Activity 1.1: Wild about Harry

SOLs: None

Objectives: Students will be able to:
- Practice communication skills
- Organize information
- Write a solution in sentences
- Develop problem-solving skills

Vocabulary: none new

Activity: Examine the problem statement on the view graph and answer in your notes the following questions:

1. What is your initial reaction? Stay in line or go?

2. Have you worked a problem like this before?

3. To help solve the problem, organize the information in the description
   a. How many customers must leave the book store before the guard allows more to enter?

   b. How many customers per minute leave the book store?

   c. How many minutes are there between groups of customers entering the book store?

   d. How long will you stand in line outside the book store?

   e. How early or late for your class will you be?

4. In a few complete sentences, write what you did to solve the problem.

Concept Summary:
Steps in problem solving:
1. Sort out the relevant information and organize it.
2. Discuss the problem with others to increase your understanding of the problem.
3. Write your solution in complete sentences to review your steps and check your answer.

Homework: none
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.2: The Handshake

SOLs: None

Objectives: Students will be able to:
- Organize information
- Develop problem solving strategies: Draw a picture, Recognize a pattern, Do a simpler problem
- Communicate problem-solving ideas

Vocabulary:
- Arithmetic sequence – a list of numbers in which consecutive numbers share a common difference
- Geometric sequence – a list of numbers in which consecutive numbers share a common ratio
- Fibonacci sequence – a list of numbers in which consecutive numbers are added to get the next number
- Inductive reasoning – arrives at a general conclusion form specific examples
- Deductive reasoning – uses laws and properties to prove/disprove conjectures

Activity: Examine the problem statement on the view graph and answer in your notes the following questions:

1. How many people were in your group?

2. How many handshakes in all were there in your group?

3. Fill in the following table:

<table>
<thead>
<tr>
<th>Number of students in group</th>
<th>Number of handshakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

4. Describe a rule for determining the number of handshakes in a group of
   A. seven students
   B. students in our class
   C. n students
Chapter 1: Introduction to Problem Solving and Mathematical Models

The Classroom Layout
5. Construct a table of values for the number of tables and the corresponding number of students

<table>
<thead>
<tr>
<th>Number of square tables in cluster</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

6. Describe the pattern that connects the number of square tables in a cluster and the total number of students that can be seated. Write a rule in sentences that will determine the total number of students that can sit in a cluster of a given number of square tables, n.

7. There are 24 students in a science course at MSHS
   A. How many tables must be put together to seat a group of 6 students?

   B. How many clusters of tables are needed for that class?

8. How would we use square table clusters in our class?

Concept Summary:
- Problem Solving Strategies
  - Discussing the problem
  - Organizing information
  - Drawing a picture
  - Recognizing patterns
  - Doing a simpler problem
- Four-step process
  - Understand the problem
  - Carry out the plan
  - Devise a plan
  - Look back at the completed solution

Homework: pg 6-9; 1-7, 9
Activity 1.3: Make Me an Offer

Objectives: Students will be able to:
- Use the basic steps for problem-solving
- Translate verbal statements into algebraic equations
- Use the basic principles of algebra to solve real-world problems
- Use formulas to solve problems

Vocabulary:
Formula – an equation involving letters as variables

Activity: Examine the problem statement on the view graph and answer in your notes the following questions:

1. “Night Train” Henson is negotiating a new contract with his team. He wants $800,000 for the year and an additional $6000 for every game he starts. His team offered $10,000 for every game he starts, but only $700,000 for a base salary. How many games would he need to start in order to make more with the team’s offer?

2. You need to open a checking account and decide to shop around for a bank. Acme bank has an account with a $10 monthly charge, plus 25 cents per check. Farmer’s bank will charge you $12 per month, with a 20 cent charge per check. How many checks would you need to write each month to make Farmer’s bank the better deal?

3. The perimeter of a rectangular pasture is 2400 feet. If the width is 800 feet, how long is the pasture?

1) Net income = Revenue − Cost
2) Net Pay = Gross Income − Deductions\(\)
3) Depreciation = (Ocost − Rvalue) ÷ Est Life
4) \( F = \left(9C \div 5\right) + 32 \)
   a) 20°C
   b) 100°C
5) \( C = \frac{5(F - 32)}{9} \)
   a) 86°F
   b) 41°F
6) Distance = rate × time
   a) 50 mph; 5 hr
   b) 25 km/min; 2 hr

Concept Summary:
- Understand the problem
- Develop a strategy for solving the problem
- Execute your strategy to solve the problem
- Look back at the completed solution

Homework: pg 17; 1-10
Activity 1.4: Proportional Reasoning

SOLs: None

Objectives: Students will be able to:
- Use proportional reasoning as a problem-solving strategy
- Write a proportion and then solve the resulting proportion

Vocabulary:
- Ratio – comparisons using quotients
- Cross Multiplication – multiplying the numerator of one ratio by the denominator of the other ratio
- Proportion – two ratios set equal to each other (solved using cross multiplication)
- Percent – a number divided by 100 (100% is the whole)
- Equivalent – numbers having the same numerical or decimal value
- Proportional Reasoning – ability to recognize when two ratios are equivalent

Key Concept:

Ways to express a ratio:
- Verbally – he made 3 out of 4 free-throws
- Fraction – he made ¾ of his foul shots
- Division – he made $4 \div 12$ or $\frac{1}{3}$ field goals
- Decimal – his probability of success was 0.33
- Percentage – he made 33% of his field goals

Proportional Problem Solving:
Solving any proportion problem generally requires the following:

\[
\frac{a}{b} = \frac{c}{d} \quad \text{Write the proportions}
\]

\[
ad = bc \quad \text{Cross-multiplication}
\]

\[
a = \frac{bc}{d} \quad \text{Solve for } a \text{ by dividing by } d
\]

Description:
The following table summarizes Michael Jordan's statistics during the six games of one of the NBA championship series.

<table>
<thead>
<tr>
<th>Game</th>
<th>Points</th>
<th>Field Goals</th>
<th>Free Throws</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>9 – 18</td>
<td>9 – 10</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>9 – 22</td>
<td>10 – 16</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>11 – 23</td>
<td>11 – 11</td>
</tr>
<tr>
<td>4</td>
<td>23</td>
<td>6 – 19</td>
<td>11 – 13</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
<td>11 – 22</td>
<td>4 – 5</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>5 – 19</td>
<td>11 – 12</td>
</tr>
</tbody>
</table>

What was his points per game average?
In which game did he score the most points?
In which game did he score the most field goals? free throws?
The following table summarizes Michael Jordan’s *free-throw* statistics from above table.

<table>
<thead>
<tr>
<th>Game</th>
<th>Verbal</th>
<th>Fraction</th>
<th>Division</th>
<th>Decimal</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. In which game was his relative free-throw performance the highest?
2. In which game was his actual free-throw performance the lowest?
3. In which game was his relative free-throw performance the lowest?

1. Equivalent Ratios: Fill in the blanks
   a. 3 out 4 is equivalent to _____ out of 12
   b. 3 out 4 is equivalent to _____ out of 32
   c. 3 out 4 is equivalent to _____ out of 100

2. Write each ratio in fraction form and reduce it:
   a. 27 out of 75
   b. 63 out of 175
   c. 36 out of 100

3. A recent survey indicated that 1 out of every 5 New York City Police (NYPD) officers holds a four-year college degree. There were approximately 41,000 NYPD officers when the survey was conducted. How many hold a four-year degree?

4. New York state has taken a leading position in raising the standards of its high school graduates. In 2003, every graduate needed to pass a series of rigorous subject-matter tests called Regents exams. Currently your cousin lives in a NY county in which only 6 out of 10 graduates receive Regent diplomas. If 5400 students in your cousin’s county earned a Regents diploma last year, how many graduated from high school?

**Concept Summary:**
Ratios can be expressed in a variety of ways
- fraction like a part / whole
- verbally like 2 out of 3
- percentage like 58% (58/100)
- decimal like 0.58

Proportions are two ratios set equal
Cross-multiplication is a method to solve for missing values in proportions

**Homework:** pg 25-7; 1-12
Activity 1.5: Fuel Economy

Objectives: Students will be able to:
Apply rates directly to solve problems
Use proportions to solve problems
Use unit of dimensional analysis to solve problems

Vocabulary:
Rate – a comparison, expressed as a quotient of two quantities that have different units of measure
Unit Analysis – uses units of measure as a guide in setting up calculations or writing an equation involving one or more rates
Direct Method – multiplies or divides directly by rates to solve problem
Proportion Method – sets up and solves a proportion

Key Concept:
• Rate is a comparison, expressed as a quotient of two quantities of different units of measure (miles per gallon)
• Methods for Solving Problems Involving Rates:
  – Direct Method: apply a know rate directly by multiplication of division to solve a problem
  – Proportion Method: Set up a proportion involving the rates and solve the proportion
  – Unit Analysis: use the units of the numbers involved to set up an equation where the units are canceled out to get the units of the answer

Direct Method:
• Identify the unit of the result
• Setup the calculation so the appropriate units will divide out, leaving the unit of the result
• Multiply or divide the numbers as usual to obtain the numerical part of the result
• Divide out the common units to obtain the unit of the answer

Proportion Method:
• Identify the known rate; write it in fractional form
• Identify the given information and the quantity to be determined
• Write a second fraction, placing the given information and the quantity x in the same positions as their units in the known rate
• Equate the two fractions to obtain a proportion
• Solve the equation for x, affixing the correct unit to the numerical results

Unit Analysis Method:
• Identify the measurement unit of the result
• Set up the sequence of multiplications so that the appropriate units divide out, leaving the appropriate measurement unit of the result
• Multiply and divide the numbers as usual to obtain the numerical part of the results
• Check that appropriate measurement units divide out, leaving the expected unit for the result

Activity:

<table>
<thead>
<tr>
<th>Make</th>
<th>City MPG</th>
<th>City Miles on 5 Gal</th>
<th>Hi-way MPG</th>
<th>Gal needed to drive 304 miles</th>
<th>Fuel tank capacity in Gal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chevy Cobalt</td>
<td>32</td>
<td>41</td>
<td></td>
<td></td>
<td>13.2</td>
</tr>
<tr>
<td>Ford Focus</td>
<td>28</td>
<td>36</td>
<td></td>
<td></td>
<td>13.2</td>
</tr>
<tr>
<td>Honda Civic</td>
<td>33</td>
<td>39</td>
<td></td>
<td></td>
<td>13.2</td>
</tr>
<tr>
<td>Hyundai Accent</td>
<td>28</td>
<td>36</td>
<td></td>
<td></td>
<td>11.9</td>
</tr>
<tr>
<td>Toyota Corolla</td>
<td>34</td>
<td>41</td>
<td></td>
<td></td>
<td>11.9</td>
</tr>
</tbody>
</table>
Chapter 1: Introduction to Problem Solving and Mathematical Models

- For each of the cars listed in the table, how many city miles can you travel per week on five gallons of gas?
- The round trip to your high school is 29 city miles, which you do five days per week. Which of the cars would get you to school for the week on 5 gallons of gas?
- The round trip each week to your summer job requires 304 highway miles. How many gallons of gas would each of the cars require? Which car would be the best?

1. **Example 1**: The gas tank of a Ford Focus holds 13.2 gallons. How many highway miles can you travel on a full tank of gas?

   The Toyota Corolla gas tank holds 11.9 gallons. Is it possible to travel as far on the highway in this car as in the Ford Focus?

2. **Example 2**: You bought the Hyundai Accent and you would like to visit your good friend in another state. The highway distance is approximately 560 miles. Solve the following problems:
   - How many gallons of gas would you need to make the round trip?
   - How many tanks of gas would you need for the trip?

3. **Example 3**: You are on a 1500 mile trip and gas stations are far apart (Montana). Your car averages 40 mpg and you are traveling at 60 mph. The fuel tank holds 12 gallons of gas and you just filled the tank. How long is it before you have to fill the tank again?
   - In terms of miles:
   - In terms of hours:

4. **Example 4**: In Canada they measure distances in kilometers (1.609 km = 1 mile) and they measure volume in liters (1 liter = 0.264 gallons).
   - If you bought 20 liter of gas, how many gallons did you buy?
   - Your car’s highway efficiency is 45 mpg. What is its fuel efficiency in kilometers per liter?

**Concept Summary**:
- Rates are a comparison of two different units of measure
- Units of measure can help setup a problem or check your answers
- Two common methods to solve problems using rates:
  1) Direct Method: Directly multiplying or dividing by rates
  2) Proportion Method: Setting up and solving a proportion

**Homework**: pg 34-6; 1-10
Activity 1.6: Hot in Texas

Objectives: Students will be able to:
- Identify input and output in situations involving two variables
- Identify a functional relationship between two variables
- Identify the independent and dependent variables
- Use a table to numerically represent a functional relationship between two variables
- Represent a functional relationship between two variables graphically
- Identify trends in data pairs that are represented numerically and graphically, including increasing and decreasing

Vocabulary:
- Variable – usually represented by a letter, a quantity that may change in value
- Function – a correspondence between an input variable and an output variable that assigns a single unique output variable to each input value
- Input – the variable that we have control over or that is independent
- Output – corresponds to a particular input value
- Independent – the input variable
- Dependent – the output variable
- Ordered Pairs – table of values or points on a graph in the form of (input value, output value)
- Numerically defined function – using ordered pairs of numbers; for each input value, there is only one output value
- Rectangular coordinate system – horizontal axis represents the input variable and the vertical axis represents the output
- Quadrants – sections of coordinate plane defined by x, y axes (starts with I in upper right and goes counterclockwise)
- Increasing – a function is increasing if its graph rises to the right
- Decreasing – a function is decreasing if its graph falls to the right
- Constant – a function is constant if its graph is horizontal

Key Concept:
- A function is a correspondence between an input variable and an output variable that assigns a single, unique output value to each input value. If x represents the input variable and y represents the output variable, then the function assigns a single, unique y-value to each x-value.
- Numerically as ordered pairs (x, y) if no two ordered pairs have the same x-value and different y-values
- Graphically, a curve represents a function if it passes what is called the vertical line test

Vertical Line Test

Increasing, Decreasing and Constant

• An increasing graph rises from left to right
• A decreasing graph falls from left to right
• A horizontal line is constant

Slopes:
**Chapter 1: Introduction to Problem Solving and Mathematical Models**

**Activity:**
It is July, and you have just moved to Midland-Odessa, Texas. The area is in the middle of a heat spell. The high temperature for each of the last 5 days has exceeded 100°F. You are sweltering and you check the internet to look at the average monthly high temperatures in Midland-Odessa compared to Marion Virginia (where you were).

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-O</td>
<td>56.8</td>
<td>63.0</td>
<td>70.9</td>
<td>78.8</td>
<td>86.8</td>
<td>92.7</td>
<td>94.3</td>
<td>92.8</td>
<td>86.1</td>
<td>77.4</td>
<td>65.9</td>
<td>58.4</td>
</tr>
<tr>
<td>Marion</td>
<td>43.3</td>
<td>48.2</td>
<td>58.1</td>
<td>66.9</td>
<td>74.5</td>
<td>80.6</td>
<td>83.7</td>
<td>83.2</td>
<td>77.9</td>
<td>67.8</td>
<td>57.4</td>
<td>47.3</td>
</tr>
</tbody>
</table>

Identify the input (domain) and output (range) variables in the table.

What is the average hi temperature for July?
for Jan?

For each input variable (month number), how many different outputs (average hi temperatures) are there?

Plot the Midlands-Odessa data as dots and the Marion data as squares

Determine if the following are functions (or just relations):
   a) The amount of property tax you have to pay is a function of the assessed value of the house

   b) The weight of a letter in ounces is a function of the postage paid for mailing the letter

**Concept Summary:**
Input variables are the independent variables; listed first in an ordered pair
Output variables are the dependent variables; listed second in an ordered pair
Functions are a special correspondence relating input to output variables so that a single unique value is assigned to each input value
Functions can be defined numerically using ordered pairs (or equations) or graphically using plotted points
Perpendicular coordinate axes (x and y) divide the plane into 4 quadrants (counterclockwise)
Functions increase if their graph rises to the right, decrease if their graph falls to the right and is constant if its graph is horizontal

**Homework:** pg 52-55; 1-9
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.7: Fill ‘er Up

SOLs: None

Objectives: Students will be able to:
- Write the equation to define a function
- Determine the domain and range of a function
- Identify the independent and dependent variables of a function

Vocabulary:
- Rational number – any number that can be expressed as the quotient of two integers
- Irrational number – any number that cannot be expressed as the quotient of two integers
- Independent variable – another name for the input variable; the variable we control
- Dependent variable – another name for the output variable; the variable we don’t control
- Domain – collection of all possible values of the independent variable
- Practical Domain – the collection of possible values for the domain that make sense in the context of the problem
- Range – the collection of all possible values of the dependent variable
- Practical Range – the collection of possible values for the range that make sense in the context of the problem
- Increment – the change between two consecutive values of the independent variable
- Function Notation – replaces the dependent variable with f(independent variable), y = f(x)

Key Concepts:

We call the collection of all possible values of the independent variable, the domain (usually x-values). The domain is usually all real numbers (x ∈ ℝ) governed by two limitations:
1) can never divide by zero
   Example 1: x cannot be equal to 3 in f(x) = 1/(x-3)  
   Domain: {x | x ≠ 3, x ∈ ℝ}
2) no negative numbers under an even power radical (square root)
   Example 2: x must be greater than or equal to 2 in f(x) = √(x-2)  
   Domain: {x | x ≥ 2, x ∈ ℝ}

We call the collection of all possible values of the dependent variable, the range (usually y-values). We have to look at each function (usually by graphing it – looking for shapes, minimums and maximums) to help us tell the range. In the examples above: range in example 1 is {y | y ≠ 0, y ∈ ℝ} and the range in example 2 is {y | y ≥ 0, y ∈ ℝ}.

Practical domain and ranges are always viewed in the **context** of the problem (what makes sense).

**Real Numbers**

Input and output values used in our course will be real numbers. A real number can be a rational number or an irrational number.

Rational numbers are any real number that can be expressed as the quotient of two integers

Irrational numbers are any real numbers that cannot be expressed as the quotient of two integers

Note: they are complementary sets (something) we will see in Stats later.

Both boxes are all real numbers
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity:
You probably need to fill your car with gas more often than you would like, so you drive around looking for the best price per gallon. What two variables determine how much the fill-up will cost?

Which one do you have any choice with?

Suppose the best price you find is $2.47 9/10 per gallon. Complete the following table:

<table>
<thead>
<tr>
<th># of gallons</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill up Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is the cost of a fill-up a function of the number of gallons pumped? Explain

Identify the independent and dependent variables.

Write an equation relation c, cost, and g, gallons pumped for the above problem

What are the domain and range in the Activity?

What are the practical domain and range?

Problem:
If you work for an hourly wage, your gross pay is a function of the number of hours you work.

a) What is the independent and dependent variables?

b) If you earn $8 per hour, complete the table:

<table>
<thead>
<tr>
<th># of hours</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Pay</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Plot the points on the graph
d) Write the equation, f(n)

e) Evaluate f(10) and explain its meaning

f) What are the practical domain and range of f?

Concept Summary:
Independent variables are the input (ones we control)
Dependent variables are the output
Domains are all possible independent variables of the function
Ranges are all possible dependent variables of the function
Practical domains and ranges depend on the context of the problem
Functional notation: dependent variable = f(independent variable)

Homework: pg 63-67; 1-11, 14
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.8: Mathematical Modeling

SOLs: None

Objectives: Students will be able to:
- Identify a mathematical model
- Solve problems using formulas as models
- Develop a function model to solve a problem
- Recognize patterns and trends between two variables using tables as models

Vocabulary:
- Mathematical Model – is a mathematical form, such as a formula, an equation, a graph or table, that fits or approximates the important features of a given situation
- Mathematical modeling – is the process of developing a mathematical model for a given situation

Activity:
After an automobile accident the investigating officers often estimate the speed of the vehicle by measuring the length of the tire skid distance. The following table gives the average skid distances for an automobile with good tires on dry pavement.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>25</th>
<th>35</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>28</td>
<td>54</td>
<td>89</td>
<td>132</td>
<td>184</td>
<td>244</td>
<td>313</td>
</tr>
</tbody>
</table>

Define the independent and dependent variables

Does it appear to be a function?

Graph it

Windchill Table

As wind speed increases, your body loses heat more rapidly, making the air feel colder than it really is. The combination of cold temperature and high wind can create a cooling effect so severe that exposed flesh can freeze.

<table>
<thead>
<tr>
<th>WIND SPEED (mph)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEMPERATURE (°F)</td>
<td>5</td>
<td>31</td>
<td>25</td>
<td>19</td>
<td>13</td>
<td>7</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>21</td>
<td>15</td>
<td>9</td>
<td>3</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>22</td>
<td>28</td>
<td>34</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>15</td>
<td>23</td>
<td>19</td>
<td>13</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>9</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>37</td>
<td>44</td>
</tr>
<tr>
<td>20</td>
<td>24</td>
<td>17</td>
<td>11</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>15</td>
<td>22</td>
<td>29</td>
<td>35</td>
<td>42</td>
</tr>
<tr>
<td>25</td>
<td>23</td>
<td>16</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>30</td>
<td>22</td>
<td>15</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>39</td>
<td>46</td>
<td>53</td>
</tr>
<tr>
<td>35</td>
<td>21</td>
<td>14</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>38</td>
<td>45</td>
<td>52</td>
<td>59</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>13</td>
<td>3</td>
<td>1</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>38</td>
<td>45</td>
<td>52</td>
<td>60</td>
<td>67</td>
</tr>
<tr>
<td>45</td>
<td>19</td>
<td>12</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>38</td>
<td>45</td>
<td>52</td>
<td>60</td>
<td>67</td>
</tr>
<tr>
<td>50</td>
<td>18</td>
<td>11</td>
<td>4</td>
<td>3</td>
<td>11</td>
<td>18</td>
<td>25</td>
<td>32</td>
<td>39</td>
<td>46</td>
<td>54</td>
<td>61</td>
<td>68</td>
</tr>
<tr>
<td>55</td>
<td>17</td>
<td>10</td>
<td>3</td>
<td>4</td>
<td>11</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>40</td>
<td>48</td>
<td>55</td>
<td>62</td>
<td>69</td>
</tr>
<tr>
<td>60</td>
<td>16</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>11</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>40</td>
<td>48</td>
<td>55</td>
<td>62</td>
<td>69</td>
</tr>
</tbody>
</table>

Frostbite occurs in 30 minutes | 16 minutes | 5 minutes

EXAMPLE: When the temperature is 15°F and the wind speed is 30 miles per hour, the windchill, or how cold it feels, is −5°F. For a Celsius version of this table, visit Almanac.com/WindChillCelsius.

—courtesy National Weather Service
Chapter 1: Introduction to Problem Solving and Mathematical Models

**Basal Energy Rate Problem:**
Basal energy rate is the daily amount of energy (in calories) needed by the body at rest to maintain body temperature and basic life processes. Basal energy differs for individuals, depending on their gender, age, height, and weight. The formula for the basal energy for men is:

\[ B = 655.096 + 9.563W + 1.85H - 4.676A \]

where B is the basal rate, W is the weight (in kg), H is the height (in cm) and A is age (in years).

a) A male patient is 70 years old, weighs 55 kg and is 172 cm tall. Is a total daily caloric intake of 1000 calories enough to keep him properly fed?

b) A man is 178 cm tall and weighs 84 kg. If his basal energy rate is 1500 calories, how old is the man?

**Wind Chill Problem:**
Examine the windchill table on page 72 and answer the following questions:

a) What is the formula used to derive the values in the table?

b) What is the windchill for an air temperature of -30°F with a 20-mph wind?

c) If the wind speed is 40 mph, what is the windchill temperature if the air temperature is -20°F?

d) What is the windchill when the temperature is 5°F with a 25-mph wind?

**Roller Rink Problem:**
As part of a community service project at your high school, you organize a fund-raiser at the neighborhood roller rink. Money raised will benefit a summer camp for children with special needs. The admission charge is $4.50 per person, $2 of which is used to pay the rink’s rental fee. The remainder is donated.

a) What are the independent and dependent variables?

b) Write an equation with x representing the number attending and y as the amount donated.

c) If the maximum capacity of the rink is 200 people, what is the most that can be raised?

**Concept Summary:**
A mathematical model is a mathematical form, like an equation, formula, graph or table that fits or approximates important features of a given situation. Mathematical models can be simple or very complex. Mathematic modeling is the process of developing a mathematical model for a given situation.

**Homework:** pg 74-7; 1-5
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.9: Fund-Raiser Revisited

SOLs: None

Objectives: Students will be able to:
- Solve an equation numerically
- Solve an equation graphically
- Distinguish between a situation represented by a graph of distinct points versus a continuous path

Vocabulary:
- Evaluated – plugging in a value for the variable
- Equation – two numerical expressions separated by an =
- Numerical method – guess a number, check and repeat
- Graphical method – plot points and interpolate the graph
- Solution – a replacement value for the variable that produces equal numerical values on both sides of the equation
- Continuous – a graph is continuous if there are no gaps, or jumps in the graph
- Discontinuity – a jump, gap or disconnect between two points on the graph

Key Concept: A graph is continuous if we can graph the entire “curve” without lifting our pencil from the paper. There are 3 types of discontinuities:
- point: where a point is missing in the curve
- infinite: where the function is undefined (denominator equal to zero!)
- jump: where we have to jump to the next point on the graph

Note the graphs of the examples of discontinuities below:

Activity: In a previous activity we found the equation y = 2.5x modeled the fund raiser with x representing the number of tickets sold and y representing the amount of money donated to summer camp fund.

a) How much money is raised if 84 tickets are sold?

b) Fill in the table below:

<table>
<thead>
<tr>
<th># of Tickets Sold</th>
<th>Amount Raised</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td></td>
</tr>
<tr>
<td>278</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 1: Introduction to Problem Solving and Mathematical Models

c) Use the table below to guess at how many tickets have to be sold to raise $200

<table>
<thead>
<tr>
<th># of Tickets Sold</th>
<th>Amount Raised</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>250</td>
</tr>
</tbody>
</table>


d) What’s the disadvantage to “guess & check”?

1. Use the graph to determine how many tickets have to be sold to raise $200

2. What’s the disadvantage to a graphical method?

Problem: The recommended weight of an adult male can be approximated by the formula, \( w = 5.5h - 220 \), where \( w \) is recommended weight and \( h \) is height in inches.

a) What is the practical domain for the weight function?

b) Complete the following table

<table>
<thead>
<tr>
<th>Ht</th>
<th>60</th>
<th>64</th>
<th>68</th>
<th>72</th>
<th>76</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Graph your values in b).

d) Write an equation that can be used to determine the height of a man who weighs 165 pounds.

e) Solve the equation by guess and check

f) Solve the equation by graphing

Concept Summary:
To solve an equation using a:
- numerical approach: use a guess, check and repeat process
- graphical approach: read the appropriate \( x, y \) coordinates on the graph of the equation
- algebra: solve for the variable in question

Graphs are continuous if you can trace the graph without having to lift your pencil from the paper (no breaks!)

Homework: pg 83-86; 1-4
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.10: Leasing a Copier

SOLs: None

Objectives: Students will be able to:
  Develop a mathematical model in the form of an equation
  Solve an equation using an algebraic approach

Vocabulary: 
  Inverse Operations – doing the opposite of what the arithmetic operation does (add, then subtract; multiply, then divide; subtract, then add; divide, then multiply)

Activity:
You are interning at a local law office, Dewey, Cheetham and Howe. You are asked by the office manager to gather some information about leasing a copy machine for the office. The sales representative at Clorox Copiers recommends a 50-copy per minute copier to satisfy the office’s copying needs. The copier leases for $455 per month, plus 1.5 cents a copy. Maintenance fees are covered in the monthly charges. The law firm would own the copier after 39 months.

  a) Identify the independent and dependent variables
  b) Write a statement determining the monthly cost
  c) Complete the table

<table>
<thead>
<tr>
<th># of copies</th>
<th>5000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly cost ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  d) Write an equation of the statement in part b
  e) What is the monthly cost of 12,000 copies?
  f) 12,000 fits in for the independent or dependent variable?
  g) If the budget only allows $800 for copier expenses, what is the copy limit?

Problems:
For each of the following, substitute the given value of y and use an algebraic approach to solve for x
  a) \( y = 3x - 5 \) where \( y = 10 \)
  b) \( y = 30 - 2x \) where \( y = 24 \)
  c) \( y = (3/4)x - 21 \) where \( y = -9 \)
  d) \( y = 15 - 2x \) where \( y = -3 \)

Concept Summary:
The goal of solving an equation, \( ax + b = c \), is to isolate the variable x on one side of the equation
  \( x = \ldots \)

  Use inverse operations to isolate the variable
  - Undo the addition by subtracting \( b \) from both sides of the equation
  - Undo the multiplication by dividing both sides of the equation by \( a \)

Homework: pg 89-93; 6-10
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.11: Comparing Energy Costs

SOLs: None

Objectives: Students will be able to:
   Develop mathematical models to solve problems
   Write and solve equations of the form $ax + b = cx + d$
   Use the distributive property to solve equations involving grouping symbols
   Solve formulas for a specified variable

Vocabulary: none new

Activity:
An architect is hired to design a house. She obtains the following information regarding the installation and operating costs of two types of heating systems: solar and electric.

<table>
<thead>
<tr>
<th>Heating System</th>
<th>Installation Cost</th>
<th>Operating Cost (per yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>$25,600</td>
<td>$200</td>
</tr>
<tr>
<td>Electric</td>
<td>$5,500</td>
<td>$1600</td>
</tr>
</tbody>
</table>

Solar Costs:
   a) Determine the 5 year total cost of the solar system
   b) Write a statement for the total cost in terms of the number of years of use
   c) Letting $x = \text{years of use}$ and $c = \text{total cost}$, write an equation from statement in b)
   d) Fill out the table:

<table>
<thead>
<tr>
<th>Years of use, $x$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost, $c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Electric Costs:
   a) Determine the 5 year total cost of the electric system
   b) Write a statement for the total cost in terms of the number of years of use
   c) Letting $x = \text{years of use}$ and $c = \text{total cost}$, write an equation from statement in b)
   d) Fill out the table:

<table>
<thead>
<tr>
<th>Years of use, $x$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost, $c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparing Solar and Electric
   a) Compare the table values from the previous slides
   b) Estimate in what year will the Electric’s cost exceed the Solar’s
   c) Write an equation that can be solved algebraically to determine what year the two costs would be equal
Chapter 1: Introduction to Problem Solving and Mathematical Models

<table>
<thead>
<tr>
<th>Car</th>
<th>MSRP</th>
<th>Annual Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord LX</td>
<td>$20,925</td>
<td>$1730</td>
</tr>
<tr>
<td>Passat GLS</td>
<td>$24,995</td>
<td>$2420</td>
</tr>
</tbody>
</table>

Cars:

a) Fill out the table below reflecting each car’s value

<table>
<thead>
<tr>
<th>Years Owned</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accord</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Passat</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Will the value of the Passat ever be lower than the Accord?

c) Write an equation to figure out when their values would be equal

d) Solve the equation

Formulas:
Solve each of the following formulas for the specified variable

a) \( A = lw \) for \( w \)

b) \( P = 2l + 2w \) for \( l \)

c) \( V = \pi r^2h \) for \( h \)

d) \( V(P + a) = k \) for \( P \)

Equations:
Solve each of the following equations for \( x \)

a) \( 2x + 9 = 5x - 12 \)

b) \( 21 - x = -3 - 5x \)

c) \( 2(x - 3) = -8 \)

d) \( 2(x - 4) + 6 = 4x - 6 \)

Concept Summary:
General strategy for solving equations for a variable
- Remove parentheses (distributive property)
- Combine like terms on the same side of equation
- Get all variables to one side and the “just” numbers to other
- Divide through by the variable’s coefficient
- Check the result in the original equation

To solve a formula for a variable, isolate that variable on one side of the equation and all other terms on the other side

Homework: pg 100-6; 4-11, 20-25
Project Activity 1.12: Summer Job Opportunities

SOLs: None

Objectives: Students will be able to:
Use problem solving skills to make decisions based on solutions of mathematical models

Vocabulary: none new

Activity: Summer Job Opportunities Worksheet

Concept Summary:
Problem solving skills can help us with real-world problems

Homework: complete worksheet for quiz grade
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.13: Graphs Tell Stories

SOLs: None

Objectives: Students will be able to:
- Describe in words what a graph tells you about a given situation
- Sketch a graph that best represents the situation described in words
- Identify increasing, decreasing and constant parts of a graph
- Identify minimum and maximum points on a graph
- Identify a function relationship graphically using the vertical line test

Vocabulary:
- Maximum point – the point where the graph changes from rising to falling
- Local Maximum Value – the y-value of the maximum point
- Minimum point – the point where the graph changes from falling to rising
- Local Minimum Value – the y-value of the minimum point
- Vertical line test – a graph represents a function if any vertical line through the graph intersects the graph no more than once

Key Concept:

Changes in Graphs
- Increasing graph rises to the right
- Decreasing graph falls to the right
- Constant graph remains horizontal

Max point
Min point

Maximums and Minimums
- Maximum or minimum point is an (x, y) location
- Maximum value is the y-value of the maximum point
  - Absolute maximum is the greatest y-value in all the range
  - Relative maximum is the greatest y-value in the neighborhood
- Minimum value is the y-value of the minimum point
  - Absolute minimum is the least y-value in all the range
  - Relative minimum is the least y-value in the neighborhood

Activity:

Graphs to Stories:
1. A person’s core body temperature (°F) in relation to time of day
   Independent:                           Dependent:
   Interpretation:

2. Performance of a simple task in relation to interest level
   Independent:                           Dependent:
   Interpretation:
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3. Net profit of a particular business in relation to time given quarterly
   Independent:   Dependent:
   Interpretation:

4. Annual gross income in relation to number of years
   Independent:   Dependent:
   Interpretation:

Stories to Graphs:

1. You drive to visit your parents, who live 100 miles away. Your average speed is 50 miles per hour. On arrival, you stay for five hours and then return home, again at an average speed of 50 miles per hour. Graph your distance from home, from the time you leave until you return home.

2. You just started a new job that pays 6 dollars per hour, with a raise of 2 dollars per hour every six months. After one and half years, you receive a promotion that gives you a wage increase of 5 dollars per hour, but your next raise won’t come for another year. Sketch a graph of your wage over your first two and half years.

Vertical Line Test:

Use the vertical line test to determine which of the following graphs represent functions

a.  
   b.  
   c.  
   d.  

Concept Summary:

Vertical line test: a vertical line can intersect the graph of a function only once
Maximum: the largest y-value on the graph; the graph increases and then decreases
Minimum: the smallest y-value on the graph; the graph decreases and then increases

Homework: pg 118-21; 1-6
Chapter 1: Introduction to Problem Solving and Mathematical Models

Activity 1.14: Heating Schedule

SOLs: None

Objectives: Students will be able to:
- Obtain a new graph from an original graph using a vertical shift
- Obtain a new graph from an original graph using a horizontal shift
- Identify vertical and horizontal shifts
- Write a new formula for a function for which its graph has been shifted vertically or horizontally

Vocabulary:
- Vertical Shift – a upward or downward shift in the graph
- Horizontal Shift – a left or right shift in the graph
- Translation – a vertical or horizontal (or both) shift of the graph of the function

Key Concept: Vertical shifts in functions occur when a value is added or subtracted from each y-value calculated. These are considered “outside” the function as illustrated below:
- Upward shift of $c$ units: $y = f(x) + c$
- Downward shift of $c$ units: $y = f(x) - c$
Shifts up or down correspond to what we naturally associate with addition and subtraction.

Horizontal shifts in functions occur when a value is added or subtracted from each x-value inputted. These are considered “inside” the function as illustrated below:
- Leftward shift of $c$ units: $y = f(x + c)$
- Rightward shift of $c$ units: $y = f(x - c)$
Shifts right or left do not correspond to what most people naturally associate with addition and subtraction.

As we studied in grade school math and in Geometry a translation, or a slide, is the vertical or horizontal (or both) shifting of a function. Picture a magnet on a refrigerator that is “slid” left or right, up or down.

An absolute value is always positive. It is defined by:
$$|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases}$$
In other words we strip off a negative sign, if the function value is negative.

Activity: The cost of fuel oil is rising and is affecting the school district’s budget. In order to save money, the building maintenance supervisor of the high school has decided to keep the building warm only during school hours. At midnight, the building temperature is 55°F. This temperature remains constant until 4 am, at which time the temperature of the building steadily increases. By 7 am, the temperature is 68°F and is maintained there until 7 pm, when the temperature begins a steady decrease. By 10 pm, the temperature is back to 55°F.
1. Graph this function.
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2. The School Board instructs the building maintenance supervisor to increase the temperature of the building by a constant 3°F.

![Graph showing temperature vs time]

Sketch the change on the same graph.
How can the second function be gotten from the first?

3. On a two-hour delay, the supervisor adapts the building’s heating function to the delay.
Sketch the same graph from the first activity.

![Graph showing temperature vs time after delay]

Sketch the two-hour delay change on the same graph.
How can the second function be gotten from the first?

**Calculator Graphing Exercise:**

Using the graphing calculator, graph the following:

a) \( y_1 = x^2 \)
   Then on the same graph do
b) \( y_2 = x^2 + 2 \)
   then change to
c) \( y_2 = x^2 - 3 \)
   then change to
d) \( y_2 = (x + 1)^2 \)
   then change to
e) \( y_2 = (x - 4)^2 \)
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Absolute Value Graphs:
Graph the following functions:  \( y = |x| \)  and compare with

\[
\begin{align*}
a) \quad y &= |x| + 5 \\
b) \quad y &= |x + 5|
\end{align*}
\]

Concept Summary:

*Vertical shifts* in functions occur when a value is added or subtracted from each \( y \)-value calculated. These are considered “outside” the function:
- Upward shift of \( c \) units: \( y = f(x) + c \)
- Downward shift of \( c \) units: \( y = f(x) - c \)

*Horizontal shifts* in functions occur when a value is added or subtracted from each \( x \)-value inputted. These are considered “inside” the function:
- Leftward shift of \( c \) units: \( y = f(x + c) \)
- Rightward shift of \( c \) units: \( y = f(x - c) \)

Homework:  pg 129-34, 1-10
Five Minute Reviews:

Activity 1:
Problem Solving Techniques:
1. What do we need to get from the information given to us about a problem?
2. What was the problem from yesterday?
3. What did we do to help solve the problem?

Activity 2:
Problem Solving Techniques:
1. How many steps were in Polya’s method?
2. What were the steps?
3. What were some other problem solving strategies?

Activity 3:
1. What is the language of Math?
2. What does the variable(s) represent?
3. In step 4 of our problem solving steps, what do we do when we get an answer?

Activity 4:
Ratios, Proportions and Percentages:
1. What method do we use to solve proportions?
2. What is each part of a proportion called?
Chapter 1: Introduction to Problem Solving and Mathematical Models

Solve the following proportions:

3. \[
\frac{6}{y} = \frac{9}{15}
\]

4. \[
\frac{x}{8} = \frac{9}{12}
\]

5. What is equivalent percentage to 11/25?

Activity 5:
Problem Solving Methods:
1. How many methods did we demonstrate last lesson?
2. Which method did you like the best?

Solve the following problems:
3. How many miles can your car go if it gets 16.9 mpg with a 25 gallon tank?

4. How long will a 635 mile trip take driving 70 miles per hour?

5. How many gallons will it take to drive 430 miles if you get 28 mpg?

Activity 6:
1. How do we test a graph to see if it is a function?
2. What is the input to \( y = f(x) \)?
3. What is the output to \( y = f(x) \)?
4. A function has _____ unique output for every input value.
5. Describe the graphs of the functions below:
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Activity 7:
1. In functional notation, what is the input variable?

2. What are the restrictions on the independent variable?

3. In function notation, what is the output variable?

4. What are the restrictions on the dependent variable?

5. What is the domain and range of \( \frac{1}{\sqrt{x-2}} \) ?

Activity 8:
1. In a wind chill chart, what are the independent variables?

2. In a wind chill chart, what is the dependent variable?

3. How do we test a graph to see if it is a function?

4. What is the domain and range of \( \sqrt{x + 2} \) ?

What is the domain and range of \( \frac{1}{x-3} \) ?

Activity 9:
1. Using your calculator solve the following:
   \[ y = 300 \quad y = 3x^2 \]

Using algebra solve the following:
2. \( 3x - 7 = 2x + 2 \)

3. \( 4x - 10 + 2x + 15 = 65 \)

4. \( 5x = (4x - 15) + 45 \)

5. \( 45 = (7x + 5) - (2x - 10) \)
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Activity 10:

1. Using your calculator solve the following:
   \[ y = 6 \quad \text{and} \quad y = e^x \]

Using algebra solve the following:
2. \[ y = 3x - 5 \text{ where } y = 10 \]

3. \[ y = 30 - 2x \text{ where } y = 24 \]

4. \[ y = (3/4)x - 21 \text{ where } y = -9 \]

5. \[ y = 15 - 2x \text{ where } y = -3 \]

Activity 11:

1. If your car depreciates at 10% a year and is now worth $12,000; how much will it be worth in two years?

2. For what value of \( x \), will Bill have more money than Tom:
   - Bill makes $40 a day plus $3 on each sale, \( x \)
   - Tom makes $55 a day but only $2 on each sale

3. Solve the following equations:
   a. \[ 4x - 12 = 2x + 14 \]
   b. \[ (2/3)x + 15 = 40 - x \]

Activity 13:

1. Which of the following graphs are functions?
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2. Identify what each of the points are in the following graph:
   a. W
   b. X
   c. Y
   d. Z

Activity 14:

1. Identify the changes in the following graphs from \( y_1 = x^2 \)
   a) \( y_2 = x^2 + 3 \)
   b) \( y_2 = x^2 - 2 \)
   c) \( y_2 = (x + 1)^2 \)
   d) \( y_2 = (x - 5)^2 \)

2. Match the following:
   Horizontal shifts outside function \( f(x + c) \)
   Vertical shifts inside function \( f(x) + c \)

3. Describe the shifts from \( y = 3x \) and \( y = 3(x - 2) + 4 \)