

Equations Review

The language of MATH is Equations!!!

Every step in solving a problem for an unknown variable should have a equation sign in it.

General concept: get the variable on one side and the numbers on the other side.

Properties of Equality (commonly used to solve problems)

Property	Result	Example
Distributive	Removes parentheses	$3(x + 2) = 3x + 3 \times 2$ $= 3x + 6$
Addition	Adds same value to both sides	$3x - 6 = 21$ $+6 = +6$ $3x = 27$
Subtraction	Subtracts same value from both sides	$4x + 7 = 23$ $-7 = -7$ $4x = 16$
Multiplication	Multiplies both sides by the same value	$\frac{1}{2} x = 8$ $2(\frac{1}{2} x = 8)$ $x = 16$
Division	Divides both sides by the same value	$4x = 16$ $\frac{4x}{4} = \frac{16}{4}$ $x = 4$
Before you can use the properties of equality (except for distribution), we may need to combine like terms (CLT):		
CLT	Group variables and numbers on each side of equation together	$3x - 9 + 2x - 8 = 23$ $5x - 17 = 23$

Find the unknown variable usually involves application of several of the properties of above to get the solution.

Common mistakes:

Combine variables and numbers

Not using the opposite to eliminate a term:

add to remove negative;

subtract to remove positive;

divide to remove the coefficient in front of the variable)

Equation Review

Basic Concept:

We want to work the equation by canceling out things on both sides (by using the properties of equality) until we get it into the final form:

variable = number

(our final answer!!)

Example:

$$\begin{aligned}
 &\frac{1}{2}x^2 + 3x - 9 = 3x - 1 \\
 &2(\frac{1}{2}x^2 + 3x - 9 = 3x - 1) \\
 &x^2 + 6x - 18 = 6x - 2 \\
 &\quad \underline{-6x \qquad -6x} \\
 &x^2 \qquad -18 = \qquad -2 \\
 &\quad \underline{+18 \qquad +18} \\
 &x^2 \qquad = \qquad 16 \\
 &\quad \sqrt{x^2} = \sqrt{16} \\
 &\quad x = 4
 \end{aligned}$$

Original Equation

Get rid of fraction ($\times 2$ both sides)

Result

Subtract 6x from both sides

Result

Add 18 to both sides

Result

Take square root of both sides

Final answer

Properties of Equality:

We can add the same thing to both sides of the “=” sign: (x below)

$$2x^2 - x = 8 - x \quad \rightarrow \quad 2x^2 - x + x = 8 - x + x \quad \rightarrow \quad 2x^2 = 8$$

We can subtract the same thing from both side of the “=” sign: (3x below)

$$5x^2 + 3x = 5 + 3x \quad \rightarrow \quad 5x^2 + 3x - 3x = 5 + 3x - 3x \quad \rightarrow \quad 5x^2 = 5$$

We can multiply both sides of the “=” sign by the same thing: (2 below)

$$\frac{1}{2}x + 5 = 9 \quad \rightarrow \quad 2 \times (\frac{1}{2}x + 5 = 9) \quad \rightarrow \quad x + 10 = 18$$

We can divide both sides of the “=” sign by the same thing: (12x below)

$$12x^2 + 24x = 48x \quad \rightarrow \quad (12x^2 + 24x = 48x) \div 12x \quad \rightarrow \quad x + 2 = 4$$

Advanced Equality Concepts (seen later in AFDA):

We can take the square root of both sides of the “=” sign:

$$x^2 = 4 \quad \rightarrow \quad \sqrt{x^2} = \sqrt{4} \quad \rightarrow \quad x = 2$$

We can take the logarithm of both sides of the “=” sign:

$$y = x^2 \quad \rightarrow \quad \log y = \log (x^2)$$

We can make both sides of the “=” sign an exponent of the same base:

$$y = x - 2 \quad \rightarrow \quad e^y = e^{(x-2)}$$

We can “undo” all of the above as well:

We can square both sides of the “=” sign:

$$\sqrt{x} = 3 \quad \rightarrow \quad (\sqrt{x})^2 = (3)^2 \quad \rightarrow \quad x = 9$$

We can get rid of the logarithms on both sides of the “=” sign: [Caution]

$$\log 3 = \log x \quad \rightarrow \quad 3 = x$$

We can get rid of the same base on both sides of the “=” sign: [Caution]

$$e^{2x-1} = e^x \quad \rightarrow \quad 2x - 1 = x$$