a monomial property. For all numbers a and b, and all integers m, n, and p, $(a^m b^n)^p = a^{mp} b^{np}$.

Examples

Simplify each expression.

a. $4x^2(5x^3)$

$$4x^2(5x^3) = (4 \cdot 5)(x^2x^3)$$
$$= 20x^{2+3}$$
$$= 20x^5$$

b. $(2x^3y)^4[(-2y)^2]^3$

$$(2x^3y)^4[(-2y)^2]^3 = (2x^3y)^4(-2y)^{3 \cdot 2}$$
$$= (2x^3y)^4(-2y)^6$$
$$= 2^4(x^3)^4y^4(-2)^6y^6$$
$$= 2^4x^{3 \cdot 4}y^4(-2)^6y^6$$
$$= 16x^{12}y^4 64y^6$$
$$= (16 \cdot 64)x^{12}(y^4y^6)$$
$$= 1024x^{12}y^{4+6}$$
$$= 1024x^{12}y^{10}$$

Practice

Simplify.

1. $a^7(a)(a^2)$ **2.** $(g^2h)(gh^4)$ **3.** $(c^5d)(c^3d^5)$ **4.** $[(3^2)^2]^2$

5. $(2m^2n^8)(2mn^9)$ **6.** $(x^2y^5)^4$ **7.** $g^5(g^3s^3)$ **8.** $(3abc)(6ab^2c^2)$

9. $(0.3u)^4$ **10.** $\left(\frac{5}{6}f\right)^2$ **11.** $-\frac{4}{5}b(15t)^2$ **12.** $(0.4j^3k^2)^2$

13. $-4(rs^4t)^2$ **14.** $(-2xy)^2(6y^8)$ **15.** $(-4y^2)^2 - (4y)^4$ **16.** $\left(\frac{1}{8}x^4\right)^2(8x^3)^2$

17. $\left(\frac{3}{4}v^3\right)^3(16v)(8w)\left(\frac{1}{9}w^4\right)$ **18.** $(2b)^4\left(\frac{1}{4}c^6\right)^3$

19. **Standardized Test Practice** Simplify $(a^2b)(ab^2)^3$.

A a^5b^6 **B** a^5b^7 **C** a^6b^6 **D** a^9b^9

Answers: **1.** a^{10} **2.** g^3h^5 **3.** c^8d^6 **4.** 6561 **5.** $4m^3n^{17}$ **6.** x^8y^{20} **7.** g^8s^3 **8.** $18a^2b^3c^3$ **9.** $0.0081u^4$ **10.** $\frac{25}{36}f^2$ **11.** $-180bt^2$ **12.** $0.16j^6k^4$ **13.** $-4r^2s^8t^2$ **14.** $24x^2y^{10}$ **15.** $-240y^4$ **16.** x^{14} **17.** $6v^{10}w^5$ **18.** $\frac{1}{4}b^4c^{18}$ **19.** B

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8-2 Dividing Monomials *(Pages 417–423)*

Quotient of Powers	You can divide powers with the same base by subtracting exponents. For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponents	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples **Simplify each expression.**

a. $\frac{a^6b^9}{a^2b^5}$

$$\frac{a^6b^9}{a^2b^5} = \left(\frac{a^6}{a^2}\right)\left(\frac{b^9}{b^5}\right)$$
$$= (a^{6-2})(b^{9-5})$$
$$= a^4b^4$$

b. $\frac{(2x^{-3})^{-3}}{(4x^2)^3}$

$$\frac{(2x^{-3})^{-3}}{(4x^2)^3} = \frac{2^{-3}x^9}{4^3x^6}$$
$$= \left(\frac{1}{4^3}\right)\left(\frac{1}{2^3}\right)\left(\frac{x^9}{x^6}\right)$$
$$= \left(\frac{1}{64}\right)\left(\frac{1}{8}\right)x^{9-6}$$
$$= \left(\frac{1}{512}\right)x^3 \text{ or } \frac{x^3}{512}$$

Practice

Simplify. Assume that no denominator is equal to zero.

1. $x^{-3}y^0z^{-2}$

2. $\frac{d^{-1}}{d^0}$

3. $\frac{4a}{a^8}$

4. $\frac{n^3}{n^{-1}}$

5. $\frac{g^7h^2}{g^5h^0}$

6. $\frac{5s^3}{40s^4}$

7. $\frac{(-u)^2v^8}{u^6v^{-3}}$

8. $\frac{a^2b^9}{a^2b^8}$

9. $\frac{16x^6y^7z^8}{-2x^4y^4z^0}$

10. $\frac{(f^{-5}g^7)^2}{(fg)^{-6}}$

11. $\frac{2rs^3}{3s^3}$

12. $\frac{(-m)^5n^7}{m^2n^7}$

13. $\frac{(j^{-4}k^5)^2}{(7j^2)^2}$

14. $\frac{26a^3}{-13a^6b^8}$

15. $\frac{18rs^0t^9}{6r^8s^7t^4}$

16. $\left(\frac{9ab^{-4}c}{6a^{-5}b^2}\right)^0$

17. Money Matters You can use the formula $P = A\left[\frac{i}{1-(1+i)^{-n}}\right]$ to find the monthly payment on a loan of A dollars that is paid back in equal monthly payments over n months. The variable i represents (annual interest rate ÷ 12). Seki has a $4,000 student loan with an 8% annual interest rate which he is scheduled to pay off in 10 years. Use the formula and a calculator to find Seki's monthly payment.

18. **Standardized Test Practice** Simplify $\frac{(x^2y)^2}{x^{-2}y^2}$.

A $\frac{1}{y}$ **B** x^2 **C** x^2y **D** x^6

Answers: **1.** $\frac{1}{x^3z^2}$ **2.** $\frac{1}{d}$ **3.** $\frac{4}{a^7}$ **4.** n^4 **5.** g^2h^2 **6.** $\frac{1}{8s}$ **7.** $\frac{v^{11}}{u^4}$ **8.** b **9.** $-8x^2y^3z^8$ **10.** $\frac{g^{20}}{f^4}$ **11.** $\frac{2r}{3}$ **12.** $-m^3$ **13.** $\frac{k^{10}}{49j^{12}}$ **14.** $-\frac{2}{a^3b^8}$ **15.** $\frac{3t^5}{r^7s^7}$ **16.** 1 **17.** $48.53 **18.** D

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8-3 Scientific Notation *(Pages 425–430)*

When dealing with very large numbers, keeping track of place value can be difficult. For this reason, it is not always desirable to express numbers in standard notation. Large numbers such as these may be expressed in **scientific notation**.

Scientific Notation	A number is expressed in scientific notation when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Examples **Express each number in scientific notation.**

a. 299,000,000

299,000,000
$= 2.99 \times 100{,}000{,}000$
$= 2.99 \times 10^8$

b. 0.0000254

0.0000254
$= 2.54 \times 0.00001$
$= 2.54 \times \frac{1}{100{,}000}$
$= 2.54 \times 10^{-5}$

Express each number in standard notation.

c. $\mathbf{3.14 \times 10^6}$

3.14×10^6
$= 3.14 \times 1{,}000{,}000$
$= 3{,}140{,}000$

d. $\mathbf{7.2 \times 10^{-4}}$

7.2×10^{-4}
$= 7.2 \times \frac{1}{10^4}$
$= 7.2 \times \frac{1}{10{,}000}$
$= 7.2 \times 0.0001$ or 0.00072

Practice

Express each number in scientific notation.

1. 3500 **2.** 0.0015 **3.** 43.8 **4.** 285,873,000
5. 0.0000000485 **6.** 0.060406 **7.** 655×10^{-5} **8.** 109×10^8

Evaluate. Express each result in scientific notation and standard notation.

9. $(5.4 \times 10^1)(4 \times 10^4)$ **10.** $(6.07 \times 10^{-3})(42 \times 10^{-1})$ **11.** $(9 \times 10^6)(20 \times 10^{-4})$

12. $\frac{2.6 \times 10^3}{6.5 \times 10^9}$ **13.** $\frac{9.5 \times 10^8}{1.9 \times 10^2}$ **14.** $\frac{3.51 \times 10^{-7}}{2.7 \times 10^2}$

15. Chemistry A mole has 6.0×10^{23} molecules. The molecular mass of water is 18 grams, or 18 moles. How many molecules are there in 18 grams of water? Express your answer in scientific notation.

16. **Standardized Test Practice** Evaluate $(1.42 \times 10^8) \div (7.1 \times 10^{-3})$.
A 2×10^4 **B** 2×10^5 **C** 2×10^{10} **D** 2×10^{11}

Answers: **1.** 3.5×10^3 **2.** 1.5×10^{-3} **3.** 4.38×10^1 **4.** 2.85873×10^8 **5.** 4.85×10^{-8} **6.** 6.0406×10^{-2} **7.** 6.55×10^{-3} **8.** 1.09×10^{10} **9.** 2.16×10^6; 2,160,000 **10.** 2.5494×10^{-2}; 0.025494 **11.** 1.8×10^4; 18,000 **12.** 4×10^{-7}; 0.0000004 **13.** 5×10^6; 5,000,000 **14.** 1.3×10^{-9}; 0.0000000013 **15.** 1.08×10^{25} **16.** C

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8-4 Polynomials *(Pages 432–436)*

Recall that a *monomial* is a number, a variable, or a product of numbers and variables. A **polynomial** is a monomial or a sum of monomials. The exponents of the variables of a polynomial must be positive. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. The **degree** of a monomial is the sum of the exponents of its variables. To find the degree of a polynomial, you must find the degree of each term. The greatest degree of any term is the degree of the polynomial. The terms of a polynomial are usually arranged so that the powers of one variable are in ascending or descending order.

Examples **Consider the expression $3x^2 + 5 + 7x$.**

a. Is the expression a polynomial and if so is it a monomial, binomial, or trinomial?

The expression is the sum of three monomials, therefore it is a polynomial. Since there are three monomials, the polynomial is a trinomial.

b. What is the degree of the polynomial?

The degree of $3x^2$ is 2, the degree of 5 is 0, and the degree of $7x$ is 1. The greatest degree is 2, so the degree of the polynomial is 2.

c. Arrange the terms of the polynomial so that the powers of x are in descending order.

$3x^2 + 7x + 5$

Practice

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, a binomial, or a trinomial.

1. $\frac{1}{80}z^3$

2. $a^8 - \frac{1}{5}a + \frac{b}{574a}$

3. $\frac{n^2}{17m}$

4. $2x + 6z - 3y$

5. $\frac{5}{d} + d^3$

6. $4st^3 + 1.2t^2 - 0.8st$

Find the degree of each polynomial.

7. $7u^3$

8. $a^8bc^2 - 9ac^2$

9. 18

10. $k^8 + h^9$

11. $2f - 9y + z - 8q$

12. $2x^3y^2z^4 - 6xy^4z^2$

Arrange the terms of each polynomial so that the powers of x are in ascending order. Then arrange them in descending order.

13. $2 + x^4 + x^2$

14. $6x - 3x^2y + 4 - 2x^8$

15. $a^2bx^6 - bcx^5 + 24 - x^2$

16. $8x^4 - 2x^8y + 4x^9 + \frac{3}{10}x^5$

17. $3a^2x^8 - 2a^2x^5 + \frac{1}{4}x^2 + \frac{1}{2}x$

18. $17xy^3 + 6x^4y - x^3y^2 + y^5$

19. **Standardized Test Practice** What is the degree of the polynomial $3x^2y - 4xy^3$?

A 1 **B** 2 **C** 3 **D** 4

Answers: **1.** yes; monomial **2.** no **3.** no **4.** yes; trinomial **5.** no **6.** yes; trinomial **7.** 3 **8.** 11 **9.** 0 **10.** 9 **11.** 1 **12.** 9 **13.** $2 + x^2 + x^4$; $x^4 + x^2 + 2$ **14.** $4 + 6x - 3x^2y - 2x^8$; $-2x^8 - 3x^2y + 6x + 4$ **15.** $24 - x^2 - bcx^5 + a^2bx^6$; $a^2bx^6 - bcx^5 - x^2 + 24$ **16.** $8x^4 + \frac{3}{10}x^5 - 2x^8y + 4x^9$; $4x^9 - 2x^8y + \frac{3}{10}x^5 + 8x^4$ **17.** $\frac{1}{2}x + \frac{1}{4}x^2 - 2a^2x^5 + 3a^2x^8$; $3a^2x^8 - 2a^2x^5 + \frac{1}{4}x^2 + \frac{1}{2}x$ **18.** $y^5 + 17xy^3 - x^3y^2 + 6x^4y$; $6x^4y - x^3y^2 + 17xy^3 + y^5$ **19.** D

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8-5 Adding and Subtracting Polynomials *(Pages 439–443)*

To add polynomials, you can group like terms and then find their sum, or you can write them in column form and then add. To subtract a polynomial, add its additive inverse, which is the opposite of each term in the polynomial.

Examples **Find each sum or difference.**

a. $(a^2 + 4a + 3) + (5a^2 - 2a - 7)$

Arrange like terms in column form and add. Follow the rules for adding signed numbers.

$$\begin{array}{r} a^2 + 4a + 3 \\ (+)\ 5a^2 - 2a - 7 \\ \hline 6a^2 + 2a - 4 \end{array}$$

b. $(12x + 7y) - (-x + 2y)$

Find the additive inverse of $-x + 2y$. Then group the like terms and add.

The additive inverse of $-x + 2y$ is $x - 2y$.

$(12x + 7y) - (-x + 2y)$
$= (12x + 7y) + (x - 2y)$
$= (12x + x) + (7y - 2y)$
$= 13x + 5y$

Try These Together

Find each sum or difference.

1.
$$\begin{array}{r} 7a + 3b - 4c \\ a + 9b + 4c \\ (+)\ -3a - 9b - 9c \\ \hline \end{array}$$

2.
$$\begin{array}{r} 2a^2 - 7a + 8 \\ 7a^2 \qquad - 2 \\ (+)\ a^2 - 2a + 1 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 5x^2 - 3x + 1 \\ (-)\ -4x^2 - 2x + 8 \\ \hline \end{array}$$

Hint: For Exercise 3, remember to add the opposite of the second term in each column.

Practice

Find each sum or difference.

4. $(a^3 - 4b^3) + (2a^3 + 5a^2b - 6b^2 + 4b^3)$

5. $(2r - 8s) - (8r + 3s)$

6. $(3x^2 + 6y + 3) + (-2x^2 + 2y - 8)$

7. $(33n + m) - 15m$

8. $(4y^2 + 3y) + (-8y^3 - 2y + 6)$

9. $(2c^2 - 9) - (4c^2 + 4c + 1)$

10. $(3q^3 + 8q) + (-5q^2 - 7q)$

11. $(5 + b + 2b^2) + (3 - 2b + 9b^2)$

12. $(5x^2y^2 - xy - 1) - (7xy + 2)$

13. $(5k^2 - 2) - (2k^2 + 6k + 1)$

14. $(6x^2 + xy - 5y^2) + (9x^2 + 4xy + 9y^2)$

15. $(ax^2 + 8ax) - (8ax^2 - 2ax + 9)$

The measure of two sides of a triangle are given. *P* represents the measure of the perimeter. Find the measure of the third side.

16. $2x - 2y, 4x - y, P = 7x + 5y$

17. $10x - 1, 8x^2 + 2, P = 15x^2 - 9x + 18$

18. **Standardized Test Practice** Find $(4x^2 + 4x - 3) - (x^2 - 8x + 2)$.

A $3x^2 + 12x - 5$ **B** $5x^2 - 4x - 1$ **C** $3x^2 - 4x - 1$ **D** $5x^2 + 12x - 5$

Answers: 1. $5a + 3b - 9c$ **2.** $10a^2 - 9a + 7$ **3.** $9x^2 - x - 7$ **4.** $3a^3 + 5a^2b - 6b^2$ **5.** $-6r - 11s$ **6.** $x^2 + 8y - 5$ **7.** $33n - 14m$ **8.** $-8y^3 + 4y^2 + y + 6$ **9.** $-2c^2 - 4c - 10$ **10.** $3q^3 - 5q^2 + q$ **11.** $11b^2 - b + 8$ **12.** $5x^2y^2 - 8xy - 3$ **13.** $3k^2 - 6k - 3$ **14.** $15x^2 + 5xy + 4y^2$ **15.** $-7ax^2 + 10ax - 9$ **16.** $x + 8y$ **17.** $7x^2 - 19x + 17$ **18.** A

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8-6 Multiplying a Polynomial by a Monomial *(Pages 444–449)*

Use the distributive property to multiply a polynomial by a monomial. You may find it easier to multiply a polynomial by a monomial if you combine all like terms in the polynomial before you multiply.

Examples

a. Find $4z^2(z^2 + 7z - 3z^2)$.

Combine like terms in the polynomial and then multiply using the distributive property.

$4z^2(z^2 + 7z - 3z^2)$
$= 4z^2(-2z^2 + 7z)$
$= 4z^2(-2z^2) + 4z^2(7z)$
$= -8z^4 + 28z^3$

b. Solve $4(n - 5) + 2 = 5(6 - n) + 3n$.

$4(n - 5) + 2 = 5(6 - n) + 3n$
$4(n) - 4(5) + 2 = 5(6) - 5(n) + 3n$
$4n - 20 + 2 = 30 - 5n + 3n$
$4n - 18 = 30 - 2n$
$6n - 18 = 30$
$6n = 48$
$n = 8$

Try These Together

Find each product.

1. $-2(2a + 8)$

2. $cd(6c^2 + 3cd)$

HINT: Use the distributive property to multiply the monomial by every term in the polynomial.

Practice

Find each product.

3. $2n(9n^2 - 2n - 12)$

4. $8g^2h(g^2 + 9h - 6gh - 2h)$

5. $8s^2(2s^2 - 4s + 4)$

6. $-\frac{1}{2}xy^2\left(\frac{2}{3}xyz + \frac{1}{3}x - 8\right)$

Simplify.

7. $u(7u - 2) + 25u$

8. $5b(-b^2 + 7b - 1) + 9(3b^3 - 6b + 2)$

9. $4r^2(3r - 7) + r(7r^2 - 5r + 2) - 4(r^2 + 9r)$

10. $\frac{1}{3}c(3c^3 + 3c - 6) + \frac{4}{3}(3c^2 - 6c)$

Solve each equation.

11. $4(-6x + 9) + 4 = -4(-5x + 12)$

12. $12(2y - 9) = 6(y - 17)$

13. $21 + \frac{3}{2}s(s - 4) = \frac{1}{2}s(3s + 36) - 12s$

14. $a(3a + 2) + a(6a + 2) + 4 = 6a\left(a + \frac{1}{2}a\right) + 9$

15. Gardening A rectangular garden is x feet wide. The length of the garden is 3 feet more than twice the width. Write a polynomial that represents the area of the garden in square feet.

16. Standardized Test Practice Simplify $-2x(3x - 4) + 6x$.

A $8x$ **B** $7x - 4$ **C** $-6x^2 - 2x$ **D** $-6x^2 + 14x$

Answers: 1. $-4a - 16$ **2.** $6c^3d + 3c^2d^2$ **3.** $18n^3 - 4n^2 - 24n$ **4.** $8g^4h - 48g^3h^2 + 56g^2h^2$ **5.** $16s^4 - 32s^3 + 32s^2$ **6.** $-\frac{1}{3}x^2y^3z - \frac{1}{6}x^2y^2 + 4xy^2$ **7.** $7u^2 + 23u$ **8.** $22b^3 + 35b^2 - 59b + 18$ **9.** $19r^3 - 37r^2 - 34r$ **10.** $c^4 + 5c^2 - 10c$ **11.** 2 **12.** $\frac{1}{3}$ **13.** $\frac{7}{4}$ **14.** $1\frac{1}{4}$ **15.** $2x^2 + 3x$ **16.** D

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8-7 Multiplying Polynomials *(Pages 452–457)*

Use the distributive property to multiply polynomials. If you are multiplying two binomials, you can use a shortcut called the **FOIL method**.

FOIL Method for Multiplying Two Binomials	To multiply two binomials, find the sum of the products of F the *First* terms O the *Outer* terms I the *Inner* terms L the *Last* terms.

Examples

a. Find $(2x + 3)(4x - 1)$.

Use the FOIL method.

$$(2x + 3)(4x - 1) = \underbrace{(2x)(4x)}_{\text{First}} + \underbrace{(2x)(-1)}_{\text{Outer}} + \underbrace{(3)(4x)}_{\text{Inner}} + \underbrace{(3)(-1)}_{\text{Last}}$$
$$= 8x^2 + (-2x) + 12x + (-3)$$
$$= 8x^2 + 10x - 3 \quad \textit{Combine like terms.}$$

b. Find $(3y + 2)(5y^2 - 2y - 4)$.

Use the Distributive Property twice.

$$(3y + 2)(5y^2 - 2y - 4)$$
$$= 3y(5y^2 - 2y - 4) + 2(5y^2 - 2y - 4)$$
$$= (15y^3 - 6y^2 - 12y) + (10y^2 - 4y - 8)$$
$$= 15y^3 + 4y^2 - 16y - 8 \quad \textit{Combine like terms.}$$

Try These Together

Find each product.

1. $(a + 7)(a + 1)$ **2.** $(d - 2)(d - 5)$ **3.** $(n + 9)(n - 9)$

HINT: Use the FOIL method to multiply binomials.

Practice

Find each product.

4. $(g + 5)(g - 2)$ **5.** $(2s - 8)(s + 2)$ **6.** $(9u - 5)(4u + 9)$

7. $(5b + 9)(9b + 3)$ **8.** $(13t - 4)(14t + 5)$ **9.** $(4r + 4s)(2r + 6s)$

10. $(2x + 7)(3x^2 + 8x - 4)$ **11.** $(h^2 - 6h + 2)(h + 1)$ **12.** $(9v^2 - v + 8)(v - 7)$

13. $(4q + 0.7)(4q - 0.4)$ **14.** $(0.6p + 9q)(0.2p + q)$ **15.** $(2w + 0.2)(9w - 0.7)$

16. $(0.1c - 8)(0.3c + 3)$ **17.** $\left(6k + \frac{1}{4}\right)\left(k - \frac{1}{2}\right)$ **18.** $\left(f - \frac{1}{3}g\right)\left(\frac{2}{3}f + 3g\right)$

19. Decorating The length of a windowless room is 1 foot more than 4 times the height. The width is 2 feet less than 3 times the height. If h is the height of the room, write a polynomial that represents the wall area, including any doors.

20. Standardized Test Practice What is the product of $(x + 1)(2x - 3)$?

A $2x^2 + x - 3$ **B** $2x^2 - x - 3$ **C** $2x^2 + 5x - 3$ **D** $2x^2 - 4$

Answers: 1. $a^2 + 8a + 7$ **2.** $d^2 - 7d + 10$ **3.** $n^2 - 81$ **4.** $g^2 + 3g - 10$ **5.** $2s^2 - 4s - 16$ **6.** $36u^2 + 61u - 45$ **7.** $45b^2 + 96b + 27$ **8.** $182t^2 + 9t - 20$ **9.** $8r^2 + 32rs + 24s^2$ **10.** $6x^3 + 37x^2 + 48x - 28$ **11.** $h^3 - 5h^2 - 4h + 2$ **12.** $9v^3 - 64v^2 + 15v - 56$ **13.** $16q^2 + 1.2q - 0.28$ **14.** $0.12p^2 + 2.4pq + 9q^2$ **15.** $18w^2 + 0.4w - 0.14$ **16.** $0.03c^2 - 2.1c - 24$ **17.** $6k^2 - 2\frac{3}{4}k - \frac{1}{8}$ **18.** $\frac{2}{3}f^2 + 2\frac{7}{9}fg - g^2$ **19.** $14h^2 - 2h$ **20.** B

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8-8 Special Products *(Pages 458–463)*

In addition to the FOIL method, other shortcuts exist for finding special products of binomials.

Square of a Sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a Difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$
Difference of Squares	$(a + b)(a - b) = (a - b)(a + b) = a^2 - b^2$

Examples

a. Find $(s + 5)^2$.

Use the square of a sum rule.

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(s + 5)^2 = (s)^2 + 2(s)(5) + (5)^2$$
$$= s^2 + 10s + 25$$

b. Find $(3g - 8)^2$.

Use the square of a difference rule.

$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(3g - 8)^2 = (3g)^2 - 2(3g)(8) + (8)^2$$
$$= 9g^2 - 48g + 64$$

c. Find $(4x + 7)(4x - 7)$.

Use the difference of squares rule.

$$(a + b)(a - b) = a^2 - b^2$$
$$(4x + 7)(4x - 7) = (4x)^2 - (7)^2$$
$$= 16x^2 - 49$$

d. Find $(6y + 9z^2)(6y - 9z^2)$

Use the difference of squares rule.

$$(a + b)(a - b) = a^2 - b^2$$
$$(6y + 9z^2)(6y - 9z^2) = (6y)^2 - (9z^2)^2$$
$$= 36y^2 - 81z^4$$

Practice

Find each product.

1. $(a + 9b)^2$

2. $(c - 5d)^2$

3. $(8m - n)^2$

4. $(7z + 7)(7z - 7)$

5. $(2g - h)(2g + h)$

6. $(8s + 8t)^2$

7. $(3u - 18v)^2$

8. $(6q + 0.4r)^2$

9. $(x^2 + y^3)^2$

10. $\left(\frac{1}{3}j^2 - k^2\right)^2$

11. $(8a^2 - 2d)(8a^2 + 2d)$

12. $(4n^2 + g^2)(4n^2 - g^2)$

13. $(6.2s + u^4)^2$

14. $(5 - b^7)(5 + b^7)$

15. $\left(\frac{3}{2}t^2 - r\right)\left(\frac{3}{2}t^2 + r\right)$

16. $\left(\frac{1}{4}c^2 - \frac{1}{3}k^3\right)^2$

17. $(2f + 1)(2f - 1)(f - 7)$

18. $(q - 2)(q + 9)(q + 2)(q - 9)$

19. **Standardized Test Practice** What is the product of $(x + 4)(x - 4)$?

A $x^2 - 8x - 16$ **B** $x^2 + 16$ **C** $x^2 - 16$ **D** $x^2 + 8x - 16$

Answers: **1.** $a^2 + 18ab + 81b^2$ **2.** $c^2 - 10cd + 25d^2$ **3.** $64m^2 - 16mn + n^2$ **4.** $49z^2 - 49$ **5.** $4g^2 - h^2$ **6.** $64s^2 + 128st + 64t^2$ **7.** $9u^2 - 108uv + 324v^2$ **8.** $36q^2 + 4.8qr + 0.16r^2$ **9.** $x^4 + 2x^2y^3 + y^6$ **10.** $\frac{1}{9}j^4 - \frac{2}{3}j^2k^2 + k^4$ **11.** $64a^4 - 4d^2$ **12.** $16n^4 - g^4$ **13.** $38.44s^2 + 12.4su^4 + u^8$ **14.** $25 - b^{14}$ **15.** $\frac{9}{4}t^4 - r^2$ **16.** $\frac{1}{16}c^4 - \frac{1}{6}c^2k^3 + \frac{1}{9}k^6$ **17.** $4f^3 - 28f^2 - f + 7$ **18.** $q^4 - 85q^2 + 324$ **19.** C

8 Chapter Review

A Lot of Lotto?

A local charity is holding a penny lottery to raise money. Work through the problems below to find out how much the charity raised.

____________ **1.** $(3x^4)(6x^3)$

____________ **2.** Divide the answer to Exercise 1 by $9x^6$.

____________ **3.** Multiply the answer to Exercise 2 by $x^2 - 3x - 1$.

____________ **4.** Add $6x^2 + 4x$ to the answer to Exercise 3.

____________ **5.** Subtract $2x^3 + x - 5$ from your answer to Exercise 4.

____________ **6.** Multiply your answer from Exercise 5 by $x + 6$.

____________ **7.** Substitute $x = 20$ into your answer to Exercise 6. Write the result in scientific notation.

____________ **8.** Multiply your answer from Exercise 7 by 3.8×10^3. Write the result in scientific notation.

Write the final answer in standard form. ______________ pennies

How many dollars did the fund-raiser generate? _______________

Answers are located in the Answer Key.

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9-1 Factors and Greatest Common Factors *(Pages 474—479)*

When two or more numbers are multiplied to form a product, each number is a *factor* of the product. **Prime numbers** are whole numbers greater than 1 that have exactly two factors, the number itself and 1. Whole numbers greater than 1 that have more than two factors are called **composite numbers**. When a whole number is expressed as a product of factors that are all prime numbers, the expression is called the **prime factorization** of the number. A monomial is in factored form when it is expressed as the product of prime numbers and no variable has an exponent greater than 1. You can use prime factorization to find the greatest **common factor (GCF)** of two or more integers, which is the greatest number that is a factor of all the integers.

Examples

a. Find the prime factorization of $72x^2$.

$72x^2$

$= 2 \cdot 36 \cdot x \cdot x$	*The least prime factor of 72 is 2.*
$= 2 \cdot 2 \cdot 18 \cdot x \cdot x$	*The least prime factor of 36 is 2.*
$= 2 \cdot 2 \cdot 2 \cdot 9 \cdot x \cdot x$	*The least prime factor of 18 is 2.*
$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x$	*The least prime factor of 9 is 3.*

The prime factorization of $72x^2$ is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x$ or $2^3 \cdot 3^2 \cdot x^2$.

b. Find the GCF of $24a^2$ and $36a$.

Find the prime factorization of each number, then circle the common factors.

$24a^2 = (2) \cdot (2) \cdot 2 \cdot (3) \cdot (a) \cdot a$

$36a = (2) \cdot (2) \cdot (3) \cdot 3 \cdot (a)$

The GCF is the product of the common factors.

$2 \cdot 2 \cdot 3 \cdot a = 12a$

The GCF is 12a.

Practice

State whether each number is *prime* or *composite*. If the number is composite, find its prime factorization.

1. 15 **2.** 27 **3.** 23 **4.** 31 **5.** 404 **6.** 1240

Find the GCF of the given monomials.

7. 18, 50 **8.** 12, 28 **9.** 56, 126

10. $5a^3$, $20a$ **11.** $12x^2y^4$, $18y^2z$ **12.** $4c$, $16c^2$, $28b^2c^5$

13. The Classroom Ms. Yip has 32, 36, and 24 students in each of her morning classes. What is the greatest number of desks can she place in each row of desks so that no row will be partially filled when the students from each of her classes are seated?

14. **Standardized Test Practice** Which list shows all the factors of 98?

A 1, 2, 4, 7, 14, 49 **B** 1, 2, 7, 8, 14, 49

C 1, 2, 7, 18, 49, 98 **D** 1, 2, 7, 14, 49, 98

Answers: 1. composite; $3 \cdot 5$ **2.** composite; $3 \cdot 3 \cdot 3$ **3.** prime **4.** prime **5.** composite; $2 \cdot 2 \cdot 101$ **6.** composite; $2 \cdot 2 \cdot 2 \cdot 5 \cdot 31$ **7.** 2 **8.** 4 **9.** 14 **10.** $5a$ **11.** $6y^2$ **12.** $4c$ **13.** 4 desks per row **14.** D

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9-2 Factoring Using the Distributive Property *(Pages 481—486)*

A polynomial is in factored form, or **factored**, when it is expressed as the product of monomials and polynomials. You can use the Distributive Property to express a polynomial in factored form. It is also possible to use the Distributive Property to factor some polynomials containing four or more terms into the product of two polynomials. This is called **factoring by grouping**.

Examples

a. Factor $9a^2b^5 - 3ab^2 + 6ab$.

$9a^2b^5 = (3) \cdot 3 \cdot (a) \cdot a \cdot (b) \cdot b \cdot b \cdot b \cdot b$

$3ab^2 = (3) \cdot (a) \cdot (b) \cdot b$

$6ab = 2 \cdot (3) \cdot (a) \cdot (b)$ *The GCF is 3ab.*

Use the Distributive Property to express the polynomial as the product of the GCF and the remaining factor of each term.

$9a^2b^5 - 3ab^2 + 6ab$

$= 3ab(3ab^4) - 3ab(b) + 3ab(2)$

$= 3ab(3ab^4 - b + 2)$

You can check this answer by using the Distributive Property. $3ab(3ab^4 - b + 2) = 9a^2b^5 - 3ab^2 + 6ab$

b. Factor $8wy + 12xy + 10wz + 15xz$.

Use the Associative Property to group together pairs of terms that have common factors.

$8wy + 12xy + 10wz + 15xz$

$= (8wy + 12xy) + (10wz + 15xz)$

Factor each pair of terms using its GCF. The GCF of the first two terms is 4y, and the GCF of last two terms is 5z.

$= 4y(2w + 3x) + 5z(2w + 3x)$

This polynomial has two terms: 4y(2w + 3x) and 5z(2w + 3x). These terms have a common factor of 2w + 3x. Use the Distributive Property to factor this polynomial.

$= (4y + 5z)(2w + 3x)$

Check this answer by using the FOIL method.

Practice

Complete. In exercises with two blanks, both blanks represent the same expression.

1. $12x + 9y = 3(\underline{\ ?\ } + 3y)$

2. $4abc + 8abc^2 = \underline{\ ?\ }(1 + 2c)$

3. $(x^2 + 2xy) + (6kx + 12ky) = x(\underline{\ ?\ }) + 6k(\underline{\ ?\ })$

4. $(12a^2 - 20ab) + (9ay - 15by) = 4a(\underline{\ ?\ }) - 3y(\underline{\ ?\ })$

Factor each polynomial.

5. $7b^2 + 42b$

6. $15m^2n - 27mn^2$

7. $10xz^2 + 30z^6$

8. $8s^3 + 24s^2q$

9. $16g + 14gh^2$

10. $36k^5 + 24k^3 - 18k$

11. $6y^3 - 21y^2 - 4y + 14$

12. $3x^3 + x^2 + 6x + 2$

13. $4w^3 + 3wz - 8w^2 - 6z$

14. Geometry The area of a rectangle is represented by $10x^3 + 15x^2 + 4x + 6$. Its dimensions are represented by binomials in x that have prime number coefficients. What are the dimensions of the rectangle?

15. Standardized Test Practice Factor the polynomial $4wf + 8w$.

A $4(wf + 2)$ **B** $4w(f + 2)$ **C** $4w(f + 8)$ **D** $w(4f + 8)$

Answers: 1. $4x$ **2.** $4abc$ **3.** $x + 2y$ **4.** $3a - 5b$ **5.** $7b(b + 6)$ **6.** $3mn(5m - 9n)$ **7.** $10z^2(x + 3z^4)$ **8.** $8s^2(s + 3q)$ **9.** $2g(8 + 7h^2)$ **10.** $6k(6k^4 + 4k^2 - 3)$ **11.** $(2y - 7)(3y^2 - 2)$ **12.** $(3x + 1)(x^2 + 2)$ **13.** $(w - 2)(4w^2 + 3z)$ **14.** $2x + 3, 5x^2 + 2$ **15.** B

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9-3 Factoring Trinomials: $x^2 + bx + c$

(Pages 489—495)

The goal of factoring quadratic trinomials is the same as factoring monomials and polynomials using the distributive property, you want to write a multiplication problem consisting of factors of the trinomial. Sometimes a trinomial can be factored into the product of two binomials. This is essentially going from a trinomial to a FOIL problem. This process can be done through trial and error, however, that may be quite time consuming. So, it may be helpful to use the following rule to help limit your trials.

> To factor a trinomial of the form $x^2 + bx + c$, find two numbers, m and n, where the sum $m + n = b$ and the product $mn = c$. Then write the trinomial $x^2 + bx + c$ as $(x + m)(x + n)$. Always use the FOIL method to check your answer. If your binomials are correct, then the product of your binomials should be the original trinomial.

Examples

a. Factor $x^2 + 10x + 21$.

$b = 10$ and $c = 21$	
$m = 7$, $n = 3$	*Find an m and an n such that $m + n = 10$ and $mn = 21$.*
$(x + 7)(x + 3)$	*Write as $(x + m)(x + n)$.*

b. Solve the equation by factoring.

$x^2 + 5x + 4 = 0$	
$m = 4$ and $n = 1$	*$m + n = 5$, $mn = 4$*
$(x + 4)(x + 1) = 0$	*$(x + m)(x + n) = 0$*
$x + 4 = 0$ or $x + 1 = 0$	*Zero Product*
$x = -4$ or $x = 0$	*Solve for x.*

Practice

Factor each trinomial.

1. $x^2 + 3x + 2$ **2.** $x^2 - x - 56$ **3.** $x^2 + 5x - 6$ **4.** $x^2 - 7x + 12$

Solve by factoring.

5. $x^2 + 12x + 20 = 0$ **6.** $x^2 - 5x - 24 = 0$ **7.** $x^2 - 18x + 80 = 0$ **8.** $x^2 + 7x - 44 = 0$

9. **Standardized Test Practice** The area of a rectangle is given by the quadratic trinomial equation $x^2 + 6x = 27$. Use factoring and the zero property to solve for x. HINT: In measurement only positive numbers are realistic answers.

$$A = lw$$
$$27 = x^2 + 6x$$
$$0 = x^2 + 6x - 27$$

A $x = 9$ units **B** $x = 6$ units **C** $x = 3$ units **D** $x = 1$ unit

Answers: **1.** $(x + 1)(x + 2)$ **2.** $(x + 7)(x - 8)$ **3.** $(x + 6)(x - 1)$ **4.** $(x - 4)(x - 3)$ **5.** $x = -2, -10$ **6.** $x = 8, -3$ **7.** $x = 8, 10$ **8.** $x = 4, -11$ **9.** C

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9-4 Factoring Trinomials: $ax^2 + bx + c$

(Pages 495—500)

Use the guess and check strategy and the FOIL method to factor a trinomial.

Example

Factor $-22x + 6x^2 - 8$.

First, rewrite the trinomial so that the terms are in descending order. Then check for a GCF.

$-22x + 6x^2 - 8 = 6x^2 - 22x - 8$

$= 2(3x^2 - 11x - 4)$ *The GCF of the terms is 2. Use the Distributive Property.*

Now factor $3x^2 - 11x - 4$.

$3x^2 - 11x - 4$ *The product of 3 and −4 is −12.*

$3x^2 + (\underline{?} + \underline{?})x - 4$

You need to find two integers whose ***product is −12*** *and whose* ***sum is −11****.*

Factors of −12	Sum of Factors	
−3, 4	−3 + 4 = 1	no
3, −4	3 + −4 = −1	no
−1, 12	−1 + 12 = 11	no
1, −12	1 + (−12) = −11	yes

Stop listing factors when you find a pair that works.

$3x^2 - 11x - 4 = 3x^2 + [1 + (-12)]x - 4$ *Select the factors 1 and −12.*

$= 3x^2 + 1x - 12x - 4$ *Simplify.*

$= (3x^2 + 1x) + (-12x - 4)$ *Group terms that have a common monomial factor.*

$= x(3x + 1) - 4(3x + 1)$ *Factor.*

$= (x - 4)(3x + 1)$ *Use the Distributive Property.*

Therefore, $6x^2 - 22x - 8 = 2(x - 4)(3x + 1)$.

Practice

Complete.

1. $b^2 + b - 6 = (b + 3)(b - \underline{?})$

2. $a^2 + 2a - 8 = (a + \underline{?})(a - 2)$

3. $x^2 - 3x - 10 = (x - \underline{?})(x + 2)$

4. $k^2 + 9k + 18 = (k + 6)(k + \underline{?})$

5. $8g^2 - 4g - 12 = (\underline{?} + 4)(2g - 3)$

6. $5n^2 - 22n + 8 = (5n - \underline{?})(n - 4)$

Factor each trinomial.

7. $x^2 + x - 12$

8. $y^2 - 5y - 14$

9. $k^2 - 15k + 50$

10. $a^2 - 4a - 12$

11. $z^2 + 11z + 24$

12. $3s^2 + 9s - 30$

13. $2x^2 + 3x - 20$

14. $9x^2 - 18x + 5$

15. $20x^2 + 17x + 3$

16. Geometry The area of a rectangle is $(6x^2 + 7x + 2)$ square inches. Find binomial expressions to represent the dimensions of this rectangle.

17. Standardized Test Practice Factor the trinomial $v^2 + 7v + 12$.

A $(v + 7)(v + 5)$ **B** $(v + 4)(v - 3)$ **C** $(v + 3)(v + 4)$ **D** $(v + 12)(v - 5)$

Answers: 1. 2 **2.** 4 **3.** 5 **4.** 3 **5.** $4g$ **6.** 2 **7.** $(x + 4)(x - 3)$ **8.** $(y - 7)(y + 2)$ **9.** $(k - 5)(k - 10)$ **10.** $(a - 6)(a + 2)$ **11.** $(z + 8)(z + 3)$ **12.** $3(s + 5)(s - 2)$ **13.** $(2x - 5)(x + 4)$ **14.** $(3x - 1)(3x - 5)$ **15.** $(4x + 1)(5x + 3)$ **16.** $3x + 2$ by $2x + 1$ **17.** C

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9-5 Factoring Differences of Squares

(Pages 501—506)

You can use the **difference of squares** rule to factor binomials that can be written in the form $a^2 - b^2$. Sometimes the terms of a binomial have common factors. If so, the GCF should always be factored out first.

Difference of Squares	$a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$

Examples

a. Factor $b^2 - 49$.

$b^2 - 49$
$= (b)^2 - (7)^2$ — *$b \cdot b = b^2$ and $7 \cdot 7 = 49$*
$= (b - 7)(b + 7)$ — *Use the difference of squares.*

b. Factor $7g^3h^2 - 28g^5$.

$7g^3h^2 - 28g^5$ — *Check for a GCF.*
$= 7g^3(h^2 - 4g^2)$ — *GCF of $7g^3h^2$ and $28g^5$ is $7g^3$.*
$= 7g^3(h - 2g)(h + 2g)$ — *$h^2 = h \cdot h$ and $4g^2 = 2g \cdot 2g$.*

Try These Together

Factor each polynomial, if possible. If the polynomial cannot be factored, write* prime*.

1. $x^2 - 4$ **2.** $y^2 + 16$ **3.** $a^2 - 144$

HINT: Both terms of the binomial must be squares. Also, the sum of two squares cannot be factored using the difference of two squares rule.

Practice

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

4. $9b^2 - 25$ **5.** $4c^2 - 7$ **6.** $4z^2 - 16$

7. $9z^2 - 19$ **8.** $-25 + 81x^2$ **9.** $v^2q^2 - 0.49r^2$

10. $a^2b^2 - 0.36c^2$ **11.** $a^2b^2c^2 - x^2y^2z^2$ **12.** $x^2y^2 - 3$

13. $t^7 - t^3u^4$ **14.** $x^5 - x^3y^2$ **15.** $64k^2 - 24$

16. Factor $\frac{4}{25}x^2 - \frac{9}{16}y^2$. (*Hint:* Find fractions that when squared equal $\frac{4}{25}$ and $\frac{9}{16}$.)

17. **Standardized Test Practice** Factor $x^2 - (y + z)^2$.

A $(x + y + z)(x - y + z)$ **B** $(x + y + z)(x + y - z)$

C $(x + y + z)(x - y - z)$ **D** $(x + y - z)(x - y + z)$

Answers: 1. $(x + 2)(x - 2)$ **2.** prime **3.** $(a - 12)(a + 12)$ **4.** $(3b + 5)(3b - 5)$ **5.** prime **6.** $4(z + 2)(z - 2)$ **7.** prime **8.** $(9x + 5)(9x - 5)$ **9.** $(vq + 0.7r)(vq - 0.7r)$ **10.** $(ab - 0.6c)(ab + 0.6c)$ **11.** $(abc - xyz)(abc + xyz)$ **12.** prime **13.** $t^3(t + u)(t - u)(t^2 + u^2)$ **14.** $x^3(x - y)(x + y)$ **15.** $8(8k^2 - 3)$ **16.** $\left(\frac{2}{5}x - \frac{3}{4}y\right)\left(\frac{2}{5}x + \frac{3}{4}y\right)$ **17.** C

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9-6 Perfect Squares and Factoring *(Pages 508—514)*

Products of the form $(a + b)^2$ and $(a - b)^2$ are called perfect squares, and their expressions are called **perfect square trinomials**.

Perfect Square Trinomials	$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$
Factoring a Perfect Square Trinomial	You can check whether a trinomial is a perfect square trinomial by checking that the following conditions are satisfied. • The first term is a perfect square. • The third term is a perfect square. • The middle term is either 2 or −2 times the product of the square root of the first term and the square root of the third term.

Example

Determine whether $4x^2 + 4xy + y^2$ is a perfect square trinomial. If so, factor it.

Check each of the following.

- *Is the first term a perfect square?* $4x^2 \stackrel{?}{=} (2x)^2$ *yes*
- *Is the last term a perfect square?* $y^2 \stackrel{?}{=} (y)^2$ *yes*
- *Is the middle term twice the product of 2x and y?* $4xy = 2(2x)(y)$ *yes*

So, $4x^2 + 4xy + y^2$ *is a perfect square trinomial.*

$$4x^2 + 4xy + y^2 = (2x)^2 + 2(2x)(y) + (y)^2$$
$$= (2x + y)^2$$

Practice

Determine whether each trinomial is a perfect square trinomial. If so, factor it. If the polynomial cannot be factored write *prime*.

1. $m^2 - 6m + 9$ **2.** $x^2 + 10x + 25$ **3.** $t^2 - 14t + 49$

4. $x^2 + 3x + 4$ **5.** $y^2 - 12y + 36$ **6.** $k^2 - 22k + 121$

Factor each polynomial. If the polynomial cannot be factored write *prime*.

7. $x^2 + 16x + 64$ **8.** $2q^2 + 30q - 8$ **9.** $x^2 + 3x + 9$

10. $4m^2 + 20m + 25$ **11.** $100h^2 - 9$ **12.** $4z^3 - 16z^2 + 16z$

13. $3x^2 + 24x + 48$ **14.** $n^2 + 1.8n + 0.81$ **15.** $7x^2 - 5.6x + 1.12$

16. Factor $\frac{1}{9}y^2 + 4y + 36$. (Hint: Check to see if the trinomial is a perfect square trinomial.)

17. **Standardized Test Practice** Factor the trinomial $5a^2 + 30a + 45$.

A $(5a + 3)^2$ **B** $5(a + 3)$ **C** $(a + 3)^2$ **D** $5(a + 3)^2$

Answers: 1. $(m - 3)^2$ **2.** $(x + 5)^2$ **3.** $(t - 7)^2$ **4.** prime **5.** $(y - 6)^2$ **6.** $(k - 11)^2$ **7.** $(x + 8)^2$ **8.** $2(q^2 + 15q - 4)$ **9.** prime **10.** $(2m + 5)^2$ **11.** $(10h - 3)(10h + 3)$ **12.** $4z(z - 2)^2$ **13.** $3(x + 4)^2$ **14.** $(n + 0.9)^2$ **15.** $7(x - 0.4)^2$ **16.** $\left(\frac{1}{3}y + 6\right)^2$ **17.** D

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9 Chapter Review

Rewind / Fast Forward

"Rewind" by factoring each polynomial completely. Then cross off the answer in the right column. "Fast forward" by multiplying your answer to check it. The letters that are left will spell an outdated technology.

Rewind

1. $18x - 9xy$

2. $4x^3 + 6x$

3. $x^2 - 64$

4. $x^2 - 16$

5. $2x^2 - 32$

6. $x^2 + 6x + 8$

7. $x^2 - 6x + 8$

8. $x^2 + x - 12$

9. $x^2 - x - 12$

10. $x^2 + 2x + xy + 2y$

11. $xy + 4y - x^2 - 4x$

12. $4x + 8y + x^2 + 2xy$

Fast Forward

$(x + 2)(y + 1)$	E
$(x - 3)(x + 4)$	N
$(x - 4)(x + 3)$	D
$(x - 4)(x - 2)$	W
$(x - 4)(x - 4)$	I
$(x - 4)(x + 4)$	N
$(x - 8)(x - 8)$	G
$(x - 8)(x + 8)$	S
$(x + 2)(x + 4)$	P
$(x + 2)(y - x)$	H
$(x + 2)(x + y)$	A
$(x + 4y)(x - 2)$	T
$(x + 4)(y - x)$	I
$(x + 4)(x + 2y)$	O
$2(2x^2 + 3x)$	T
$2(x - 4)(x + 4)$	S
$2(x + 8)(x - 8)$	R
$2(x^2 + 16)$	A
$2x(2x^2 + 3)$	O
$3x(6 - 3y)$	C
$9x(2 - y)$	E
$9x(2 - xy)$	K

Answers are located in the Answer Key.

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10-1 Graphing Quadratic Functions *(Pages 524–530)*

Quadratic Function	A **quadratic function** is a function that can be written in the form $f(x) = ax^2 + bx + c$ where $a \neq 0$. The graph of a quadratic function is a **parabola**. a is positive: parabola opens upward and vertex is a **minimum** point of the function a is negative: parabola opens downward and vertex is a **maximum** point of the function
Axis of Symmetry	Parabolas have **symmetry**, which means that when they are folded in half on a line that passes through the vertex, each half matches the other exactly. This line is called the **axis of symmetry**. Axis of symmetry for graph of $y = ax^2 + bx + c$, where $a \neq 0$, is $x = -\frac{b}{2a}$.

Example

Given the equation $y = x^2 - 2x + 3$, find the equation for the axis of symmetry, the coordinates of the vertex, and graph the equation.

In the equation $y = x^2 - 2x + 3$, a = 1 and b = −2. Substitute these values into the equation for the axis of symmetry.

axis of symmetry: $x = -\frac{b}{2a}$

$$x = -\frac{-2}{2(1)} \text{ or } 1$$

Since you know the line of symmetry, you know the x-coordinate for the vertex is 1.

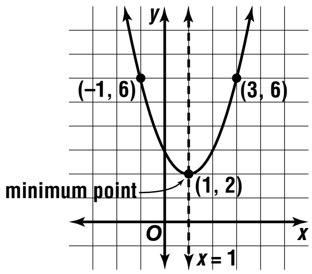
$y = x^2 - 2x + 3$

$y = 1 - 2 + 3$ *or 2 Replace x with 1.*

Coordinates of vertex: *(x, y) = (1, 2)*

Graph the vertex and the line of symmetry, x = 1.

Using the equation, you can find another point on the graph. The point (3, 6) is 2 units right of the axis of symmetry. Since the graph is symmetrical, if you go 2 units left of the axis and 6 units up, you will find a third point on the graph, (−1, 6). Repeat this for several other points. Then sketch the parabola.



Practice

Write the equation of the axis of symmetry and find the coordinates of the vertex of the graph of each equation. State if the vertex is a *maximum* or *minimum*. Then graph the equation.

1. $y = x^2 + 10x + 24$

2. $y = -x^2 - 6x + 7$

3. $y = x^2 - 2x + 1$

4. $y = -3x^2 - 18x - 24$

5. $y = x^2 + x - 6$

6. $y = 2x^2 - 18$

7. $y = -x^2 + 1$

8. $y = 3x^2$

9. $y = x^2 + 2x + 1$

10. **Standardized Test Practice** What is the vertex of the graph of $y = 1 - 4x + 2x^2$?

A (2, 1) **B** (−2, 17) **C** (1, −1) **D** (−1, 7)

Answers: 1–9. For graphs, see Answer Key. **1.** $x = -5$; (−5, −1); minimum **2.** $x = -3$; (−3, 16); maximum **3.** $x = 1$; (1, 0); minimum **4.** $x = -3$; (−3, 3); maximum **5.** $x = -0.5$; (−0.5, −6.25); minimum **6.** $x = 0$; (0, −18); minimum **7.** $x = 0$; (0, 1); maximum **8.** $x = 0$; (0, 0); minimum **9.** $x = -1$; (−1, 0); minimum **10.** C

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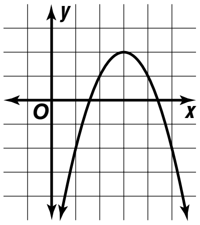
10-2 Solving Quadratic Equations by Graphing *(Pages 533–538)*

The solutions of a quadratic equation are called the **roots** of the equation. You can find the real number roots by finding the x-intercepts or **zeros** of the related quadratic function. Quadratic equations can have two distinct real roots, one distinct root, or no real roots. These roots can be found by graphing the equation to see where the parabola crosses the x-axis.

Examples

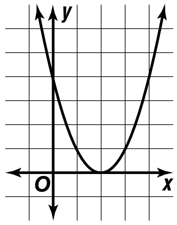
Describe the real roots of the quadratic equations whose related functions are graphed below.

a.



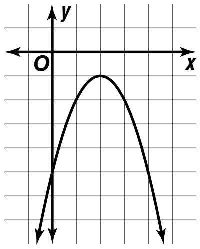
The parabola crosses the x-axis twice. One root is between 1 and 2, and the other is between 4 and 5.

b.



Since the vertex of the parabola lies on the x-axis the function has one distinct root, 2.

c.

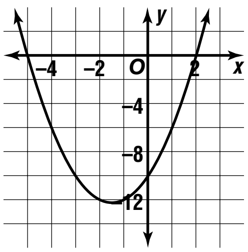


This parabola does not intersect the x-axis, so there are no real roots. The solution set is ∅.

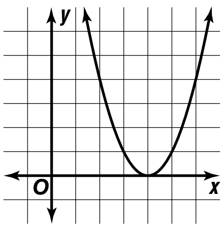
Practice

State the real roots of each quadratic equation whose related function is graphed below.

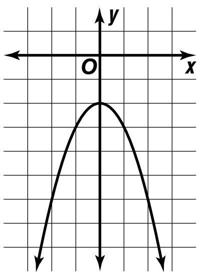
1.



2.



3.



Solve each equation by graphing. If integral roots cannot be found, state the consecutive integers between which the roots lie.

4. $x^2 + 2x - 3 = 0$

5. $-m^2 + 8m - 16 = 0$

6. $-g^2 + 4g - 5 = 0$

7. $4k^2 - 8k + 4 = 0$

8. $h^2 - 3 = 0$

9. $n^2 - 4n + 6 = 0$

10. $w^2 + 2w = 0$

11. $-v^2 + 6v - 7 = 0$

12. $t^2 - 4 = 0$

13. **Standardized Test Practice** The real roots of a quadratic equation correspond to the __?__ of the graph of the related function.

A x-intercepts **B** y-intercepts **C** vertex **D** maximum

Answers: **1.** −5, 2 **2.** 4 **3.** ∅ **4.** −3, 1 **5.** 4 **6.** ∅ **7.** 1 **8.** between 1 and 2; between −1 and −2 **9.** ∅ **10.** −2, 0 **11.** between 1 and 2; between 4 and 5 **12.** −2, 2 **13.** A

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10-3 Solving Quadratic Equations by Completing the Square *(Pages 539–544)*

You can solve some quadratic equations by taking the square root of each side. To do so, the quadratic expression on one side of the equation must be a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, use the method called **completing the square**.

Completing the Square	To complete the square for a quadratic expression of the form $x^2 + bx$, follow the steps below. 1. Find $\frac{1}{2}$ of b, the coefficient of x. 2. Square the result of step 1. 3. Add the result of step 2 to $x^2 + bx$, the original expression.

Examples

a. Find the value of c that makes $x^2 + 12x + c$ a perfect square.

1. *Find $\frac{1}{2}$ of 12.* $\frac{12}{2} = 6$
2. *Square the result of step 1.* $6^2 = 36$
3. *Add the result of step 2 to $x^2 + 12x$.* $x^2 + 12x + 36$

So, $c = 36$.
Notice that $x^2 + 12x + 36 = (x + 6)^2$.

b. Solve $x^2 + 16x - 10 = 0$ by completing the square.

Notice that $x^2 + 16x - 10$ is not a perfect square.

$x^2 + 16x - 10 = 0$

$x^2 + 16x = 10$ — *Add 10 to each side.*

$x^2 + 16x + 64 = 74$ — *Since $\left(\frac{16}{2}\right)^2$ is 64, add 64 to each side.*

$(x + 8)^2 = 74$ — *Factor $x^2 + 16x + 64$.*

$x + 8 = \pm\sqrt{74}$ — *Take the square root of each side.*

$x = -8 \pm \sqrt{74}$

Solution set: $\{-8 + \sqrt{74}, -8 - \sqrt{74}\}$

Practice

Find the value of c that makes each trinomial a perfect square.

1. $y^2 + 8y + c$ **2.** $a^2 + 6a + c$ **3.** $x^2 + 10x + c$

4. $x^2 + 9x + c$ **5.** $s^2 + 11s + c$ **6.** $z^2 + 7z + c$

Solve each equation by completing the square. Leave irrational roots in simplest radical form.

7. $x^2 + 8x + 12 = 0$ **8.** $y^2 + 6y - 15 = 0$ **9.** $z^2 + 12z - 25 = 0$

10. $a^2 + 14a - 18 = 0$ **11.** $x^2 + 10x + 16 = 0$ **12.** $x^2 + 18x + 17 = 0$

13. **Standardized Test Practice** Which expression shows the solutions of $x^2 + 16x + 32 = 0$?

A $8 + 4\sqrt{2}$ **B** $-8 + 4\sqrt{2}$ **C** $8 \pm 4\sqrt{2}$ **D** $-8 \pm 4\sqrt{2}$

Answers: 1. 16 **2.** 9 **3.** 25 **4.** 20.25 **5.** 30.25 **6.** 12.25 **7.** −2, −6 **8.** $-3 \pm 2\sqrt{6}$ **9.** $-6 \pm \sqrt{61}$ **10.** $-7 \pm \sqrt{67}$ **11.** −2, −8 **12.** −1, −17 **13.** D

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10-4 Solving Quadratic Equations by Using the Quadratic Formula *(Pages 546–552)*

You can use the quadratic formula to solve any quadratic equation involving any variable.

The Quadratic Formula	The solutions of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example

Use the Quadratic Formula to solve $x^2 - 2x - 5 = 0$.

In the equation $x^2 - 2x - 5 = 0$, a = 1, b = −2, and c = −5.
Substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = \frac{2 + \sqrt{24}}{2} \text{ or } x = \frac{2 - \sqrt{24}}{2}$$

$x \approx 3.45$ $\quad$ $x \approx -1.45$ $\quad$ *Use a calculator.*

The solutions are approximately 3.45 and −1.45.

Practice

Solve each equation by using the Quadratic Formula. Approximate irrational roots to the nearest hundredth.

1. $x^2 + 6x + 8 = 0$
2. $n^2 - 12n + 32 = 0$
3. $c^2 + 4c + 8 = 0$
4. $p^2 + 4p - 1 = 0$
5. $d^2 - 2d - 15 = 0$
6. $5h^2 + 4h + 4 = 0$
7. $3e^2 - 6e + 3 = 0$
8. $2m^2 + 8m + 2 = 0$
9. $g^2 - 3g + 2 = 0$
10. $4k^2 + 2k + 3 = 0$
11. $3f^2 - 11f - 4 = 0$
12. $4v^2 + 12v + 9 = 0$
13. $x^2 - 12x = -27$
14. $3x^2 + 6x = 1$
15. $3x - 1 = -x^2$
16. $2x(x + 1) = -5$
17. $x^2 = 2(4x - 1)$
18. $2(x^2 + 3) = 3x$

19. Automotive Sales Mark decided that the price of a car tire is a quadratic function of the radius of the tire. He modeled this using the equation $p = -r^2 + 36r - 255$, where p is the price of the tire in dollars and r is the radius of the tire in inches. Find the price that the model predicts for a tire of radius 14 inches. Then find the price the model predicts for a tire of radius 16 inches.

20. **Standardized Test Practice** For a certain quadratic equation, the value of $b^2 - 4ac$ is −8. How many real number roots does the equation have?

A 3 roots **B** 2 roots **C** 1 root **D** 0 roots

Answers: 1. −4, −2 **2.** 4, 8 **3.** no real roots **4.** −4.24; 0.24 **5.** −3, 5 **6.** no real roots **7.** 1 **8.** −3.73, −0.27 **9.** 1, 2 **10.** no real roots **11.** $-\frac{1}{3}$, 4 **12.** −1.5 **13.** 3, 9 **14.** −2.15, 0.15 **15.** −3.3, 0.3 **16.** no real roots **17.** 0.26, 7.74 **18.** none **19.** \$53; \$65 **20.** D

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10-5 Exponential Functions *(Pages 554–560)*

Exponential Function	An exponential function is a function that can be described by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$.

You can use ordered pairs to graph exponential functions. When you've graphed enough ordered pairs, connect the points to form a smooth curve. The y-intercept of an exponential function is the y-coordinate of the point at which the graph crosses the y-axis.

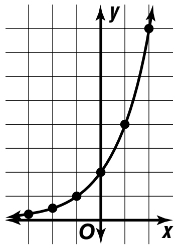
Examples

a. Graph the equation $y = 2^{x+1}$ and state the y-intercept.

Make a table of values and then graph the function.

x	−3	−2	−1	0	1	2
y	0.25	0.5	1	2	4	8

The y-intercept is 2.



b. Determine whether the data in the table display exponential behavior.

x	3	5	7	9
y	3	9	27	91

The domain values are at regular intervals of 2.

3 → 9 → 27 → 91

×3 ×3 ×3

Since the domain values are at regular intervals and the range values have a common factor, the data are probably exponential.

Practice

Graph each function. State the y-intercept.

1. $y = 2^x$

2. $y = 2^{x-3}$

3. $y = 2^x - 3$

4. $y = \left(\frac{1}{2}\right)^x$

5. $y = \left(\frac{1}{2}\right)^{x+1}$

6. $y = \left(\frac{1}{2}\right)^x + 1$

Determine whether each set of data displays exponential behavior.

7.

x	5	10	15	20
y	3	6	9	12

8.

x	2	4	6	8
y	5	25	125	625

9.

x	4	5	6	7
y	40	35	30	25

10.

x	10	20	30	40
y	64	32	16	8

11. **Standardized Test Practice** Compare the graphs of $y = 2^x$ and $y = 2^x + 1$.

A The graph of $y = 2^x$ is steeper than the graph of $y = 2^x + 1$.
B The graph of $y = 2^x + 1$ is steeper than the graph of $y = 2^x$.
C The graph of $y = 2^x + 1$ is the graph of $y = 2^x$ translated 1 unit up.
D The graph of $y = 2^x + 1$ is the graph of $y = 2^x$ translated 1 unit down.

Answers: **1–6.** For graphs, see Answer Key. **1.** 1 **2.** $\frac{1}{8}$ **3.** −2 **4.** 1 **5.** $\frac{1}{2}$ **6.** 2 **7.** no **8.** yes **9.** no **10.** yes **11.** C

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10-6 Growth and Decay *(Pages 561–565)*

General Equation for Exponential Growth	$A = C(1 + r)^t$ in which the initial amount C increases by the same percent r over a given period of time t.
General Equation for Exponential Decay	$A = C(1 - r)^t$ in which the initial amount C decreases by the same percent r over a given period of time t.
Compound Interest Equation	$A = P\left(1 + \frac{r}{n}\right)^{nt}$ where A = amount of the investment over a period of time, P = principal (initial amount of investment), r = annual rate of interest expressed as a decimal, n = number of times the interest is compounded each year, and t = number of years (may be expressed as a fraction) the money is invested.

Example

If a city with a population of 125,000 is decreasing at a rate of 1.15% per year, what will its population be after 10 years?

$A = C(1 - r)^t$ — *General equation for exponential decay.*

$A = 125{,}000(1 - 0.0115)^{10}$ — *$C = 125{,}000$, $r = 0.0115$, and $t = 10$.*

$A \approx 111{,}347$ — *In ten years the population will be about 111,347.*

Practice

Determine whether each exponential equation represents growth or decay.

1. $y = 20(0.85)^x$ **2.** $y = 20(1.025)^x$ **3.** $y = 20(0.682)^x$

4. Finance Determine the final amount for each investment.

- **a.** \$500 invested at 7.5% per year compounded monthly for 2 years
- **b.** \$500 invested at 7.5% per year compounded yearly for 2 years
- **c.** \$500 invested at 6.25% per year compounded daily for 3 years
- **d.** \$500 invested at 6.25% per year compounded monthly for 3 years
- **e.** \$500 split into two investments: \$400 invested at 8% per year compounded quarterly for 2 years and \$100 invested at 10.75% per year compounded yearly for 1 year

5. Standardized Test Practice Due to decline in industry in a particular city, the enrollment at the local high school is also declining. Since 1995, the school lost students at an annual rate of 1.95%. Given that the enrollment in 1995 was 1020 students, which equation can be used to find out what the enrollment will be in the year 2015 if the school continues to loose students at the same rate?

A $A = 1020(1 - 0.195)^{20}$ **B** $A = 1020(1 - 0.195)^{15}$

C $A = 1020(1 - 0.0195)^{20}$ **D** $A = 1020(1 - 0.0195)^{15}$

Answers: 1. decay **2.** growth **3.** decay **4a.** \$580.65 **4b.** \$577.81 **4c.** \$603.11 **4d.** \$602.82 **4e.** \$579.41 **5.** C

10-7 Geometric Sequences *(Pages 567–572)*

A sequence of numbers such as 2, 4, 8, 16, 32,... forms a **geometric sequence**. Each number in a geometric sequence increases or decreases by a common factor r, called the **common ratio**.

Geometric Sequence	A geometric sequence can be written in the form of a, ar, ar^2, ar^3, ar^4,... where $r \neq 0$ or 1.
Calculating the *n*th term	The nth term of a geometric series with initial term a_1 and common ratio r is calculated by $a_n = a_1 \cdot r^{n-1}$.

Examples

a. Determine if the sequence is geometric.

−1, 3, −9, 27,...

$\frac{27}{-9} = -3$ — *Find the common ratio.*

$(-1)(-3), 3(-3)$ — *Test for each element.*

Yes, the sequence is geometric.

b. Find the 12th term of the sequence 4, 16, 64, 256,....

$a_n = a_1 \cdot r^{n-1}$ — *Formula for the nth term.*

$\frac{16}{4} = 4$ — *Find the common ratio.*

$a_{12} = 4 \cdot 4^{12-1}$ — *Substitute.*

$a_{12} = 4 \cdot 4^{11}$ — *Simplify.*

$a_{12} = 4 \cdot 4{,}194{,}304$ — *Multiply.*

$a_{12} = 16{,}777{,}216$ — *Multiply.*

Practice

Find the next three terms in each sequence.

1. $\frac{1}{2}, -1\frac{1}{2}, 4\frac{1}{2}, -13\frac{1}{2}$,...

2. −2, −15, −112.5, −843.75,...

3. 1, 6, 36, 216,...

4. 56, 28, 14, 7,...

5. 64, −48, 36, −27,...

6. 2, 22, 242, 2662,...

7. Find the 10th term of the geometric sequence whose first term is 3 and common ratio is −2.

8. Find the 9th term of 25, 12.5, 6.25, 3.125,....

9. A geometric sequence begins with 5 and has a common ratio of $-\frac{1}{4}$. What is the sequence's 4th term?

10. **Standardized Test Practice** The 15th term of a geometric sequence is 32,768. Which choice shows the possible first term and the possible common ratio?

A 2, 2 **B** 4, 3 **C** 15, 4 **D** 8, −4

Answers: **1.** $40\frac{1}{2}, -121\frac{1}{2}, 364\frac{1}{2}$ **2.** −6328.125, −47460.9375, −355957.03125 **3.** 1296, 7776, 46656 **4.** 3.5, 1.75, 0.875 **5.** 20.25, −15.1875, 11.390625 **6.** 29282, 322102, 3543122 **7.** −1536 **8.** 0.09765625 **9.** −0.078125 **10.** A

10

Chapter Review

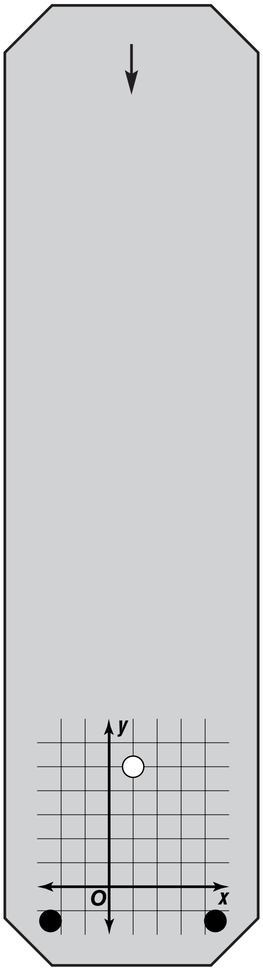
Quadratic Mini Golf

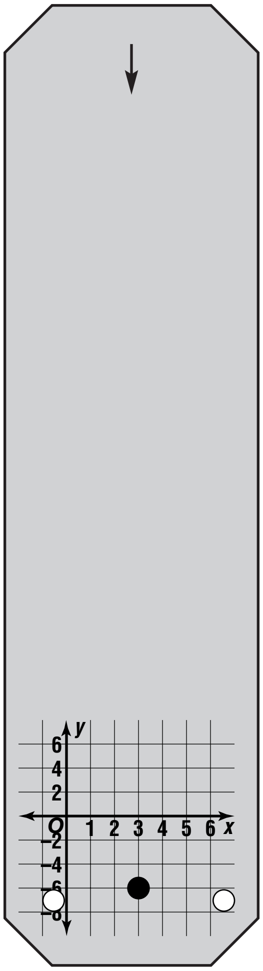
Below is a map of four holes on a miniature golf course. The object of this mini-golf game is to use the graph of a quadratic equation to build a bumper around the holes that will make it easier to sink your putts. You want to putt your golf ball into a black hole. If your ball goes into a white hole, you lose it. The distances shown on the golf course below are units that correspond to the units on a coordinate grid. From the four equations below, pick the one whose graph will make your putt easier for each hole.

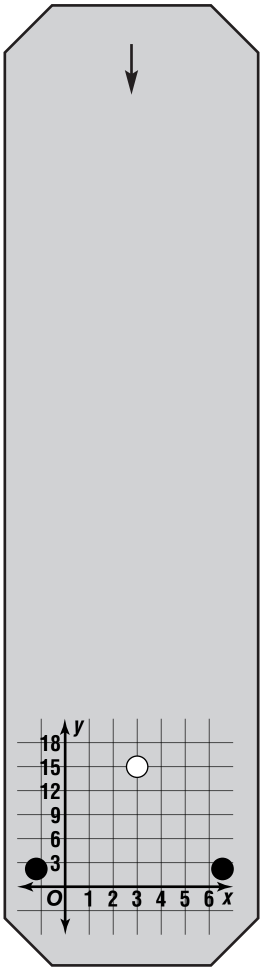
a. $y = 4x^2 - 8x + 4$ **b.** $y = -x^2 + 2x + 5$

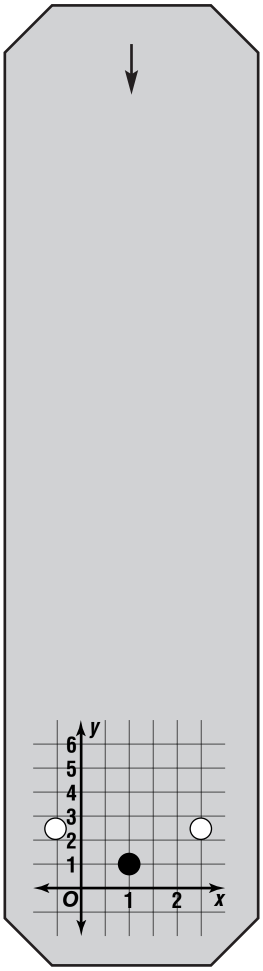
c. $y = x^2 - 6x + 1$ **d.** $y = -2x^2 + 12x$

Hole 1	**Hole 2**	**Hole 3**	**Hole 4**
Equation:	Equation:	Equation:	Equation:
______________	______________	______________	______________









Answers are located in the Answer Key.

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11-1 Simplifying Radical Expressions

(Pages 586–592)

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
Quotient Property of Square Roots	For any numbers a and b where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
Rationalizing the Denominator	Use the following steps (called **rationalizing the denominator**) to remove a radical from the denominator of a fraction. $\frac{\sqrt{a}}{\ } = \frac{\sqrt{a}}{\ } \cdot \frac{\sqrt{b}}{\ }$ or $\frac{\sqrt{ab}}{\ }$, where $a \geq 0$ and $b > 0$
Conjugates	The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called **conjugates** of each other. You can use the fact that $\left(a\sqrt{b} + c\sqrt{d}\right)\left(a\sqrt{b} - c\sqrt{d}\right) = a^2b - c^2d$ to produce a product without radicals.
Radicals and Absolute Values	When finding the principal square root of an expression containing variables, be sure that the result is not negative. Use absolute value to ensure nonnegative results where necessary. $\sqrt{x^2} = \lvert x \rvert$ $\sqrt{x^3} = x\sqrt{x}$ $\sqrt{x^4} = x^2$ $\sqrt{x^5} = x^2\sqrt{x}$ $\sqrt{x^6} = \lvert x^3 \rvert$
Simplest Radical Form	A radical expression is in simplest form when the following three conditions have been met. 1. No **radicands** (the expressions under the radical signs) have perfect square factors other than 1. 2. No radicands contain fractions. 3. No radicals appear in the denominator of a fraction.

Try These Together

Simplify. Leave in radical form and use absolute value symbols when necessary.

1. $\sqrt{84}$ **2.** $\sqrt{90}$ **3.** $\sqrt{125x^2}$

HINT: Find the prime factorization of the number under the radical sign, then simplify the perfect squares. For example, $\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$.

Practice

Simplify. Leave in radical form and use absolute value symbols when necessary.

4. $\sqrt{156}$ **5.** $\sqrt{270}$ **6.** $\sqrt{800}$ **7.** $\sqrt{44b^5}$

8. $\sqrt{\frac{b^2}{49}}$ **9.** $\sqrt{\frac{36}{z^3}}$ **10.** $\sqrt{\frac{16}{x^2}}$ **11.** $2x\sqrt{8} \cdot 8x\sqrt{7}$

12. Standardized Test Practice Simplify $\left(y + \sqrt{5}\right)\left(y - \sqrt{5}\right)$.

A $y^2 - 2\sqrt{5} - 5$ **B** $y^2 - 2\sqrt{5}$ **C** $y^2 - \sqrt{10}$ **D** $y^2 - 5$

Answers: **1.** $2\sqrt{21}$ **2.** $3\sqrt{10}$ **3.** $5|x|\sqrt{5}$ **4.** $2\sqrt{39}$ **5.** $3\sqrt{30}$ **6.** $20\sqrt{2}$ **7.** $2b^2\sqrt{11b}$ **8.** $\frac{|b|}{7}$ **9.** $\frac{6\sqrt{z}}{z^2}$ **10.** $\frac{4}{|x|}$ **11.** $32x^2\sqrt{14}$ **12.** D

Chapter 11

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11-2 Operations With Radical Expressions

(Pages 593–597)

Radical expressions in which the radicands are alike can be added or subtracted in the same way that monomials are added or subtracted. If the radicals in a radical expression are not in simplest form, simplify them first. Then use the distributive property wherever possible to further simplify the expression. You can also use the FOIL method to multiply radical expressions with different radicands.

Examples

a. Simplify $3\sqrt{11} + 2\sqrt{7} - 5\sqrt{7} + 9\sqrt{11}$.

$3\sqrt{11} + 2\sqrt{7} - 5\sqrt{7} + 9\sqrt{11}$
$= (2 - 5)\sqrt{7} + (3 + 9)\sqrt{11}$
$= -3\sqrt{7} + 12\sqrt{11}$

b. Simplify $2\sqrt{12} + 4\sqrt{3}$.

$2\sqrt{12} + 4\sqrt{3} = 2\left(\sqrt{2^2}\sqrt{3}\right) + 4\sqrt{3}$
$= 2\left(2\sqrt{3}\right) + 4\sqrt{3}$
$= 4\sqrt{3} + 4\sqrt{3}$
$= 8\sqrt{3}$

Try These Together

Simplify.

1. $3\sqrt{6} + \sqrt{6}$ **2.** $14\sqrt{5} - 2\sqrt{5}$ **3.** $4\sqrt{18} + 2\sqrt{8}$

HINT: Make sure the radicals are in simplest form first, then use the distributive property to further simplify the expression.

Practice

Simplify.

4. $3\sqrt{7} + 4\sqrt{7} - 3\sqrt{7}$ **5.** $4\sqrt{13} - 2\sqrt{13} + 6\sqrt{13}$ **6.** $2\sqrt{7x} + 3\sqrt{7x}$

7. $5\sqrt{3a} + 4\sqrt{3a}$ **8.** $2\sqrt{c} + 6\sqrt{c} - 3\sqrt{c}$ **9.** $4\sqrt{8} + 3\sqrt{8} + 2\sqrt{8}$

10. $\sqrt{16} + \sqrt{24} + \sqrt{9}$ **11.** $\sqrt{20} + \sqrt{28} - \sqrt{25}$ **12.** $\sqrt{30} + \sqrt{40} - \sqrt{12}$

13. $2\sqrt{2} + 2\sqrt{\frac{1}{2}}$ **14.** $6\sqrt{50} + 3\sqrt{3}$ **15.** $2\sqrt{72} - 3\sqrt{50}$

16. Sailing Before modern navigational tools, old sailing ships would have a small platform on top of the front mast called a crow's nest. Sailors in the crow's nest could see land or other ships that were much farther away than the sailors on deck. The equation $d = \sqrt{\frac{3h}{2}}$ can be used to find the distance d in miles a person h feet high above the water can see. If the deck was 20 feet above the water and the crow's nest was another 32 feet above the deck, about how much farther could sailors in the crow's nest see than those on deck? Round to the nearest tenth of a mile.

17. Standardized Test Practice Simplify $6\sqrt{3x} + 4\sqrt{3x} - \sqrt{3x}$.

A $9\sqrt{3x}$ **B** $9\sqrt{x}$ **C** $10\sqrt{3x}$ **D** $27\sqrt{x}$

Answers: 1. $4\sqrt{6}$ **2.** $12\sqrt{5}$ **3.** $16\sqrt{2}$ **4.** $4\sqrt{7}$ **5.** $8\sqrt{13}$ **6.** $5\sqrt{7x}$ **7.** $9\sqrt{3a}$ **8.** $5\sqrt{c}$ **9.** $18\sqrt{2}$ **10.** $7 + 2\sqrt{6}$

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11-3 Radical Equations *(Pages 598–603)*

Equations that contain radicals with variables in the radicand are called **radical equations**. To solve a radical equation, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

Examples

a. Solve $\sqrt{x} - 4 = -2$.

$\sqrt{x} - 4 = -2$	
$\sqrt{x} = 2$	*Add 4 to each side.*
$(\sqrt{x})^2 = 2^2$	*Square each side.*
$x = 4$	*Evaluate.*

Check the solution.

$\sqrt{x} - 4 = -2$

$\sqrt{4} - 4 = -2$

$2 - 4 = -2$

$-2 = -2$

b. Solve $\sqrt{2x - 4} = x - 2$.

$\sqrt{2x - 4} = x - 2$	
$(\sqrt{2x - 4})^2 = (x - 2)^2$	*Square each side.*
$2x - 4 = x^2 - 4x + 4$	*Simplify.*
$0 = x^2 - 6x + 8$	*Subtract.*
$0 = (x - 4)(x - 2)$	*Factor.*
$x = 4$ or $x = 2$	*Use the Zero Product Property.*

Check your solutions.

$\sqrt{2x - 4} = x - 2$	$\sqrt{2x - 4} = x - 2$
$\sqrt{2(4) - 4} = 4 - 2$	$\sqrt{2(2) - 4} = 2 - 2$
$\sqrt{4} = 2$	$\sqrt{0} = 0$
$2 = 2$	$0 = 0$

Try These Together

Solve each equation. Check your solution

1. $\sqrt{x} = \sqrt{3}$ **2.** $\sqrt{y} = \sqrt{6}$ **3.** $\sqrt{a} = 3\sqrt{5}$

HINT: Isolate the radical and then square both sides to eliminate the radical.

Practice

Solve each equation. Check your solution.

4. $\sqrt{y} - 4 = 0$ **5.** $\sqrt{c} + 4 = 0$ **6.** $\sqrt{s} + 2 = 0$

7. $\sqrt{3t + 1} = 6$ **8.** $\sqrt{2x - 2} = 4$ **9.** $16 - 5\sqrt{2y} = 1$

10. $3 + 2\sqrt{m} = 7$ **11.** $5 + 3\sqrt{4x} = 8$ **12.** $\sqrt{a - 3} = a - 5$

13. $\sqrt{x + 6} = x + 4$ **14.** $3 + \sqrt{a - 3} = 6$ **15.** $15 + \sqrt{y - 12} = 33$

16. Physics The period T of a pendulum is the time it takes to make one complete swing. At the Earth's surface, $T = 2\pi\sqrt{\frac{L}{32}}$, where T is measured in seconds and L is the length of the pendulum in feet. To the nearest tenth, how long is a pendulum with a period of 2 seconds?

17. Standardized Test Practice Solve the equation $\sqrt{x + 7} = 2\sqrt{2}$.

A 1 **B** 2 **C** 7 **D** 8

Answers: 1. 3 **2.** 6 **3.** 45 **4.** 16 **5.** no solution **6.** no solution **7.** $11\frac{2}{3}$ **8.** 9 **9.** $4\frac{1}{2}$ **10.** 4 **11.** $\frac{1}{4}$ **12.** 7 **13.** −2 **14.** 12 **15.** 336 **16.** 3.2 ft **17.** A

11-4 The Pythagorean Theorem *(Pages 605–610)*

You can use the **Pythagorean Theorem** to find the length of any side of a right triangle if the lengths of the other two sides are known. A corollary to this theorem can be used to determine whether a triangle is a right triangle.

Pythagorean Theorem	If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.
Corollary to the Pythagorean Theorem	If c is the measure of the longest side of a triangle and $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

Examples

a. Find the length of leg b of a right triangle if the length of leg a is 24 and the length of the hypotenuse is 30.

$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$30^2 = 24^2 + b^2$	*Substitute.*
$900 = 576 + b^2$	*Evaluate.*
$324 = b^2$	*Subtract 576 from each side.*
$\sqrt{324} = b$	*Take square root of each side.*
$18 = b$	*Simplify.*

The length of leg b is 18 units.

b. The lengths of the sides of a triangle are 14 m, 12 m, and 10 m. Is the triangle a right triangle?

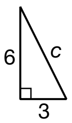
$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$14^2 \stackrel{?}{=} 12^2 + 10^2$	*Substitute.*
$196 \stackrel{?}{=} 144 + 100$	*Evaluate.*
$196 \neq 244$	*Add.*

The triangle is not a right triangle.

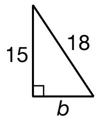
Practice

Find the length of each missing side. Round to the nearest hundredth.

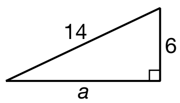
1.



2.



3.



If c is the measure of the hypotenuse of a right triangle, find each missing measure. Round answers to the nearest hundredth.

4. $a = 12, b = 32, c = \underline{\ ?\ }$

5. $a = 7, b = 10, c = \underline{\ ?\ }$

6. $a = 16, c = 52, b = \underline{\ ?\ }$

7. $a = 2, b = 4, c = \underline{\ ?\ }$

8. $b = 18, c = \sqrt{740}, a = \underline{\ ?\ }$

9. $a = 5, b = \sqrt{10}, c = \underline{\ ?\ }$

10. Art Jessica is making a collage of rectangles for her art project. The largest rectangle is 12 inches long and 8 inches wide. What is the length of a diagonal of the rectangle?

11. Standardized Test Practice Jamal and Gloria start hiking from the same point. After Bill hikes 7 miles due east and Jamal hikes 4 miles due north, how far apart are the two hikers?

A 5.29 mi **B** 5.40 mi **C** 8.06 mi **D** 9.25 mi

Answers: 1. $c = 6.71$ **2.** $b = 9.95$ **3.** $a = 12.65$ **4.** $c = 34.18$ **5.** $c = 12.21$ **6.** $b = 49.48$ **7.** $c = 4.47$ **8.** $a = 20.40$ **9.** $c = 5.92$ **10.** about 14.42 in. **11.** C

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11-5 The Distance Formula *(Pages 611–615)*

You can use the Distance Formula, which is based on the Pythagorean Theorem, to find the distance between any two points on the coordinate plane.

The Distance Formula	The distance between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the following formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Examples

a. Find the distance between (2, 3) and (6, 8).

Let $x_1 = 2$, $y_1 = 3$, $x_2 = 6$, *and* $y_2 = 8$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(6 - 2)^2 + (8 - 3)^2}$

$d = \sqrt{4^2 + 5^2}$

$d = \sqrt{16 + 25}$

$d = \sqrt{41}$ *or about 6.4 units.*

b. Find the value of *a* if (*a*, 3) and (2, −1) are 5 units apart.

Let $x_1 = a$, $y_1 = 3$, $x_2 = 2$, $y_2 = -1$, *and* $d = 5$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(2 - a)^2 + (-1 - 3)^2}$

$5 = \sqrt{(-a + 2)^2 + (-4)^2}$

$5 = \sqrt{a^2 - 4a + 4 + 16}$

$5 = \sqrt{a^2 - 4a + 20}$

$5^2 = \left(\sqrt{a^2 - 4a + 20}\right)^2$

$25 = a^2 - 4a + 20$

$0 = a^2 - 4a - 5$

$0 = (a + 1)(a - 5)$ *Factor.*

$a = -1$ *or* $a = 5$ *Zero product property*

Practice

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

1. (4, 6), (1, 5) **2.** (15, 4), (10, 10) **3.** (−7, −2), (11, 3)

4. (6, 13), (2, 15) **5.** (25, 11), (18, 6) **6.** $(12, 3\sqrt{5}), (6, 2\sqrt{5})$

Find the value of *a* if the points with the given coordinates are the indicated distance apart.

7. $(1, 3), (a, -9); d = 13$ **8.** $(-5, a), (3, -7); d = 10$ **9.** $(-9, 3), (-2, a); d = \sqrt{74}$

10. Geometry Find the perimeter of square *QRST* if two of the vertices are *Q*(5, 9) and *R*(−4, −3).

11. Standardized Test Practice Find the distance between the points whose coordinates are $(2\sqrt{7}, 4\sqrt{5})$ and $(\sqrt{7}, 2\sqrt{20})$.

A $\sqrt{5}$ **B** $\sqrt{7}$ **C** $\sqrt{32}$ **D** $\sqrt{70}$

Answers: 1. $\sqrt{10}$ or 3.16 **2.** $\sqrt{61}$ or 7.81 **3.** $\sqrt{349}$ or 18.68 **4.** $2\sqrt{5}$ or 4.47 **5.** $\sqrt{74}$ or 8.60 **6.** $\sqrt{41}$ or 6.40 **7.** −4 or 6

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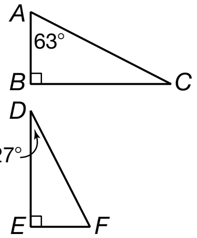
11-6 Similar Triangles *(Pages 616–621)*

Two figures are **similar (~)** if they have the same shape, but not necessarily the same size.

Similar Triangles	• If the corresponding angles of two triangles have equal measures, the triangles are similar. The sides opposite the corresponding angles are corresponding sides. • If two triangles are similar, the measures of their corresponding sides are proportional, and the measures of their corresponding angles are equal.

Examples

a. Determine whether the pair of triangles shown at the right are similar.



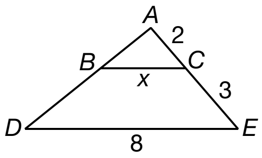
Two triangles are similar if the measures of their corresponding angles are equal.

$m\angle C = 180° - (90° + 63°)$
$= 27°$

$m\angle F = 180° - (90° + 27°)$
$= 63°$

Since corresponding angles have equal measures, triangle ABC is similar to triangle FED, or $\triangle ABC \sim \triangle FED$.

b. In the figure below, $\triangle ABC \sim \triangle ADE$. Find the value of x.



Write a proportion matching corresponding sides of each triangle.

$\frac{BC}{DE} = \frac{AC}{AE}$

$\frac{x}{8} = \frac{2}{2+3}$

$(2 + 3)(x) = 8(2)$ *Find the cross products.*

$5x = 16$

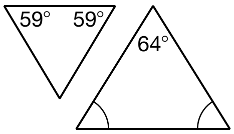
$\frac{5x}{5} = \frac{16}{5}$

$x = 3.2$

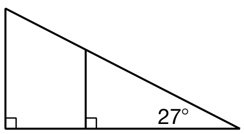
Practice

Determine whether each pair of triangles is similar.

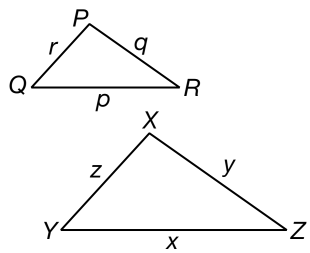
1.



2.



Triangle *PQR* is similar to triangle *XYZ*. For each set of measures given, find the measures of the remaining sides.



3. $p = 4, q = 3.5, r = 3, x = 8$

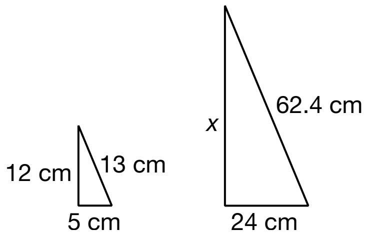
4. $p = 5, q = 5, r = 2, z = 3$

5. $x = 20, y = 18, z = 16, q = 9$

6. $x = 22.5, y = 18, z = 15, r = 10$

7. **Standardized Test Practice** The triangles in the figure at the right are similar. Find the value of x.

A 24 cm **B** 48 cm
C 57.6 cm **D** 67.6 cm



Answers: **1.** no **2.** yes **3.** $y = 7, z = 6$ **4.** $x = 7.5, y = 7.5$ **5.** $p = 10, r = 8$ **6.** $p = 15, q = 12$ **7.** C

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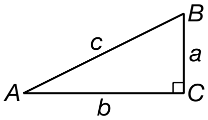
11-7 Trigonometric Ratios *(Pages 623–630)*

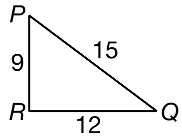
In a right triangle, the side opposite the right angle is the longest side. This side is called the **hypotenuse**. The other two sides are the **legs**.

Definition of Trigonometric Ratios	sine of $\angle A = \dfrac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}$	$\sin A = \dfrac{a}{c}$
	cosine of $\angle A = \dfrac{\text{measure of leg adjacent } \angle A}{\text{measure of hypotenuse}}$	$\cos A = \dfrac{b}{c}$
	tangent of $\angle A = \dfrac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent } \angle A}$	$\tan A = \dfrac{a}{b}$



Examples

a. Find the sine, cosine, and tangent of angle *Q*.



$\sin Q = \dfrac{\text{opposite leg}}{\text{hypotenuse}} = \dfrac{9}{15}$ or 0.6

$\cos Q = \dfrac{\text{adjacent leg}}{\text{hypotenuse}} = \dfrac{12}{15}$ or 0.8

$\tan Q = \dfrac{\text{opposite leg}}{\text{adjacent leg}} = \dfrac{9}{12}$ or 0.75

b. Find the measure of angle *P*, $m\angle P$, to the nearest degree.

$\sin P = \dfrac{\text{opposite leg}}{\text{hypotenuse}} \Rightarrow \sin P = \dfrac{12}{15}$ or 0.8

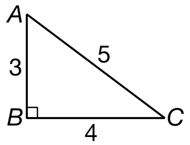
Use a scientific calculator to find the angle measure with a sine of 0.8.

***Enter:** 0.8 [2nd] [SIN^{-1}]   **Result:** 53.13010235   So, $m\angle P \approx 53°$.*

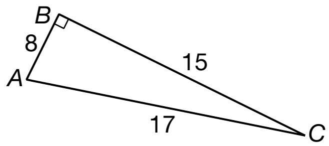
Practice

For each triangle, find sin *C*, cos *C*, and tan *C* to the nearest thousandth. Use a calculator to find the value of each trigonometric ratio to the nearest ten thousandth if necessary.

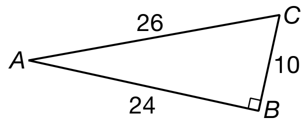
1.



2.



3.



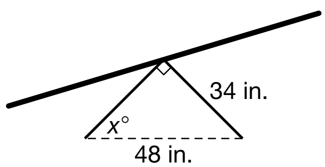
4. sin 14°   **5.** cos 68°   **6.** tan 80°   **7.** cos 60°   **8.** sin 85°

Use a calculator to find the measure of each angle to the nearest degree.

9. $\sin B = 0.8192$ **10.** $\cos M = 0.7660$ **11.** $\tan W = 0.2309$

12. $\cos Y = 0.7071$ **13.** $\sin P = 0.9052$ **14.** $\tan K = 0.2675$

15. **Standardized Test Practice** Which equation can be used to find the measure of the angle measuring $x°$ under the seesaw?



A $\sin(x°) = \dfrac{48}{34}$ **B** $\cos(x°) = \dfrac{48}{34}$

C $\sin(x°) = \dfrac{34}{48}$ **D** $\tan(x°) = \dfrac{34}{48}$

Answers: 1. $\sin C = \frac{3}{5}$; $\cos C = \frac{4}{5}$; $\tan C = \frac{3}{4}$ **2.** $\sin C = \frac{8}{17}$; $\cos C = \frac{15}{17}$; $\tan C = \frac{8}{15}$ **3.** $\sin C = \frac{12}{13}$; $\cos C = \frac{5}{13}$; $\tan C = \frac{12}{5}$ **4.** 0.2419 **5.** 0.3746 **6.** 5.6713 **7.** 0.5 **8.** 0.9962 **9.** 55° **10.** 40° **11.** 13° **12.** 45° **13.** 65° **14.** 15° **15.** C

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11

Chapter Review

Radical Roof

The staff at Monsoon High School stores its math textbooks in the storage buildings below. The books are evenly divided among all of the storage buildings. However, the rainy season is fast approaching and some of the storage buildings will leak when it rains. With your parent, help the staff of Monsoon High School find out which roofs will leak before the rains begin. Simplify the expressions on each building. If the simplified expression contains a radical sign (roof), then the storage building will not leak. If the expression does not contain a radical sign, then the building will leak. Mark the leaky buildings with a big X so the staff will know to move the textbooks out of those buildings.

Simplify each expression.

1. $\sqrt{64}$

2. $\sqrt{80x^2}$

3. $\sqrt{72ab^5c^2}$

4. $\frac{\sqrt{18}}{\sqrt{2}}$

5. $\frac{\sqrt{12}}{\sqrt{2}}$

6. $2\sqrt{5} + 3\sqrt{7} - 5\sqrt{5} + \sqrt{7}$

7. $3\sqrt{2} + \sqrt{5} - \sqrt{2} - \sqrt{5} - 2\sqrt{2}$

8. $\sqrt{3}(2 + \sqrt{3})$

9. $(2\sqrt{5} - 7)(\sqrt{5} + 6) - 5\sqrt{5}$

10. $\frac{5}{\sqrt{3} + 6}$

Answers are located in the Answer Key.

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12-1 Inverse Variation *(Pages 642–647)*

A situation in which y *decreases* as x *increases* is called an **inverse variation**. In this situation y varies inversely as x or y is inversely proportional to x. Solutions to an inverse variation can be expressed as the **product rule**. The product rule states that for any two solutions (x_1, y_1) and (x_2, y_2), $x_1, y_1 = x_2 y_2$ and $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

Inverse Variation	If y varies inversely as x, then as x increases y decreases, or as x decreases y increases. An inverse variation can be described by the equation $xy = k$, where $k \neq 0$.
Product Rule	For solutions (x_1, y_1) and (x_2, y_2), $x_1y_1 = x_2y_2$ and $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

Example

Solve for *x*.

If y varies inversely as x and $y_1 = 5$ when $x_1 = 9$, find x_2 when $y_2 = 15$.

Method 1

$x_1 y_1 = x_2 y_2$	*Use the product rule.*
$9 \cdot 5 = x_2 \cdot 15$	*Substitute.*
$45 = x_2 \cdot 15$	*Simplify.*
$3 = x_2$	*Divide both sides by 15.*

Method 2

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$	*Use a proportion.*
$\frac{9}{x_2} = \frac{15}{5}$	*Substitute.*
$45 = 15x_2$	*Cross multiply.*
$3 = x_2$	*Divide both sides by 15.*

Practice

Write an inverse variation equation that relates *x* and *y*. Assume that *y* varies inversely as *x*. Then solve.

1. If $y = 10$ when $x = 7$, find y when $x = 5$.
2. If $y = 21$ when $x = 10$, find y when $x = 4$.
3. If $y = 17.5$ when $x = 12$, find y when $x = 8$.
4. If $y = 5$ when $x = 5$, find x when $y = 2$.
5. If $y = 13$ when $x = -3$, find x when $y = -3.9$.
6. Find the value of y when $x = 5$ if $y = 8$ when $x = 10$.
7. Find the value of y when $x = \frac{3}{4}$ if $y = 27$ when $x = \frac{1}{4}$.
8. If $x = 2.1$ when $y = 7.2$ find x when $y = 7.56$.
9. **Standardized Test Practice** Assuming that y varies inversely as x, find the value of x when $y = -17$ if $y = -12$ when $x = -8\frac{1}{2}$.

   A $x = -12\frac{1}{24}$ **B** $x = -24$ **C** $x = -6$ **D** $x = -\frac{1}{6}$

Answers: 1. 14 **2.** 52.5 **3.** 26.25 **4.** 12.5 **5.** 10 **6.** 16 **7.** 9 **8.** 2 **9.** C

Chapter 12

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12-2 Rational Expressions

(Pages 648–653)

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. Any values of the variable that result in a denominator of zero must be excluded from the domain of the variable. These are called **excluded values** of the rational expression. To simplify a rational expression, eliminate (by dividing) any common factors of the numerator and denominator using the GCF.

Example

Simplify $\frac{b-3}{b^2-2b-3}$ and state the excluded values of b.

$\frac{b-3}{b^2-2b-3} = \frac{b-3}{(b-3)(b+1)}$ *Factor the denominator.*

$b - 3 = 0$ and $b + 1 = 0$ *Exclude the values for which $b - 3 = 0$ and $b + 1 = 0$.*

$b = 3$ $\quad b = -1$

Therefore, b cannot equal 3 or −1.

$\frac{b-3}{(b-3)(b+1)} = \frac{\overset{1}{\cancel{b-3}}}{\underset{1}{\cancel{(b-3)}}(b+1)}$ *Simplify the fraction by dividing by the GCF, b − 3.*

$= \frac{1}{b+1}, b \neq -1, 3$

Try These Together

Simplify and state the excluded values of the variables.

1. $\frac{7a^3}{14a}$

HINT: Find the exclude values before you simplify the expression.

2. $\frac{x^2+3x+2}{x^2-4x-5}$

HINT: Factor both the numerator and the denominator.

Practice

Simplify and state the excluded values of the variables.

3. $\frac{6x^2y}{30x}$ **4.** $\frac{9x^4y^2z}{x^6y}$ **5.** $\frac{20xyz^3}{60x^2yz^3}$ **6.** $\frac{8a}{a^2+3a}$

7. $\frac{12x}{3x+6}$ **8.** $\frac{10x-5x^2}{2x^2}$ **9.** $\frac{x^2-25}{x-5}$ **10.** $\frac{b^2-4}{4b-8}$

11. $\frac{3x+3}{x^2-1}$ **12.** $\frac{a+7}{a^2+9a+14}$ **13.** $\frac{x^2+6x+8}{6x+24}$ **14.** $\frac{y^2+7y+6}{y^2+5y-6}$

15. Standardized Test Practice Simplify the rational expression $\frac{2x^2-98}{8x-56}$.

A $4(x+7)$ **B** $4(x-7)$ **C** $\frac{x^2-49}{x-7}$ **D** $\frac{x+7}{4}$

Answers: 1. $\frac{a^2}{2}, a \neq 0$ **2.** $\frac{x+2}{x-5}, x \neq -1, 5$ **3.** $\frac{xy}{5}, x \neq 0$ **4.** $\frac{9yz}{x^2}, x \neq 0, y \neq 0$ **5.** $\frac{1}{3x}, x \neq 0, y \neq 0, z \neq 0$ **6.** $\frac{8}{a+3}, a \neq 0, -3$ **7.** $\frac{4x}{x+2}, x \neq -2$ **8.** $\frac{10-5x}{2x}, x \neq 0$ **9.** $x+5, x \neq 5$ **10.** $\frac{b+2}{4}, b \neq 2$ **11.** $\frac{3}{x-1}, x \neq -1, 1$ **12.** $\frac{1}{a+2}, a \neq -7, -2$ **13.** $\frac{x+2}{6}, x \neq -4$ **14.** $\frac{y+1}{y-1}, y \neq -6, 1$ **15.** D

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12-3 Multiplying Rational Expressions

(Pages 655–659)

To multiply rational expressions, you can divide by the common factors either *before* or *after* you multiply the expressions. From this point on, you may assume that no denominator has a value of 0.

Example

Multiply $\frac{2x^2(3x-2)}{3x^2+x-2} \cdot \frac{1}{4x}$.

$\frac{2x^2(3x-2)}{3x^2+x-2} \cdot \frac{1}{4x} = \frac{2x^2(3x-2)}{(3x-2)(x+1)} \cdot \frac{1}{4x}$ *Factor the denominator.*

$= \frac{\overset{1\ x}{\cancel{2x^2}}\overset{1}{\cancel{(3x-2)}}}{\underset{1}{\cancel{(3x-2)}}(x+1)} \cdot \frac{1}{\underset{2\ 1}{\cancel{4x}}}$ *Divide by the GCF of 2x(3x − 2) before multiplying.*

$= \frac{x}{2(x+1)}$ or $\frac{x}{2x+2}$ *Multiply. Then, simplify the denominator.*

Try These Together

1. Multiply $\frac{ab^2}{12} \cdot \frac{6}{b}$.

HINT: Divide both numerator and denominator by the same quantity—their greatest common factor.

2. Multiply $(x-8) \cdot \frac{4}{x^2-64}$.

HINT: Write $x - 8$ as $\frac{x-8}{1}$.

Practice

Find each product. Assume that no denominator has a value of 0.

3. $\frac{15a}{b^3} \cdot \frac{2b^4}{3}$

4. $\frac{3x^4yz^2}{24y^2} \cdot \frac{4}{x}$

5. $16abc \cdot \frac{ab}{bc^2}$

6. $\frac{25mn^2}{4n} \cdot \frac{10n^3}{5m}$

7. $(2x+8) \cdot \frac{7}{x+4}$

8. $\frac{12(a-1)}{3a} \cdot \frac{a^2}{a-1}$

9. $\frac{x+2}{5} \cdot \frac{2}{x^2+2x}$

10. $\frac{x^2-9}{x-3} \cdot \frac{9x-6}{3}$

11. $\frac{2x-10}{3x} \cdot \frac{6x^2}{x^2-25}$

12. $\frac{x^2+16}{x} \cdot \frac{x}{x+4}$

13. $\frac{4x+2}{2x+6} \cdot \frac{6}{2x^2+7x+3}$

14. $\frac{x^2+2x-15}{x^2+4x} \cdot \frac{x^2}{x+5}$

15. $\frac{y^2-36}{y+3} \cdot \frac{y-4}{y^2+2y-24}$

16. $\frac{3x+12}{x^2-x-2} \cdot \frac{2x-2}{6x+24}$

17. $\frac{3x^2-6x-9}{x^2-x-2} \cdot \frac{x^2-4}{6x+12}$

18. **Standardized Test Practice** Multiply $\frac{x^2+14x+49}{x^2-49} \cdot \frac{x-7}{x+7}$.

A $x+7$ **B** $\frac{1}{x+7}$ **C** 1 **D** $x-2$

Answers: 1. $\frac{ab}{2}$ **2.** $\frac{4}{x+8}$ **3.** $10ab$ **4.** $\frac{x^3z^2}{2y}$ **5.** $\frac{16a^2b}{c}$ **6.** $\frac{25n^4}{2}$ **7.** 14 **8.** $4a$ **9.** $\frac{2}{5x}$ **10.** $3x^2+7x-6$ **11.** $\frac{4x}{x+5}$ **12.** $\frac{x^2+16}{x+4}$ **13.** $\frac{6}{x^2+6x+9}$ **14.** $\frac{x^2-3x}{x+4}$ **15.** $\frac{y-6}{y+3}$ **16.** $\frac{x-1}{x^2-x-2}$ **17.** $\frac{x-3}{2}$ **18.** C

12-4 Dividing Rational Expressions *(Pages 660–664)*

To divide algebraic rational expressions, multiply by the reciprocal of the divisor (the second fraction).

Example

Find $\frac{x^2 - 4}{5x} \div \frac{x + 2}{x - 2}$.

$\frac{x^2 - 4}{5x} \div \frac{x+2}{x-2} = \frac{x^2-4}{5x} \cdot \frac{x-2}{x+2}$ — *The reciprocal of $\frac{x+2}{x-2}$ is $\frac{x-2}{x+2}$.*

$= \frac{\overset{1}{\cancel{(x+2)}}(x-2)}{5x} \cdot \frac{x-2}{\underset{1}{\cancel{x+2}}}$ — *Factor. Then divide by the common factor x + 2.*

$= \frac{(x-2)(x-2)}{5x}$ or $\frac{x^2 - 4x + 4}{5x}$ — *Multiply.*

Try These Together

1. Find $\frac{5m^2}{10} \div \frac{3m^5}{12}$.

HINT: First rewrite, multiplying by the reciprocal of the second fraction. Then divide by the greatest common factor.

2. Find $\frac{3a - 15}{a + 4} \div (a - 5)$.

HINT: The reciprocal of a − 5 is $\frac{1}{a-5}$.

Practice

Find each quotient. Assume that no denominator has a value of 0.

3. $\frac{8x}{3yz^2} \div \frac{4xy}{3yz}$

4. $10bc^2 \div \frac{2abc}{8b}$

5. $\frac{x-5}{8} \div \frac{x-5}{32}$

6. $\frac{x-8}{x+3} \div \frac{x+2}{x+2}$

7. $\frac{4x^2+4}{2} \div \frac{x^3+x}{x}$

8. $\frac{b^2-25}{4} \div (b+5)$

9. $\frac{n^2-1}{3} \div \frac{n+1}{3n+3}$

10. $\frac{4b^5}{b+3} \div \frac{4b^2}{5b+15}$

11. $\frac{2k+10}{k-3} \div \frac{2}{k-3}$

12. $\frac{8}{y+2} \div \frac{y-2}{y^2-4}$

13. $\frac{x+1}{x^2+8x+7} \div \frac{4}{2x+14}$

14. $\frac{x^2+x-6}{2x} \div \frac{x+3}{4x^2+8x}$

15. $\frac{n^2-9}{n-3} \div \frac{n+3}{n^2+7n+12}$

16. $\frac{x+1}{x^2+2x+1} \div \frac{x-3}{x+1}$

17. $\frac{4m}{m-6} \div \frac{m^2+2m}{m^2-4m-12}$

18. **Standardized Test Practice** Find the quotient $\frac{x+1}{2} \div \frac{x^2+6x+5}{4}$.

A $\frac{2}{x+5}$ **B** $2(x+5)$ **C** $\frac{1}{2}(x+5)$ **D** $\frac{x+5}{2}$

Answers: **1.** $\frac{2}{m^3}$ **2.** $\frac{3}{a+4}$ **3.** $\frac{2}{yz}$ **4.** $\frac{40bc}{a}$ **5.** 4 **6.** $\frac{x-8}{x+3}$ **7.** 2 **8.** $\frac{b-5}{4}$ **9.** n^2-1 **10.** $5b^3$ **11.** $k+5$ **12.** 8 **13.** $\frac{1}{2}$ **14.** $2x^2-8$ **15.** $n^2+7n+12$ **16.** $\frac{1}{x-3}$ **17.** 4 **18.** A

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12-5 Dividing Polynomials *(Pages 666–671)*

To divide a polynomial by a *monomial*, divide each term of the polynomial by the monomial. To divide a polynomial by a *binomial*, first try factoring the dividend. If you cannot factor the dividend, use long division.

Examples

a. Find $(5x^2 - 3xy + 2y^2) \div 2xy$.

$\frac{5x^2 - 3xy + 2y^2}{2xy}$ *Rewrite as a fraction.*

$= \frac{5x^2}{2xy} - \frac{3xy}{2xy} + \frac{2y^2}{2xy}$ *Divide each term by 2xy.*

$= \frac{5x}{2y} - \frac{3}{2} + \frac{y}{x}$ *Simplify each term.*

The quotient is $\frac{5x}{2y} - \frac{3}{2} + \frac{y}{x}$.

b. Find $(t^2 - 5t + 10) \div (t + 3)$.

Since the dividend, $t^2 - 5t + 10$, cannot be factored, use long division.

$$\begin{array}{r} t \phantom{{}- 5t + 10} \\ t + 3\overline{)t^2 - 5t + 10} \\ (-)\ \underline{t^2 + 3t} \phantom{{}+ 10} \\ -8t \phantom{{}+ 10} \end{array}$$

$t^2 \div t = t$
Multiply t and t + 3.
Subtract.

$$\begin{array}{r} t - 8 \\ t + 3\overline{)t^2 - 5t + 10} \\ (-)\ \underline{t^2 + 3t} \phantom{{}+ 10} \\ -8t + 10 \\ (-)\ \underline{-8t - 24} \\ 34 \end{array}$$

Multiply −8 and t + 3.
Subtract.

The quotient is t − 8 with remainder 34 or $t - 8 + \frac{34}{t + 3}$.

Try These Together

1. Find $(x^3 + 4x - 8) \div 2x$.
HINT: Divide each term of the dividend by 2x.

2. Find $(y^2 + 7y + 10) \div (y + 2)$.
HINT: Factor the dividend, $y^2 + 7y + 10$.

Practice

Find each quotient.

3. $(k^2 - 12k + 6) \div 3k$

4. $(x^2 + 7x + 10) \div (x + 2)$

5. $(x^2 - 5x + 6) \div (x - 3)$

6. $(a^2 - 3a - 4) \div (a + 1)$

7. $(2y^2 + 10y + 8) \div (y + 4)$

8. $(x^2 + 8x + 14) \div (x + 1)$

9. $(2b^2 - 5b + 8) \div (b - 2)$

10. $(2x^2 + 9x + 3) \div (x + 3)$

11. $\frac{t^2 - 6t + 16}{8t}$

12. $\frac{2n^2 + 6n + 3}{n + 3}$

13. $\frac{x^2 + 5x + 6}{x + 1}$

14. $\frac{6x^2 + x - 10}{2x - 3}$

15. $\frac{y^3 - 4y^2 + 2y + 8}{y + 1}$

16. $\frac{x^3 + x - 2}{x - 1}$

17. **Standardized Test Practice** Find $(3x^2 + 6x + 9) \div 3x$.

A $3x + 3$ **B** $3x + 2$ **C** $x + 3 + \frac{3}{x}$ **D** $x + 2 + \frac{3}{x}$

Answers: **1.** $\frac{x^2}{2} + 2 - \frac{4}{x}$ **2.** $y + 5$ **3.** $\frac{k}{3} - 4 + \frac{2}{k}$ **4.** $x + 5$ **5.** $x - 2$ **6.** $a - 4$ **7.** $2y + 2$ **8.** $x + 7 + \frac{7}{x + 1}$ **9.** $2b - 1 + \frac{6}{b - 2}$ **10.** $2x + 3 - \frac{6}{x + 3}$ **11.** $\frac{t}{8} - \frac{3}{4} + \frac{2}{t}$ **12.** $2n + \frac{3}{n + 3}$ **13.** $x + 4 + \frac{2}{x + 1}$ **14.** $3x + 5 + \frac{5}{2x - 3}$ **15.** $y^2 - 5y + 7 + \frac{1}{y + 1}$ **16.** $x^2 + x + 2$ **17.** D

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12-6 Rational Expressions with Like Denominators *(Pages 672–677)*

To add or subtract rational expressions with like denominators, add or subtract the numerators and then write the sum or difference over the common denominator. To subtract a quantity, add its additive inverse. Remember to simplify your answer, if necessary, by dividing by the GCF.

Examples

a. Find $\frac{7t}{9} - \frac{2t-1}{9}$.

$$\frac{7t}{9} - \frac{2t-1}{9} = \frac{7t-(2t-1)}{9}$$
$$= \frac{5t+1}{9}$$

b. Find $\frac{6y-3}{2y-1} + \frac{5y+1}{1-2y}$.

The denominator of the second expression can be rewritten. $1 - 2y = -(-1 + 2y)$ or $-(2y - 1)$.

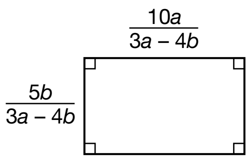
$$\frac{6y-3}{2y-1} + \frac{5y+1}{1-2y} = \frac{6y-3}{2y-1} - \frac{5y+1}{2y-1}$$
$$= \frac{6y-3-(5y+1)}{2y-1}$$
$$= \frac{y-4}{2y-1}$$

Practice

Find each sum or difference. Express in simplest form.

1. $\frac{9}{3m} + \frac{-12}{3m}$
2. $\frac{-5x}{21} + \frac{12x}{21}$
3. $\frac{3}{x} - \frac{9}{x}$
4. $\frac{t+2}{4} - \frac{t}{4}$
5. $\frac{y+3}{2} + \frac{4y-6}{2}$
6. $\frac{2x}{8} - \frac{-14x}{8}$
7. $\frac{3c}{4c+1} + \frac{c+1}{4c+1}$
8. $\frac{7k}{k+2} - \frac{6k}{k+2}$
9. $\frac{-2}{x-5} + \frac{x-3}{x-5}$
10. $\frac{3n}{2n-3} + \frac{n-6}{3-2n}$
11. $\frac{3d-2}{2} + \frac{d+4}{2}$
12. $\frac{a}{a+4} - \frac{8+a}{a+4}$
13. $\frac{2n}{5n+5} - \frac{n-1}{5n+5}$
14. $\frac{x-4}{1-x} + \frac{2x-5}{x-1}$
15. $\frac{x-9}{x+2} - \frac{2x-12}{x+2}$

16. **Standardized Test Practice** Which of the following is an expression for the perimeter of the rectangle?



A $\frac{15ab}{3a-4b}$ **B** $\frac{20a+10b}{3a-4b}$

C $\frac{15ab}{9a-8b}$ **D** $\frac{10a-5b}{3a-4b}$

Answers: 1. $\frac{-1}{m}$ 2. $\frac{x}{3}$ 3. $\frac{-6}{x}$ 4. $\frac{1}{2}$ 5. $\frac{5y-3}{2}$ 6. $2x$ 7. 1 8. $\frac{k}{k+2}$ 9. 1 10. $\frac{2n+6}{2n-3}$ 11. $2d+1$ 12. $\frac{-8}{a+4}$ 13. $\frac{1}{5}$ 14. 1 15. $\frac{-x+3}{x+2}$ 16. B

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12-7 Rational Expressions with Unlike Denominators *(Pages 678–683)*

The **least common multiple** (LCM) of two or more positive whole numbers is the least positive number that is a common multiple of all the numbers. To add or subtract rational expressions with unlike denominators, first rename the fractions so the denominators are alike, using the least common denominator of the fractions. You may need to factor one or both of the denominators first. The **least common denominator** (LCD) is the LCM of the denominators.

Examples

a. Find $\frac{5}{2y} + \frac{4}{3y^2}$.

List the prime factors of 2y and $3y^2$ to find the LCD.

$2y = 2 \cdot y \qquad 3y^2 = 3 \cdot y \cdot y$

Use each prime factor the greatest number of times it appears in each of the factorizations.

LCD: $2 \cdot 3 \cdot y \cdot y$ *or* $6y^2$

Change each rational expression into an equivalent expression with the LCD.

$$\frac{5}{2y} + \frac{4}{3y^2} = \frac{5(3y)}{2y(3y)} + \frac{4(2)}{3y^2(2)}$$
$$= \frac{15y}{6y^2} + \frac{8}{6y^2} \text{ or } \frac{15y + 8}{6y^2}$$

b. Find $\frac{x}{x-1} - \frac{5}{x-2}$.

LCD: $(x - 1)(x - 2)$

$$\frac{x}{x-1} - \frac{5}{x-2}$$
$$= \frac{x(x-2)}{(x-1)(x-2)} - \frac{5(x-1)}{(x-1)(x-2)}$$
$$= \frac{x^2 - 2x}{(x-1)(x-2)} - \frac{5x - 5}{(x-1)(x-2)}$$
$$= \frac{x^2 - 2x - (5x - 5)}{(x-1)(x-2)}$$
$$= \frac{x^2 - 7x + 5}{(x-1)(x-2)}$$

Practice

Find each sum or difference. Express in simplest form.

1. $\frac{1}{2x} - \frac{2}{10x}$

2. $\frac{1}{7x} + \frac{2}{x}$

3. $\frac{10}{xy^2} + \frac{5}{y^2}$

4. $\frac{9}{a^3} - \frac{7}{a}$

5. $\frac{2}{3x + 6} + \frac{5}{x + 2}$

6. $\frac{7}{2x - 8} - \frac{2}{x - 4}$

7. $\frac{2x}{x + 1} + \frac{x}{4x + 4}$

8. $\frac{5}{x + 6} + \frac{3}{x + 3}$

9. $\frac{4}{x + 3} - \frac{5x}{x - 3}$

10. $\frac{7x}{x^2 - 16} + \frac{2}{x + 4}$

11. $\frac{x}{x - 10} - \frac{3}{x^2 - 100}$

12. $\frac{4x}{x - 1} + \frac{-x}{x^2 + 5x - 6}$

13. **Standardized Test Practice** Find $\frac{3}{x^2 + x - 20} + \frac{2}{x + 5}$.

A $\frac{5}{x - 4}$ **B** $\frac{5}{x^2 + x - 20}$ **C** $\frac{2x - 5}{x - 4}$ **D** $\frac{2x - 5}{x^2 + x - 20}$

Answers: **1.** $\frac{3}{10x}$ **2.** $\frac{15}{7x}$ **3.** $\frac{5x + 10}{xy^2}$ **4.** $-\frac{7a^2 + 9}{a^3}$ **5.** $\frac{17}{3x + 6}$ **6.** $\frac{3}{2x - 8}$ **7.** $\frac{9x}{4x + 4}$ **8.** $\frac{8x + 33}{x^2 + 9x + 18}$ **9.** $\frac{-5x^2 - 11x - 12}{x^2 - 9}$ **10.** $\frac{9x - 8}{x^2 - 16}$ **11.** $\frac{x^2 + 10x - 3}{x^2 - 100}$ **12.** $\frac{4x^2 + 23x}{x^2 + 5x - 6}$ **13.** D

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12-8 Mixed Expressions and Complex Fractions *(Pages 684–689)*

A **mixed expression** is an algebraic expression that contains a monomial and a rational expression. Simplifying a mixed expression is similar to the process used in rewriting a mixed number as an improper fraction.

Simplifying a Complex Fraction	Any complex fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$, can be expressed as $\frac{ad}{bc}$.

Example

Simplify $\dfrac{3 + \frac{6}{x}}{\frac{x+2}{4}}$**.**

$$\frac{3 + \frac{6}{x}}{\frac{x+2}{4}} = \frac{\frac{3(x)}{x} + \frac{6}{x}}{\frac{x+2}{4}}$$ *The LCD of the numerator is x.*

$$= \frac{\frac{3x+6}{x}}{\frac{x+2}{4}}$$ *Add to simplify the numerator.*

$$= \frac{3x+6}{x} \cdot \frac{4}{x+2}$$ *Multiply by the reciprocal of the divisor.*

$$= \frac{3(x+2)}{x} \cdot \frac{4}{x+2}$$ *Factor to simplify before multiplying.*

$$= \frac{3\overset{1}{\cancel{(x+2)}}}{x} \cdot \frac{4}{\underset{1}{\cancel{x+2}}}$$ *Divide by the common factor of x + 2.*

$$= \frac{12}{x}$$ *Multiply.*

Practice

Write each mixed expression as a rational expression.

1. $x - \frac{4}{x}$

2. $4 - \frac{2}{x+7}$

3. $9 - \frac{n+4}{n-1}$

4. $3 + \frac{x+5}{x^2-25}$

Simplify.

5. $\dfrac{\frac{a}{b}}{\frac{2a}{b^5}}$

6. $\dfrac{\frac{xyz}{x^2}}{\frac{y^5z}{x^4}}$

7. $\dfrac{m + \frac{5}{m}}{\frac{m+7}{m}}$

8. $\dfrac{t + \frac{3}{t-2}}{2 + \frac{4}{t-2}}$

9. **Standardized Test Practice** Simplify $\dfrac{\frac{x}{x+2}}{\frac{1}{x-5}}$.

A $\frac{x+1}{2x-3}$ **B** $\frac{x^2-5x}{x+2}$ **C** $\frac{x}{x^2-3x-10}$ **D** $\frac{2x-5}{x+3}$

Answers: **1.** $\frac{x^2-4}{x}$ **2.** $\frac{4x+26}{x+7}$ **3.** $\frac{8n-13}{n-1}$ **4.** $\frac{3x-14}{x-5}$ **5.** $\frac{b^4}{2}$ **6.** $\frac{y^4}{x^3}$ **7.** $\frac{m^2+5}{m+7}$ **8.** $\frac{t^2-2t+3}{2t}$ **9.** B

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12-9 Solving Rational Equations *(Pages 690–695)*

A **rational equation** is an equation that contains rational expressions. To solve a rational equation, multiply each side of the equation by the LCD of the rational expressions in the equation. Doing so can yield results that are not solutions to the original equation, called **extraneous solutions** or "false" solutions. To eliminate extraneous solutions, be sure no solution is an excluded value of the original equation.

Example

Solve $\frac{a}{a+1} + \frac{3a+4}{a+1} = 3$.

$$\frac{a}{a+1} + \frac{3a+4}{a+1} = 3$$

$(a+1)\left(\frac{a}{a+1} + \frac{3a+4}{a+1}\right) = (a+1)3$ *Multiply each side by the LCD, a + 1.*

$(a+1)\frac{a}{a+1} + (a+1)\frac{3a+4}{a+1} = (a+1)3$ *Use the Distributive Property.*

$a + 3a + 4 = 3a + 3$ *Multiply.*

$4a + 4 = 3a + 3$ *Add.*

$a + 4 = 3$ *Subtract 3a from each side.*

$a = -1$ *Subtract 4 from each side.*

Since −1 is an excluded value of the original equation, −1 is an extraneous solution. Thus, this equation has no solution.

Practice

Solve each equation.

1. $\frac{2}{3y} + \frac{4}{y} = \frac{1}{3}$

2. $n - 4 = \frac{5}{n}$

3. $\frac{-3}{x} = 7 + 2x$

4. $\frac{1}{t} = \frac{3}{t-6}$

5. $\frac{x-2}{x} + (x+7) = \frac{-9}{x}$

6. $2x = \frac{4x}{x-2}$

7. $\frac{k+8}{k} - \frac{k-4}{k} = 3$

8. $\frac{a+1}{a} = \frac{a+1}{a-4}$

9. $\frac{n-3}{n-1} + \frac{2n}{n-1} = 2$

10. $\frac{w+5}{w+6} + \frac{w}{4} = \frac{1}{4}$

11. $\frac{x}{x+2} = \frac{1}{x}$

12. $\frac{n-1}{n} = \frac{n+1}{n+3}$

13. $\frac{x}{8} + \frac{2}{x} = \frac{x}{4}$

14. $\frac{y+3}{y+2} = 1 - \frac{y+1}{y+2}$

15. $\frac{c+4}{c-2} - 3 = \frac{c}{4}$

16. **Standardized Test Practice** Solve $\frac{x}{3} - \frac{1}{x} = \frac{2}{x}$.

A $x = -3$ **B** $x = 3$ **C** $x = -3, 3$ **D** no solution

Answers: **1.** 14 **2.** −1, 5 **3.** −3, $-\frac{1}{2}$ **4.** −3 **5.** −7, −1 **6.** 0, 4 **7.** 4 **8.** −1 **9.** no solution **10.** −7, −2 **11.** −1, 2 **12.** 3 **13.** −4, 4 **14.** no solution **15.** −10, 4 **16.** C

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12 Chapter Review

Connect the Dots

Imagine that you have just won the vacation of a lifetime in a raffle. Complete this puzzle to find out how you will be traveling to your destination. First simplify each expression completely. Then connect the dots following the instructions in the box at the right.

1. $\frac{9x^2}{3xy}$

2. $\frac{x^2 + 5x}{3x + 15}$

3. $\frac{x - 2}{2} \cdot \frac{x + 4}{x^2 - 4}$

4. $\frac{x^2 - x}{x^2 - 1} \div \frac{x}{x + 1}$

5. $(x^3 + 5x^2 + 5x - 3) \div (x + 3)$

6. $\frac{18}{3 - x} - \frac{6x}{3 - x}$

7. $\frac{1}{5x} - \frac{3}{7x}$

8. $\frac{x^2 + 3x}{x + 3} + \frac{1}{x - 3}$

Connect the answers to each problem in the following order:

Connect #1 to #2.
Connect #3 to #4.
Connect #5 to #6.
Connect #2 to #7.
Connect #5 to #3.
Connect #7 to #8.
Connect #4 to #6.

$\frac{-8}{35x}$ •

• $\frac{x^2}{x^2 + 2x + 1}$

$\frac{3}{x}$ •

$\frac{x + 5}{3}$ •

• $\frac{x - 2}{2}$

• $\frac{22}{35x}$

• $\frac{x^2 + 3}{x^2 - 9}$

• $\frac{-2}{35x}$

$\frac{x - 2}{2x - 8}$ •

• x

• $\frac{x^2 + 2x - 8}{2x^2 - 8}$

$\frac{3x}{xy}$ •

$\frac{3x}{y}$ •

• $\frac{x}{3}$

$\frac{x + 4}{2x + 4}$ •

• $\frac{x^2 - 3x + 1}{x - 3}$

• 1

• $x^2 + 2x + 1$

• $\frac{x}{3y}$

• $x^2 - 2x + 1$

• $x^2 + 2x - 1$

• 6

• $x^2 - 2x - 1$

Answers are located in the Answer Key.

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13-1 Sampling and Bias *(Pages 708–713)*

In order to estimate some characteristic within a large group, or **population**, a smaller group that is a subset of the population, known as a **sample**, is often used. Samples can be random or biased. A **random sample** of a population is selected with the goal of finding a representation of the entire population without any preference.

Types of Random Samples	
Simple Random Sample	A sample that is as likely to be chosen as any other sample from the population.
Stratified Random Sample	The population is divided into similar, nonoverlapping groups. Then a simple sample is chosen from each group.
Systematic Random Sample	Each item is selected according to a specific time or unit interval.

In a **biased sample** of a population the sample is chosen with some favoritism.

Types of Biased Samples	
Convenience Sample	Any sample that consists of population items that are easily accessed.
Voluntary Response Sample	A sample which includes only willing participants.

Example

On an assembly line, every 100th hinge is removed and inspected. Identify the sample and suggest a population from which the sample was selected, then classify the sample.

sample- The sample is every 100th hinge.

population- The population consists of every hinge on the assembly line.

classification- This is a systematic random sample.

Practice

Classify each sample.

1. The school newspaper asks the students to write letters stating their preferred candidate for student council president.

2. The name of every 7th grade student is placed in one hat and the name of every 8th grade student is placed in another hat, then 5 names are drawn from each hat.

3. A math teacher selects every 5th person in class to complete a problem on the board.

4. Shannon wants to find out which music group is the most liked by students in her school. As a sample, she asks the 10 girls on her basketball team.

5. **Standardized Test Practice** The name of every employee at a company is placed in one bin. Fifteen names are drawn for a prize. Classify this sample.

A Simple Random **B** Stratified Random
C Convenience **D** Voluntary Response

Answers: 1. Voluntary Response **2.** Stratified Random **3.** Systematic Random **4.** Convenience **5.** A

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13-2 Introduction to Matrices *(Pages 715–721)*

A rectangular arrangement of numbers, or **elements**, is called a **matrix**. Its **dimensions**, or the number of **rows** by the number of **columns**, describe a matrix. Two or more matrices with the same dimensions can be added or subtracted by performing the operation to corresponding elements. Any matrix can be multiplied by a constant called a *scalar*. The process of multiplying a matrix with a constant is called **scalar multiplication**. In scalar multiplication each element is multiplied by the constant.

Example

If $A = \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 10 \\ 13 & -7 \end{bmatrix}$ find $A + B$ and $5A$.

$A + B = \begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 10 \\ 13 & -7 \end{bmatrix}$	*Substitution*	$5A = 5\begin{bmatrix} 1 & 7 \\ 5 & 2 \end{bmatrix}$	*Substitution*
$A + B = \begin{bmatrix} 1 + (-4) & 7 + 10 \\ 5 + 13 & 2 + (-7) \end{bmatrix}$	*Matrix addition*	$5A = \begin{bmatrix} 5(1) & 5(7) \\ 5(5) & 5(2) \end{bmatrix}$	*Scalar multiplication*
$A + B = \begin{bmatrix} -3 & 17 \\ 18 & -5 \end{bmatrix}$	*Simplify.*	$5A = \begin{bmatrix} 5 & 35 \\ 25 & 10 \end{bmatrix}$	*Simplify.*

Practice

If $A = \begin{bmatrix} 15 & 10 & 9 \\ -2 & -6 & 3 \end{bmatrix}$, $B = [-7 \quad 5]$, and $C = \begin{bmatrix} 17 & 5 & 10 \\ 11 & -3 & -1 \end{bmatrix}$, find each sum, difference, or product.

1. $A + B$
2. $A + C$
3. $C - A$
4. $A - C$
5. $3A$
6. $-2B$
7. $0.5C$
8. $2A + 3C$
9. $B + C$
10. $-C - A$
11. $-2B + 5C$
12. $3A - 4C$

13. **Standardized Test Practice** Find $-3A - B$ if $A = \begin{bmatrix} -5 & 6 & 2 \\ 1 & 0 & -1 \\ -2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 7 \\ 10 & 8 & 6 \\ -6 & 9 & -1 \end{bmatrix}$.

A $\begin{bmatrix} 17 & -12 & 1 \\ 7 & 8 & 9 \\ 0 & -3 & -16 \end{bmatrix}$ **B** $\begin{bmatrix} -7 & 0 & -5 \\ -9 & -8 & -7 \\ 4 & -5 & 6 \end{bmatrix}$

C $\begin{bmatrix} 13 & -24 & -13 \\ -13 & -8 & -3 \\ 12 & -21 & -14 \end{bmatrix}$ **D** $\begin{bmatrix} 13 & -24 & -13 \\ -13 & -8 & -3 \\ 12 & -21 & -4 \end{bmatrix}$

Answers: **1.** Not possible **2.** $\begin{bmatrix} 32 & 15 & 19 \\ 9 & -9 & 2 \end{bmatrix}$ **3.** $\begin{bmatrix} 2 & -5 & 1 \\ 13 & 3 & -4 \end{bmatrix}$ **4.** $\begin{bmatrix} -2 & 5 & -1 \\ -13 & -3 & 4 \end{bmatrix}$ **5.** $\begin{bmatrix} 45 & 30 & 27 \\ -6 & -18 & 9 \end{bmatrix}$ **6.** $[14 \quad -10]$ **7.** $\begin{bmatrix} 8.5 & 2.5 & 5 \\ 5.5 & -1.5 & -0.5 \end{bmatrix}$ **8.** $\begin{bmatrix} 81 & 35 & 48 \\ 29 & -21 & 3 \end{bmatrix}$ **9.** Not possible **10.** $\begin{bmatrix} -32 & -15 & -19 \\ -9 & 9 & -2 \end{bmatrix}$ **11.** Not possible **12.** $\begin{bmatrix} -23 & 10 & -13 \\ -50 & -6 & 13 \end{bmatrix}$ **13.** C

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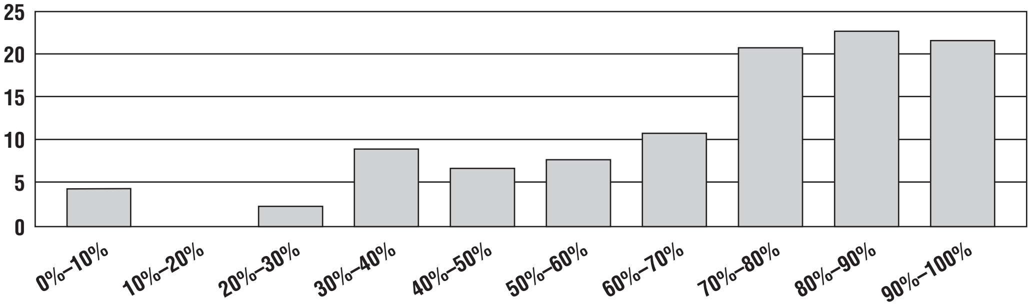
13-3 Histograms *(Pages 722–728)*

A **histogram** is a special type of bar graph in which the data are organized into intervals. The **frequency**, or number of values in each interval, determines the height of each bar in a histogram. The frequency can be found in a **frequency table**, which displays each interval's amount of data. When analyzing a histogram, note that the horizontal axis shows the range of data separated into **measurement classes** and the vertical axis shows the frequency.

Practice

Mrs. Jackson has a total of 100 students who participated in a stock market game. The students followed one stock each for a span of two weeks, then recorded the stocks current value compared to the stocks original value. For example, a stock that was originally $10.00 per share and is now $9.00 per share would be worth 90% of its original value. The histogram displays the percent of value of the stocks monitored by Mrs. Jackson's students. Use this information to answer the following questions.



1. In which interval does the median appear?

2. When looking at the distribution of the data, are there any intervals with no data? If so, which interval has no data?

3. When looking at the distribution of the data, would you say that the data are symmetrical? Why or why not?

4. How many students have a current stock value that is 50%−60% of the original value?

5. Which interval has the most elements?

6. **Standardized Test Practice** According to the information in the histogram, which of the following is a true statement?

A The value of every stock is less than or equal to the original value.
B Most stocks are currently 50%−60% of their original value.
C Some stocks increased in value over the two-week span.
D The 0%−10% interval contains the least amount of data.

Answers: 1. 70%–80% **2.** Yes, 10%–20% **3.** No, the distribution is skewed to the right. **4.** 7 **5.** 80%–90% **6.** A

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13-4 Measures of Variation *(Pages 731–736)*

Finding Measures of Variation	• The **range** of a set of data is the difference between the greatest and the least values of the set. • The **quartiles** in a set of data are the values that divide the data into four equal parts. • The *median* (Q_2) separates the data into two equal parts. • The **lower quartile** (Q_1) is the median of the lower half of the data. • The **upper quartile** (Q_3) is the median of the upper half of the data. • $Q_3 - Q_1$ is the **interquartile range**.

Example

Find the range, median, upper quartile, lower quartile, and interquartile range for 10, 14, 17, 18, 21, 25, 27, 28.

Notice that the data is already arranged in order from least to greatest.

range: *28 − 10 or 18*

lower quartile (Q_1): *(14 + 17) ÷ 2 or 15.5*

median (Q_2): *(18 + 21) ÷ 2 or 19.5*

upper quartile (Q_3): *(25 + 27) ÷ 2 or 26.*

interquartile range: *$Q_3 - Q_1$, or 26 − 15.5 or 10.5.*

Try These Together

Find the range, median, upper quartile, lower quartile, and interquartile range for each set of data.

1. 2, 5, 8, 7, 2, 1

2. 40°, 44°, 52°, 48°, 47°, 51°, 40°

HINT: First arrange the data in order. Then find the median. If the median is an item in the set of data, it is not included in either the upper or lower half of the data.

Practice

Find the range, median, upper quartile, lower quartile, and interquartile range for each set of data.

3.

Stem	Leaf
9	1
8	5 8 9
7	0 2 5 9
6	3

9|1 = 91 km

4.

Stem	Leaf
10	5
11	6 8 9
12	0 0 1 1 5 7

10|5 = 105 yd

5.

Stem	Leaf
2	0 0 5
3	0 5 5 5
4	0 5
5	5
6	0

2|0 = $20

6. School The following is a list of grades on an algebra test.
80, 92, 95, 70, 88, 60, 76, 90, 84, 74, 64, 98

a. Find the median and upper quartile.

b. What is the lowest grade in the top 25% of the data?

c. What is the lowest grade in the top 50% of the data?

7. Standardized Test Practice Which of the following does *not* represent approximately 25% of a data set?

A data below lower quartile **B** data between lower quartile and median

C data above upper quartile **D** data between upper and lower quartiles

Answers: 1. 7; 3.5; 7; 2; 5 **2.** 12; 47; 51; 40; 11 **3.** 28; 79; 88.5; 71; 17.5 **4.** 22; 120; 121; 118; 3 **5.** 40; 35; 45; 25; 20 **6a.** 82; 91 **6b.** 92 **6c.** 84 **7.** D

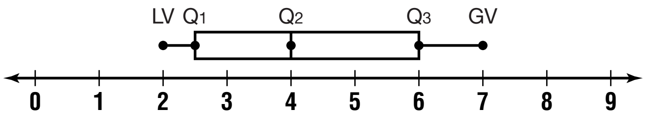
13-5 Box-and-Whisker Plots *(Pages 737–742)*

Drawing Box-and-Whisker Plots	1. Arrange data in numerical order. 2. Compute the *quartiles*: Q1, Q2, and Q3. The *median* (Q2) is the middle value of the data. The *upper quartile* (Q1) is the median of the lower half of the data and the *upper quartile* (Q3) is the median of the upper half of the data. 3. Find the **extreme values**. These are the *least value* (LV) and the *greatest value* (GV) of the data. 4. Draw a number line and choose a scale that includes the extreme values. Above the number line, draw dots corresponding to LV, Q1, Q2, Q3, and GV. Draw a box to designate the data between Q1 and Q3. Draw a vertical line through Q2. 5. Draw a segment from Q1 to LV and from Q3 and GV. These two segments are the **whiskers** of the plot.

Example

Draw a box-and-whisker plot for this data: 2, 2, 3, 4, 4, 5, 6, 6, 7.

The median, or Q_2, is 4. The LV is 2 and GV is 7. Q_1 is $(2 + 3) \div 2$ or 2.5. Q_3 is $(6 + 6) \div 2$ or 6.



Practice

1. Recreation The table shows the number of state parks in selected states.

State Parks in Midwest States													
State	**No.**	**State**	**No.**	**State**	**No.**	**State**	**No.**	**State**	**No.**	**State**	**No.**	**State**	**No.**
IA	53	IL	62	IN	23	KS	24	MI	68	MN	66	MO	47
ND	11	NE	8	OH	73	OK	47	SD	11	WI	51		

a. Make a box-and-whisker plot of the data.

b. Which half of the data is more widely dispersed?

2. Entertainment The running time in minutes of early and recent Academy Award Best Picture winners are listed in the table at the right.

1928–1947	139, 104, 103, 130, 112, 110, 105, 132, 179, 117, 127, 222, 130, 118, 139, 102, 126, 100, 170, 118
1980–1999	121, 122, 197, 162, 178, 142, 195, 131, 118, 181, 99, 128, 140, 113, 161, 158, 132, 188, 123, 124

a. Make a box-and-whisker plot of the data for each group of years.

b. Did the lengths vary more in early or recent years?

3. **Standardized Test Practice** About how much of the data does the box contain in a box-and-whisker plot?

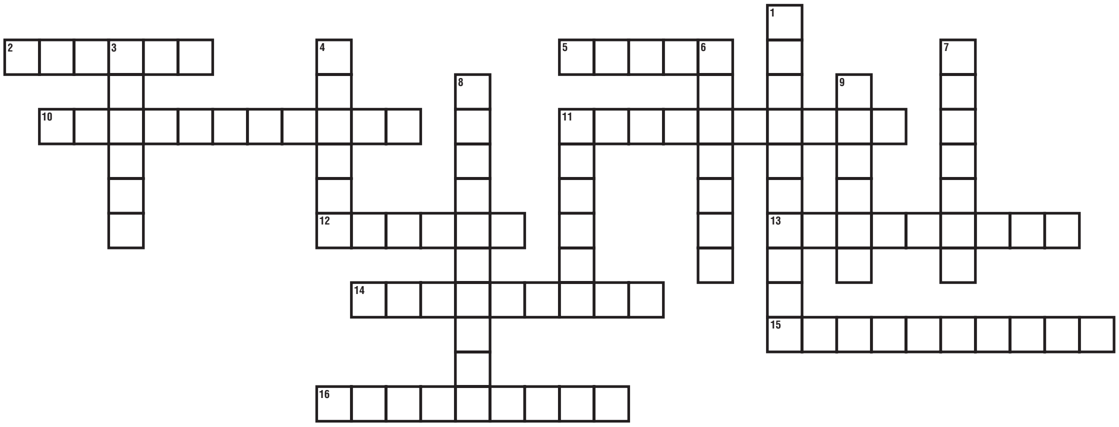
A one quarter **B** one half **C** all of the data **D** none of these

Answers: 1a. See Answer Key. **1b.** lower half **2a.** See Answer Key. **2b.** recent years **3.** B

13 Chapter Review

Crossword Puzzle

Complete the crossword puzzle.



Across

2. rectangular array of numbers
5. the difference between the greatest and least values in a set of data
10. a type of biased sample where the items are selected because of easy access
11. a type of random sample where items are selected according to a specific time or item interval
12. a portion of a larger group
13. the number of values in a measurement class
14. a bar graph in which the data are organized into equal intervals
15. the number of rows and columns of a matrix
16. a type of biased sample involving people who want to participate

Down

1. a type of random sample where the population is first divided into similar, nonoverlapping groups
3. a sample selected so that it represents the entire population
4. the survey where the entire population is included
6. an entry in a matrix
7. a value that is much less or much greater than the rest of the data
8. the large group represented by a sample
9. a sample that favors one or more parts of a population
11. the number in the multiplication of a number times a matrix

Answers are located in the Answer Key.

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14-1 Counting Outcomes *(Pages 754–758)*

Chapter 14

Tree diagrams and the Fundamental Counting Principle are two methods of calculating the total number of possible outcomes for any situation. A **tree diagram** is a picture that creates a list of every possible outcome. This list is called a **sample space** and each individual element of the sample space is called an **event**. The **Fundamental Counting Principle** uses multiplication to find the total number of outcomes.

Fundamental Counting Principle	If an event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by event N can occur in $m \cdot n$ ways.

A **factorial** may be used to find the total number of outcomes of a scenario with descending amounts of choices. The factorial of n, written as $n!$, is calculated by $n \cdot (n - 1) \cdot (n - 2) \cdot ... \cdot 3 \cdot 2 \cdot 1$.

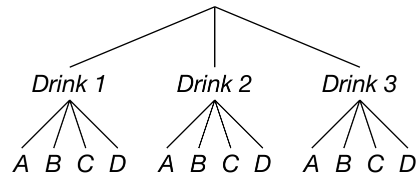
Examples

a. How many lunches can you choose from 3 different drinks and 4 different sandwiches?

Letter the different sandwiches A, B, C, and D.

A tree diagram shows 12 as the number of outcomes.

You could also use the Fundamental Counting Principle.



number of types of drinks	×	number of types of sandwiches	=	number of possible outcomes
3	×	4	=	12

There are 12 possible outcomes.

b. Find the value of 5!.

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$5! = 120$

c. How many ways can you place 8 books on a shelf?

$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$8! = 40{,}320$

Practice

Use a tree diagram or the Fundamental Counting Principle to find the total number of outcomes.

1. A restaurant menu has a special where you can select from 3 meats, 2 vegetables and 2 drinks.

2. A soccer team's kit consists of 2 jerseys, 2 pairs of shorts, and 2 pairs of socks.

3. A pizza shop offers 10-inch, 12-inch, and 16-inch sizes with thin, thick, deep dish, or garlic crust. Also, the customer can choose a topping from extra cheese, pepperoni, sausage, mushroom, and green pepper.

4. **Standardized Test Practice** In how many ways can a group of 10 people form a line for an amusement park ride?

A 100,000 **B** 3,628,800 **C** 1,814,400 **D** 403,200

Answers: 1. 12 **2.** 8 **3.** 60 **4.** B

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14-2 Permutations and Combinations

(Pages 760–767)

An arrangement in which order is important is called a **permutation**. Arrangements or listings where the order is not important are called **combinations**. Working with these arrangements, you will use **factorial** notation. The symbol 5!, or 5 factorial, means $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. The expression $n!$ means the product of all counting numbers beginning with n and counting backwards to 1. The definition of 0! is 1.

Working with Permutations and Combinations	The symbol ${}_7P_3$ means the number of permutations of 7 things taken 3 at a time. To find ${}_7P_3$ use the formula ${}_nP_r = \frac{n!}{(n-r)!}$, or $\frac{7!}{(7-3)!} \cdot \frac{5040}{24} = 210$. The symbol ${}_7C_3$ means the number of combinations of 7 things taken 3 at a time. To find ${}_7C_3$ use the formula ${}_nC_r = \frac{n!}{(n-r)!r!}$, or $\frac{7!}{(7-3)!3!} \cdot \frac{5040}{144} = 35$.

Examples

a. Find ${}_5P_3$

${}_5P_3 = 5 \cdot 4 \cdot 3$ or 60

$${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

b. Find ${}_5C_3$

First find the value of ${}_5P_3$ or $\frac{5!}{(5-3)!3!}$.

From Example A, you know that ${}_5P_3$ is 60.

Divide 60 by 3!. This is $\frac{60}{6}$ or 10.

c. Fred plans to buy 4 tropical fish from a tank at a pet shop. Does this situation represent a permutation or a combination? Explain.

This situation represents a combination. The only thing that matters is which fish he selects. The order in which he selects them is irrelevant.

Practice

Tell whether each situation represents a permutation or combination.

1. a stack of 18 tests

2. two flavors of ice cream out of 31 flavors

3. 1st-, 2nd-, and 3rd-place winners

4. 20 students in a single file line

How many ways can the letters of each word be arranged?

5. RANGE

6. QUARTILE

7. MEDIAN

Find each value.

8. ${}_5P_2$

9. ${}_{10}P_3$

10. 7!

11. 9!

12. ${}_7C_2$

13. ${}_{12}C_3$

14. $\frac{5!2!}{3!}$

15. $\frac{8!4!}{7!3!}$

16. **Standardized Test Practice** If there are 40 clarinet players competing for places in the district band, how many ways can the 1st and 2nd chairs be filled?

A 40!

B $40 \cdot 39$

C $\frac{40 \cdot 39}{2!}$

D 2

Answers: 1. permutation **2.** combination **3.** permutation **4.** permutation **5.** 120 ways **6.** 40,320 ways **7.** 720 ways **8.** 20 **9.** 720 **10.** 5040 **11.** 362,880 **12.** 21 **13.** 220 **14.** 40 **15.** 32 **16.** B

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14-3 Probability of Compound Events *(Pages 769–776)*

A **compound event** consists of two or more simple events. When one event *does not* affect the others, we say that these are **independent events**. If the outcome of an event *does* affect the outcome of another event, we say that these are **dependent events**.

Examples

A bag contains 4 red marbles, 5 blue marbles, and 3 green marbles. Two marbles are picked at random. Find each probability.

a. 2 red marbles if the first marble is returned before the second is chosen

Since the first marble is returned before the second one is chosen, the events are independent.

$P(\text{red}) = \frac{4}{12}$ or $\frac{1}{3}$

$P(\text{red, then red}) = \frac{1}{3} \cdot \frac{1}{3}$ or $\frac{1}{9}$

b. 2 red marbles if the first marble is *not* returned before the second is chosen

Since the first marble is not *returned before the second one is chosen, the events are dependent.*

$P(\text{red}) = \frac{4}{12}$ or $\frac{1}{3}$

$P(\text{red after one red is selected}) = \frac{3}{11}$

$P(\text{red, then red}) = \frac{1}{3} \cdot \frac{3}{11}$ or $\frac{1}{11}$

Practice

1. **School** Eva forgot to study one of the chapters for her history test so she had to guess on two multiple-choice questions which each had four answer choices. What is the probability that she got both questions correct?

2. During a magic trick, a magician randomly selects two cards from a standard deck of cards.
   - **a.** Find the probability both cards are clubs if the first card is returned to the deck before the second card is selected.
   - **b.** Find the probability both cards are clubs if the first card is not returned to the deck before the second card is selected.

3. **Gift Wrapping** A gift-wrapping service offers the following choices.
   Paper: Sunflowers, Stripes, Spirals, Silver, Plaid
   Ribbon: White, Silver, Yellow, Gold
   - **a.** What is the probability that a customer who chooses at random will choose sunflower paper and yellow ribbon?
   - **b.** If you choose at random, what is the probability of selecting paper with either stripes or spirals with white ribbon?

4. **Standardized Test Practice** The probability that Tara will make a free throw is $\frac{3}{4}$. What is the probability that Tara will make her next two free throws?

   A $\frac{3}{4}$ **B** $\frac{1}{2}$ **C** $\frac{9}{16}$ **D** $\frac{3}{8}$

Answers: 1. $\frac{1}{16}$ 2a. $\frac{1}{16}$ 2b. $\frac{1}{17}$ 3a. $\frac{1}{20}$ 3b. $\frac{1}{10}$ 4. C

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14-4 Probability Distributions *(Pages 777–781)*

A **random variable** is a variable whose value is the numerical outcome of a random event. The probability of every possible value of the random variable is called a **probability distribution**. Probability distributions have the following properties.

1. The probability for each random variable x is $0 \leq x \leq 1$.
2. The sum of the probabilities for each value x is 1.
3. The probability for any compound event is equal to the sum of the probabilities of each individual event.

Examples **The owner of a bicycle shop recorded the number of bicycles owned by each of his customers. The results are shown in the table.**

Number of Bicycles	Number of Customers
1	13
2	21
3	17
4	11
5+	2

a. Find the probability that a randomly chosen person owns 3 bicycles.

$P(X = 3) = \frac{17}{64}$ — *The number of customers with 3 bicycles divided by the total number of people surveyed*

$P(X = 3) = 0.265625$

$P(X = 3) = 26.5625\%$

b. Find the probability that a randomly chosen person owns at least 4 bicycles.

$P(X \geq 4) = \frac{13}{64}$

$P(X \geq 4) = 0.203125$

$P(X \geq 4) = 20.3125\%$

Practice

Use the probability distribution table to answer the following questions.

X = Number of Bicycles	$P(X)$
1	0.203125
2	0.328125
3	0.265625
4	0.171875
5+	0.03125

1. What is the probability that a randomly chosen person has less than 3 bicycles?

2. What is the probability that a randomly chosen person has at least 3 bicycles?

3. **Standardized Test Practice** What is the probability that a randomly chosen person will have at least 1 bicycle?

A 20.3125% **B** 79.6875% **C** 100% **D** 120.3125%

Answers: 1. 53.125% **2.** 46.875% **3.** C

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14-5 Probability Simulations *(Pages 782–788)*

The type of probability that you have used so far is **theoretical probability**, which is calculated by dividing the number of favorable outcomes by the number of total possible outcomes. Probability can also apply to the actual data that is collected by conducting an experiment. This type of probability is called **experimental probability**. Experimental probability is a ratio that compares the **relative frequency**, or the number of times a favorable outcome occurred, with the total number of times the experiment was conducted. Performing an experiment many times, recording data, and analyzing results is called an **empirical study**. When conducting an empirical study with an event that may be unrealistic to perform, you can use a **simulation**, or similar experiment with the same probability as the desired experiment.

Calculating Theoretical Probability	$P(\text{event}) = \dfrac{\text{the number of favorable outcomes}}{\text{the number of possible outcomes}}$
Calculating Experimental Probability	$P(\text{event}) = \dfrac{\text{the relative frequency of favorable events}}{\text{total number of events}}$

Examples

A Number Cube is Rolled 20 Times	
Number Rolled	**Frequency**
1	2
2	5
3	3
4	8
5	1
6	1

a. What is the theoretical probability of rolling a 6 on a number cube?

$P(6) = \frac{1}{6}$

$P(6) = 16.\overline{6}\%$

b. According to the data, what is the experimental probability of rolling a 6 on a number cube?

$P(6) = \frac{1}{20}$

$P(6) = 5\%$

Practice

A card is drawn from a standard deck of 52 playing cards. This process is repeated a total of 100 times. The results have been recorded in the table. Use this information for Exercises 1–3.

Clubs	22
Diamonds	17
Hearts	31
Spades	30

1. What is the experimental probability of drawing a club?

2. What is the experimental probability of drawing a diamond or a spade?

3. **Standardized Test Practice** What is the theoretical and experimental probability of drawing a heart or a club?

A 50%, 53% **B** 25%, 31% **C** 25%, 22% **D** 50%, 48%

Answers: 1. 22% **2.** 47% **3.** A

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14 Chapter Review

Hidden Picture

Find each value.

1. 5!

2. 8!

3. ${}_{10}P_4$

4. ${}_7P_2$

5. ${}_5C_3$

6. ${}_8C_5$

7. the number of sandwiches that can be made if a person must choose one out of 3 different types of meat, one out of 2 different types of cheese, and one out of 4 different types of bread

8. the number of outfits that can be worn if a person must choose one out of 5 pairs of slacks, one out of 6 shirts, and one out of 2 jackets

9. the number of ways 6 children can form a line

10. the number of ways to choose the first 3 batters from 9 baseball players

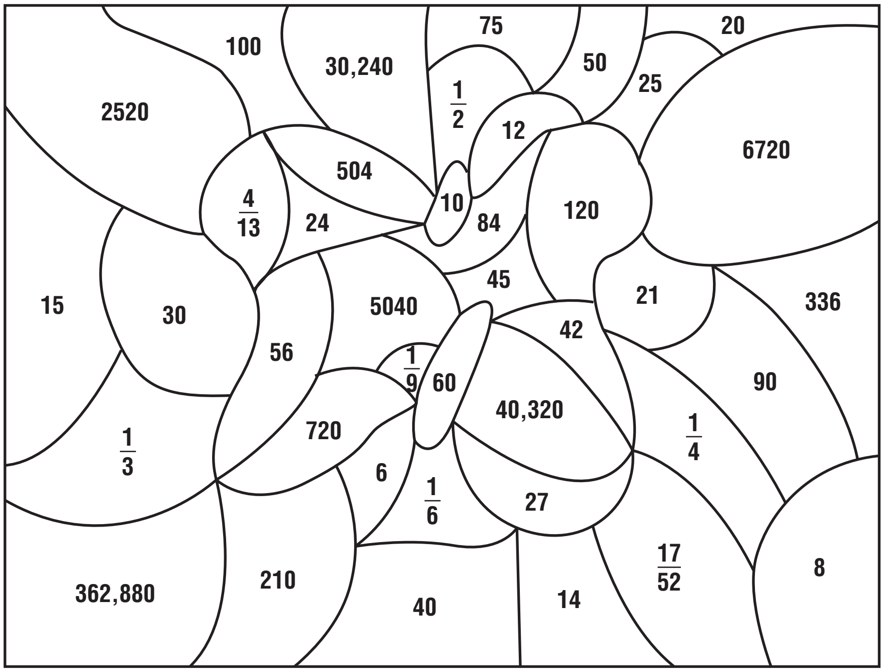
11. the number of ways to choose 2 committee members from 10 students

12. the number of ways to choose 3 types of candy bars out of 9 types of candy bars to sell for the band fundraiser

13. the probability that a card chosen at random from a standard deck of cards is either an ace or a club

14. the probability that a die is rolled twice and both times the number is less than 3

Shade in each region containing an answer to the Exercises 1–14. What do you see?



Answers are located in the Answer Key.

Answer Key

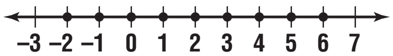
Lesson 1-6

19. $6(g + a) + 3g = 6g + 6a + 3g$
distributive property
$= 6g + 3g + 6a$
commutative property (+)
$= (6g + 3g) + 6a$
associative property (+)
$= (6 + 3)g + 6a$
distributive property
$= 9g + 6a$
substitution (=)

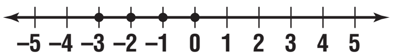
Chapter 1 Review

1. $5 \cdot 1 + 6 \div 2 = 8$ **2.** $3 \cdot (8 + 2) = 30$
3. $9 \div (5 - 8 \div 4)^2 = 1$
4. $2x + 7x + 6y + 8x = 17x + 6y$
5. $5(n + 1) + 3(n + 6) = 8n + 23$
6. $6 + 5 = 5 + 6$; commutative property (+)

Lesson 2-1

5.


6.


7.


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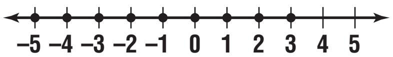

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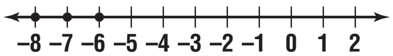
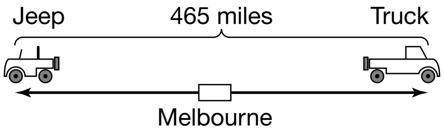


Chapter 2 Review

1a. 1°C **1b.** 5°C **1c.** −7°C **1d.** 3°C **1e.** −2°C **1f.** 21°C **2.** Berlin: 34°F; London: 42°F; Montreal: 18°F; Paris: 38°F; Beijing: 28°F; Sao Paulo: 74°F **3.** Sao Paulo, Brazil, is in the southern hemisphere, and December is summertime in the southern hemisphere. **4.** Answers will vary.

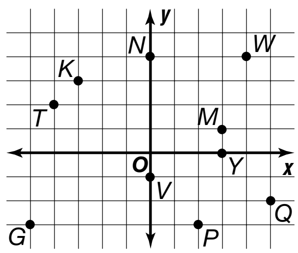
Lesson 3-9

2.


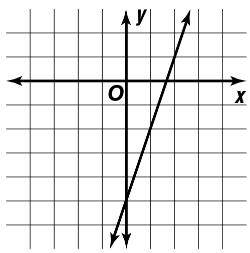
Chapter 3 Review

1. −3 **2.** 5 **3.** −12 **4.** 6 **5.** 7 **6.** −30
7. −7 **8.** 31 **9.** −9 **10.** 4 **11.** 8
Solution to puzzle: algebra is fun

Lesson 4-1

10–19.


Lesson 4-5

2. $y = 3x - 5$


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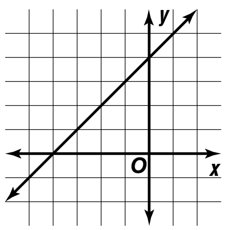

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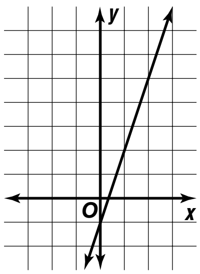

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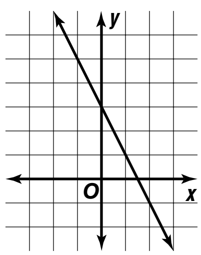
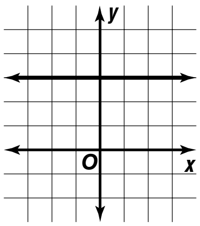


Answer Key

12.


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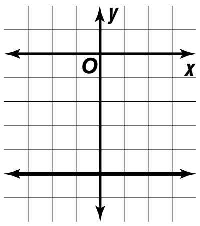

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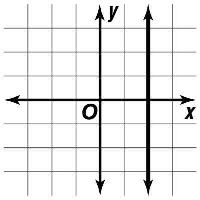

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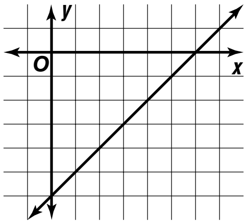

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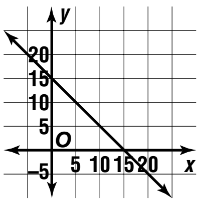

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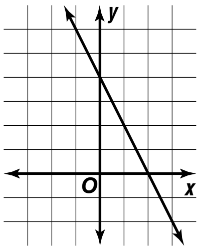
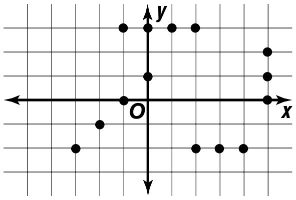


Chapter 4 Review

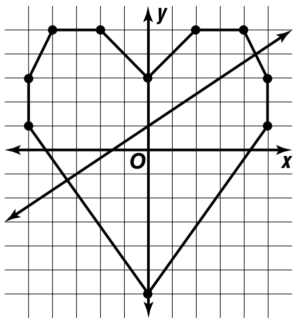
Clue 1: $D = \{0, 5\}$; $R = \{1, 3\}$
Clue 2: $\{(0, 5), (-2, 4), (-2, 3)\}$
Clue 3: The points are: $(-2, -1)$, $(-1, 0)$, and $(2, 3)$.
Clue 4: no; yes; yes
Clue 5: 4



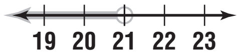
Chapter 5 Review

1. $m = -2$ **2.** $m = 0$ **3.** undefined slope **4.** $y = 5$ **5.** $x = -5$ **6.** Sample answer: $y - 3 = 2(x + 5)$. **8.** $m = -\frac{7}{5}$

10. $y = -x + 3$



Lesson 6-1

1.


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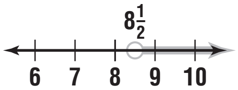

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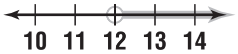

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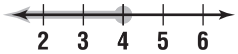

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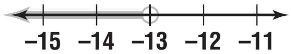
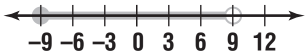


6.


Lesson 6-4

1.


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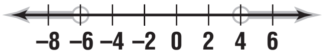

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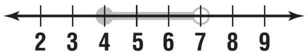

4.


5.


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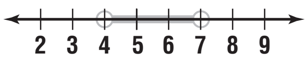

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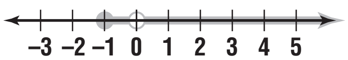
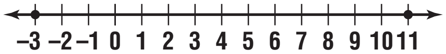
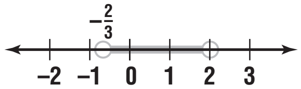
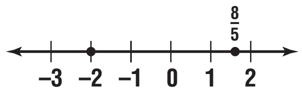


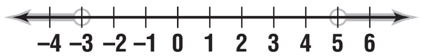
Answer Key

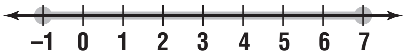
Lesson 6-5

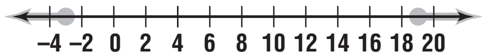
1. 

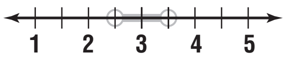
2. 

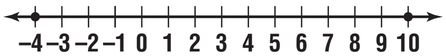
3. 

4. 

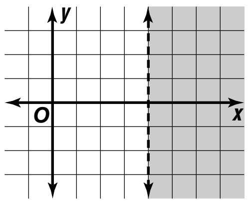
5. 

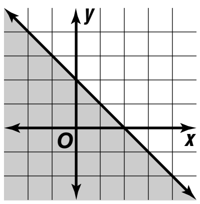
6. 

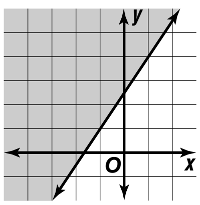
7. 

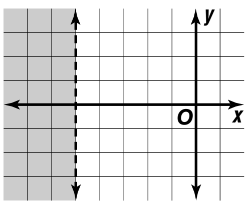
8. 

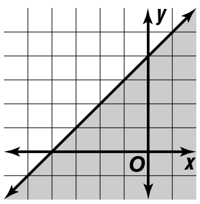
Lesson 6-6

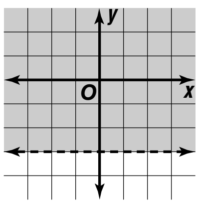
3. 

4. 

5. 

6. 

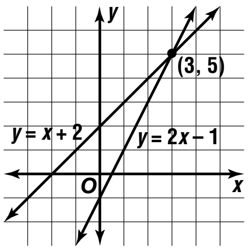
7. 

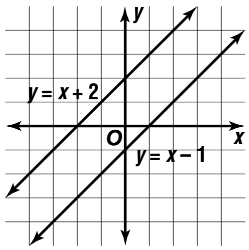
8. 

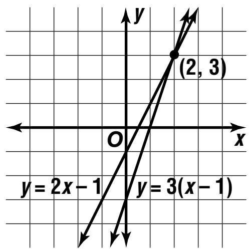
Chapter 6 Review

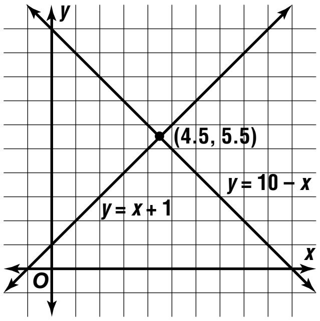
1. $x \leq 4$ **2.** $t > 4$ **3.** $n < 4$ **7.** $w < 5$ and $w > -3$ **9.** $p \geq 5$ or $p \leq -3$
NY YANKEES TICKETS

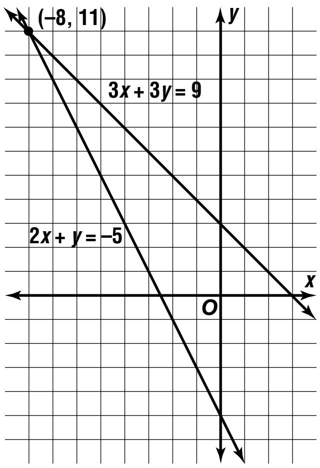
Lesson 7-1

1. 

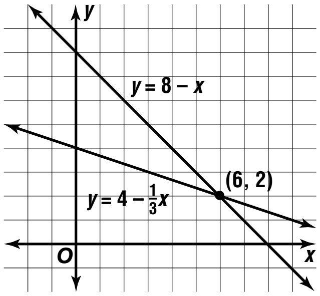
2. 

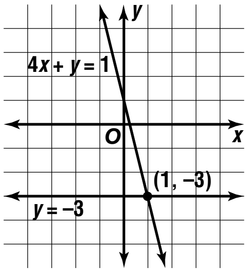
3. 

4. 

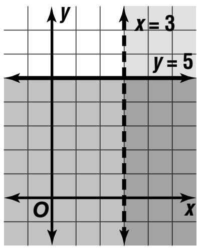
5. 

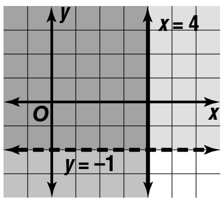
Answer Key

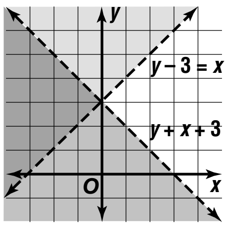
6. 

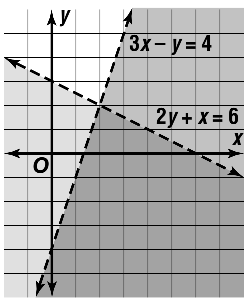
7. 

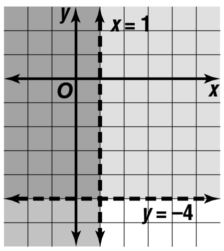
Lesson 7-5

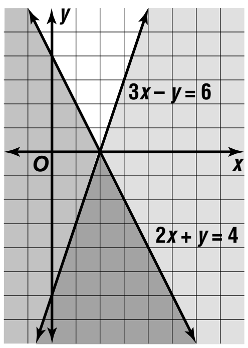
1. 

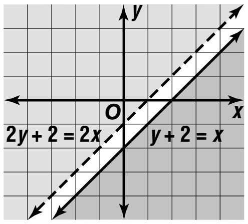
2. 

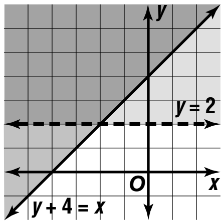
3. 

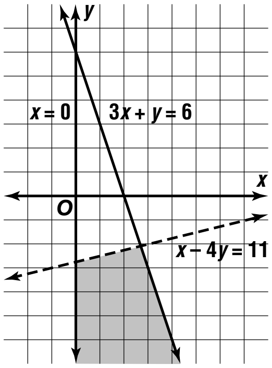
4. 

5. 

6. 

7. 

8. 

9. 

Chapter 7 Review

Gold: (−2, 4); Silver: (4, −3); Diamonds: no solution; Jewels: (−1, 0); The gold is nearest to the starting point, (0, 5).

Chapter 8 Review

1. $18x^7$ **2.** $2x$ **3.** $2x^3 - 6x^2 - 2x$
4. $2x^3 + 2x$ **5.** $x + 5$ **6.** $x^2 + 11x + 30$
7. 6.5×10^2 **8.** 2.47×10^6
2,470,000; $24,700

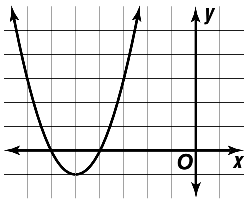
Chapter 9 Review

1. $9x(2 - y)$ **2.** $2x(2x^2 + 3)$
3. $(x - 8)(x + 8)$ **4.** $(x - 4)(x + 4)$
5. $2(x - 4)(x + 4)$ **6.** $(x + 2)(x + 4)$
7. $(x - 4)(x - 2)$ **8.** $(x - 3)(x + 4)$
9. $(x - 4)(x + 3)$ **10.** $(x + 2)(x + y)$
11. $(x + 4)(y - x)$ **12.** $(x + 4)(x + 2y)$
The outdated technology: EIGHT TRACK

Answer Key

Lesson 10-1

1.



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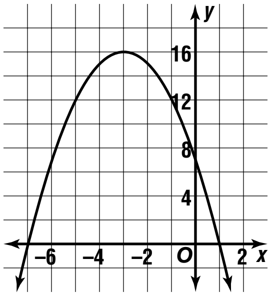

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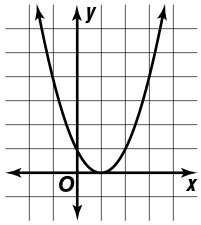

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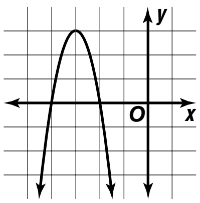

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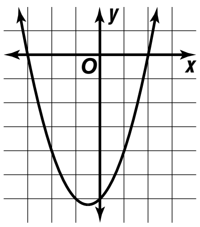

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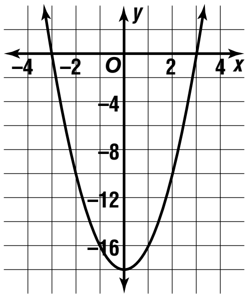

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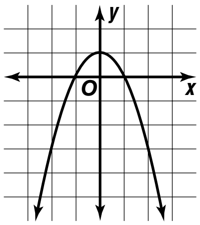

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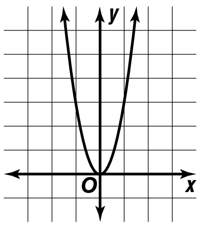

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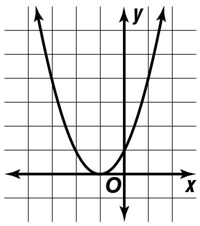
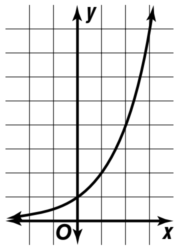


Lesson 10-5

1.



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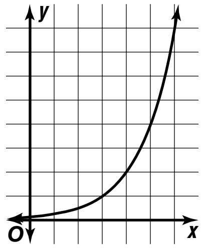

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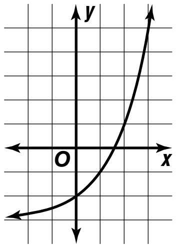

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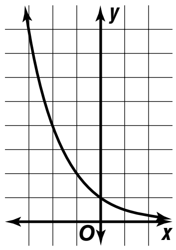

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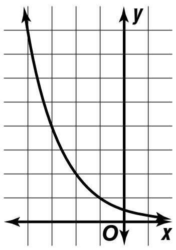

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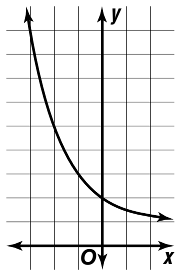
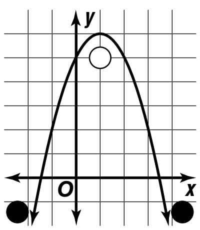
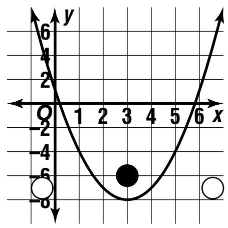


Chapter 10 Review

Hole 1: $y = -x^2 + 2x + 5$

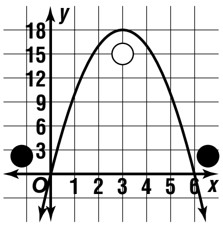


Hole 2: $y = x^2 - 6x + 1$

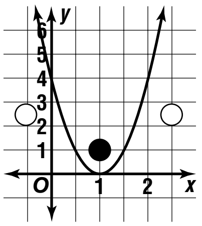


Answer Key

Hole 3: $y = -2x^2 + 12x$



Hole 4: $y = 4x^2 - 8x + 4$

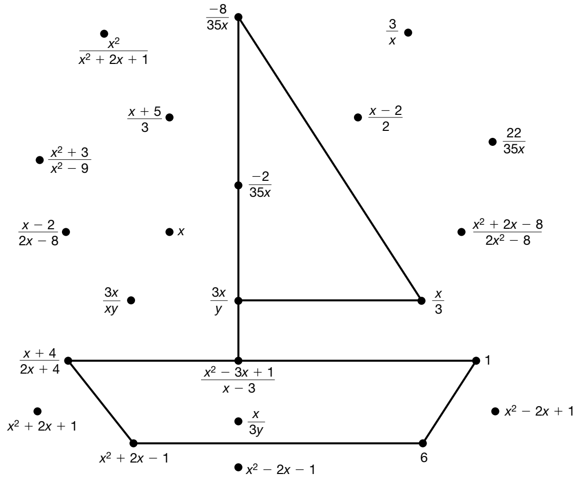


Chapter 11 Review

1. 8 (X—Leaky Roof) **2.** $4|x|\sqrt{5}$
3. $6b^2|c|\sqrt{2ab}$ **4.** 3 (X—Leaky Roof)
5. $\sqrt{6}$ **6.** $-3\sqrt{5} + 4\sqrt{7}$
7. 0 (X—Leaky Roof) **8.** $3 + 2\sqrt{3}$
9. -32 (X—Leaky Roof) **10.** $\frac{30 - 5\sqrt{3}}{33}$

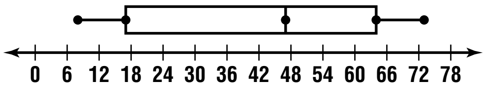
Chapter 12 Review

1. $\frac{3x}{y}$ **2.** $\frac{x}{3}$ **3.** $\frac{x + 4}{2x + 4}$ **4.** 1 **5.** $x^2 + 2x - 1$
6. 6 **7.** $\frac{-8}{35x}$ **8.** $\frac{x^2 - 3x + 1}{x - 3}$



Lesson 13-5

1a.



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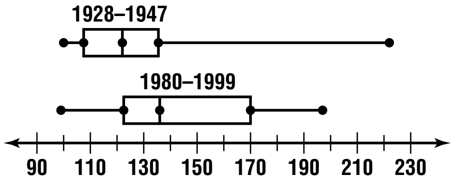
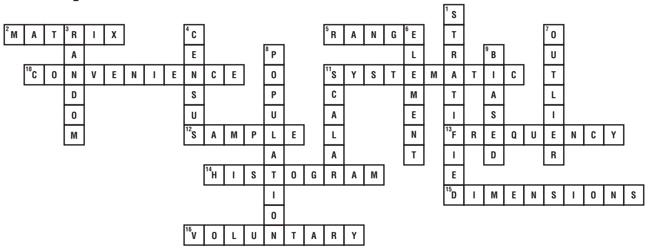
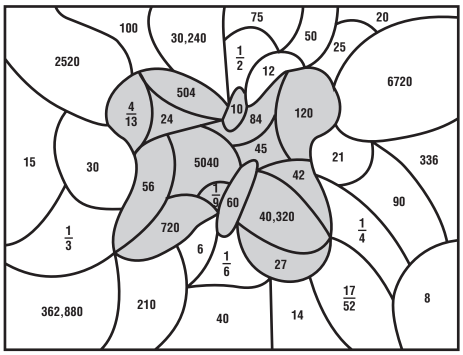


Chapter 13 Review



Chapter 14 Review

1. 120 **2.** 40,320 **3.** 5040 **4.** 42 **5.** 10
6. 56 **7.** 24 **8.** 60 **9.** 720 **10.** 504
11. 45 **12.** 84 **13.** $\frac{4}{13}$ **14.** $\frac{1}{9}$



a butterfly