Asymptotes Review Notes

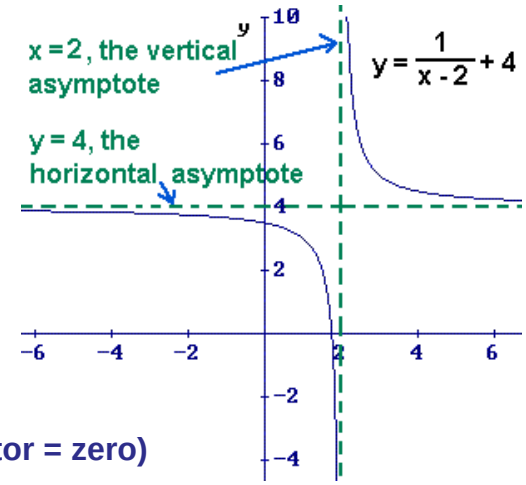
Horizontal Asymptotes:

- **Where the graph approaches as x gets very large (negatively or positively)**
  We find this by graphing the function on our calculator and estimating the horizontal line the graph approaches as x gets large (in calculus they take a limit of the function as x approaches positive or negative infinity).

Example: $y = \dfrac{1}{x - 2} + 4$

Graph: In this graph the curve approaches the horizontal line $y = 4$ from below when x is large negatively. It approaches the same horizontal line $y = 4$ from above when x is large positively. It has a vertical asymptote of $x = 2$.

Example of Graph:



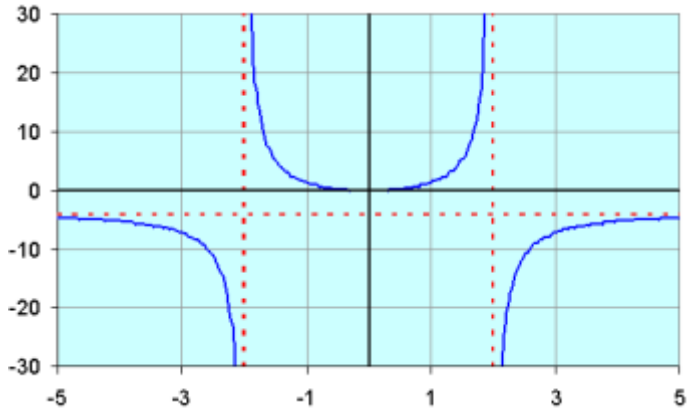
Vertical Asymptotes:

- **Where the domain is undefined (because denominator = zero)**
  Zeros for the denominator can be found using the same techniques as on the x-intercepts worksheet

Example: $y = \dfrac{4x^2}{(2 - x)(2 + x)}$

Graph: In this graph the denominator is equal to zero at $x = 2$ and $x = -2$; therefore the function is undefined at those points and it has vertical asymptotes at $x = 2$ and $x = -2$. It has a horizontal asymptote of $y = -4$.

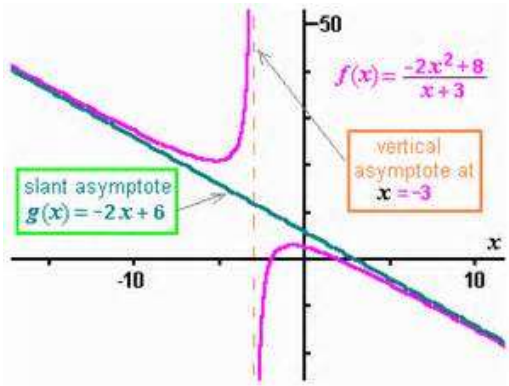
Example of Graph:



Slant-wise Asymptotes:

This type of asymptote is not seen in AFDA, but is seen in pre-calculus and calculus. When we have a rational function $f(x)$ in the form of a polynomial $g(x)$ divided by another polynomial $h(x)$ and the order (highest exponential power) of $g(x)$ is greater than $h(x)$ by 1, then we get a slant-wise asymptote in the form of $y = mx + b$. The graph to the right illustrates an example of a slant-wise asymptote (found using long division to get the equation of the line).

Example of Graph:



Form: $y = f(x) = \dfrac{g(x)}{h(x)}$

Example: $y = \dfrac{-2x^2 + 8}{x + 3}$

Graph: slant-wise asymptote of $y = -2x + 6$, a vertical asymptote of $x = -3$ and a domain of $x \neq -3$

Asymptotes Example Problems **Name:** _____

1. **Find the zeros of the following functions:**

   a) $y = 3x - 12$

   b) $y = 2x + 7$

   c) $y = 3x - 8$

   d) $y = 2x + 5$

2. **Find the zeros for the following functions in factored form:**

   a) $y = (x - 2)(x + 5)$

   b) $y = (x + 2)(x - 9)$

   c) $y = x(x + 7)(x - 3)$

   d) $y = (3x - 5)(2x + 7)$

3. **Find the horizontal and vertical asymptotes and domains of the following functions:**

   a) $y = \dfrac{2}{x - 7}$

   b) $y = \dfrac{-7}{(x - 3)(x + 4)}$

   c) $y = \dfrac{1}{x + 3} + 1$

   d) $y = \dfrac{5}{(x + 2)(x - 5)}$

4. **Find the horizontal and vertical asymptotes and domains and ranges of the following functions from their graphs:**