Probability – Conditional and Two-way Tables

Probability Rules for any Probabilistic Model:
1) Sum of all P(Events) = 1
2) All probabilities must be $0 \leq$ P(Events) ≤ 1
3) P(Event) + P(Event's Compliment) = 1
4) P(certainty) = 1 and   P(impossibility) = 0

Conditional Probability: Finding the probability of an event given that something else has already happened (or is true). P(A | B) is read what is the probability of A given that B has occurred. It is governed by the following formula:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

obviously P(B) $\neq$ 0 (since it has occurred)

Example: What is the P(A|B), if P(A) = 0.6, P(B) = 0.3, P(A and B) = 0.2?

$$P(A|B) = P(A \text{ and } B) / P(B) = 0.2/0.3 = 0.67$$

Two-Way or ContingencyTables: A table listing two or more characteristics of a group.

With tables, you have to cover up the areas that are not true (from what was given – already true) and then figure out the total possible and the event occurrences. In the examples below, we will use the table below listing employees years of service:

Years	0-4	5-9	10-14	14+	Totals
Males	12	6	17	21	56
Females	8	9	13	14	44
Totals	20	15	30	35	100

1. What is the probability of randomly selecting a female employee?

   44 female employees out of 100 total;   P(F) = 44/100 = 0.44

2. Given that the employee is male, what is the probability that they have less than 4 years of experience?

   Only look at the male row! 12 males in 0-4 out of 56 total;   P(0-4|M) = 12/56 = 0.21

3. Given that the employee has between 10 and 14 years of experience, what is the probability that the employee is female?

Only look at the 10-14 yr column! 13 females out of 30 in 10-14;   P(F|10-14) = 13/30 = 0.43

4. Given that the employee has more than 14 years of experience, what is the probability that the employee is male?

   Only look at the 14+ yr column! 21 males out of 35 in 14+;   P(M|14+) = 21/35 = 0.60

5. What is the probability of randomly selected an employee with less than 14 years of experience given that they are female?

   Use complement!   P($\leq$14|F) = 1 – P(>14|F) = 1 – 14/44 = 30/44 = 0.68

Conditional Probability Worksheet

Given the following information: P(D) = 0.7, P(E) = 0.2, and P(D and E) = 0.15.

1. Find the P(D|E)

2. Find the P(E|D)

Given the following table of grades from Mrs. Hardcase's English classes:

Grades	A	B	C	D	F	Totals
Males	12	6	17	14	7	56
Females	8	9	13	8	6	44
Totals	20	15	30	22	13	100

3. What is the probability that a randomly selected student got a A or B?

4. What is the probability that an "A" student is male?

5. What is the probability that if a student was female that they got a passing grade?

6. What is the probability of a male student given that they failed?

7. What is the probability of a randomly selected student is male?

8. What is the probability of a female student given that they got a "B"?

9. What is the probability of a randomly selected student passing Mrs. Hardcase's class?