Chapter 10 Circles

Addressed or Prepped VA SOL:

G.4 The student will construct and justify the constructions of
h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

G.11 The student will solve problems, including practical problems, by applying properties of circles. This will include determining
a) angle measures formed by intersecting chords, secants, and/or tangents;
b) lengths of segments formed by intersecting chords, secants, and/or tangents;
c) arc length; and
d) area of a sector.

G.12 The student will solve problems involving equations of circles.

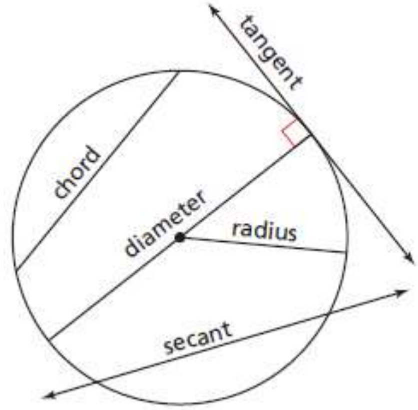
SOL Progression

Middle School:
- Solve two-step equations
- Use the Pythagorean Theorem to find the distance between two points in the coordinate plane.
- Solve real-world problems

Algebra I:
- Solve linear equations in one variable
- Multiply binomials
- Solve quadratic equations using square roots and by completing the square
- Graph points and functions in the coordinate plane

Geometry:
- Identify chords, diameters, radii, secants, and tangents of circles
- Find angle and arc measures
- Use inscribed angles and polygons and circumscribed angles
- Use properties od chords tangents, and secants to solve problems
- Write equations of circles

Chapter 10 Circles



Section 10-1: Lines and Segments that Intersect Circles

SOL: G.11.a and .b

Objective:

Identify special segments and lines
Draw and identify common tangents
Use properties of tangents

Vocabulary:

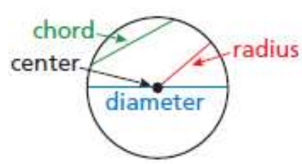
Center – the central point of a circle
Chord – any segment that endpoints are on the circle
Circle – the set of all points in a plane equidistant for a given point called the center of the circle
Circumference – is the perimeter of the circle (once around the outside) C = 2πr = dπ
Common tangent – a line or segment that is tangent to two coplanar circles
Concentric circles – coplanar circle that have a common center
Diameter – a chord that contains the center of the circle
Point of tangency – the point that the circle and tangent intersect
Radius – any segment that endpoints are the center and a point on the circle; ½ diameter
Secant – a line that intersects a circle in two points
Tangent – a line in the plane of a circle that intersects the circle in exactly one point
Tangent circles – coplanar circle that intersect in one point

Core Concepts:

Core Concept

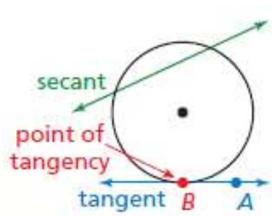
Lines and Segments That Intersect Circles

A segment whose endpoints are the center and any point on a circle is a **radius**.



A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points.

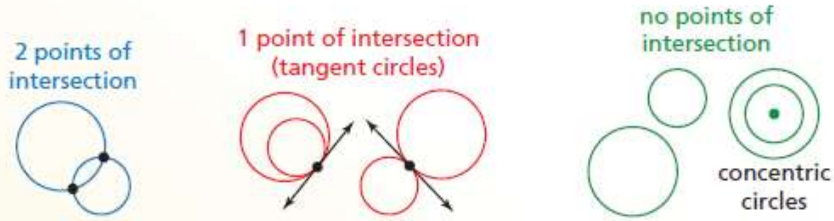


A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.

Core Concept

Coplanar Circles and Common Tangents

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.

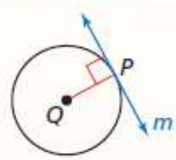


A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Theorems

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

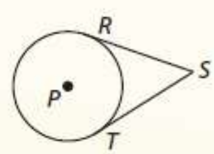


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 480

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.



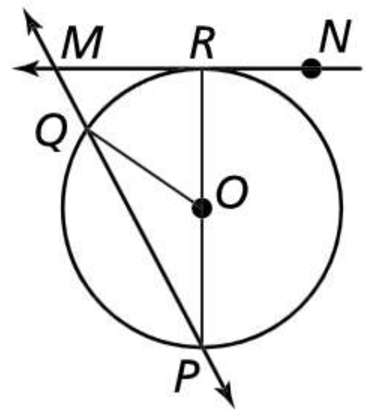
If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 480

Examples:

Example 1:

Tell whether the line, ray, or segment is best described as a radius, chord, diameter, secant or tangent of circle O.

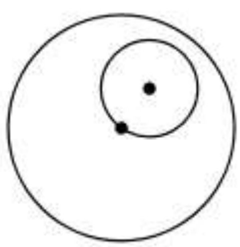


a. $\overline{PR}$
b. $\overleftrightarrow{MN}$
c. $\overleftrightarrow{PQ}$
d. $\overline{QO}$

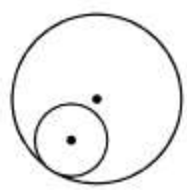
Example 2:

Tell how many common tangents the circles have and draw them.

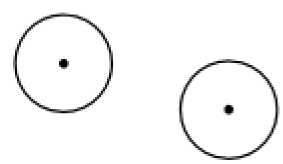
a.



b.

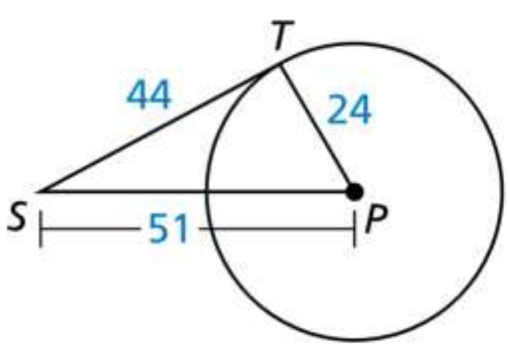


c.



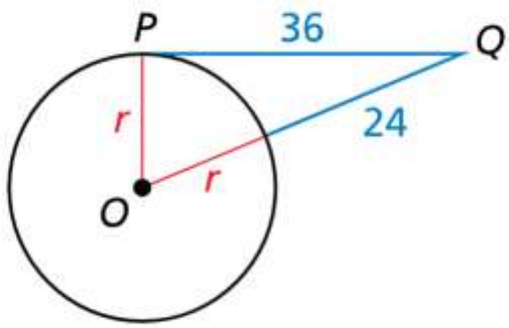
Example 3:

Is $\overline{ST}$ tangent to $\odot P$?



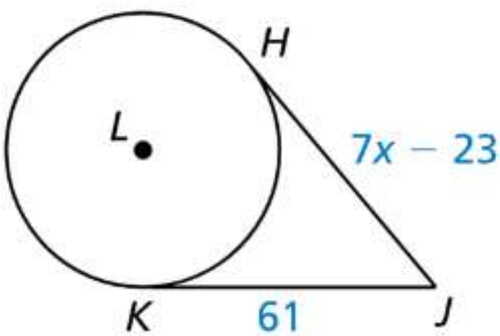
Example 4:

In the diagram, point P is a point of tangency. Find the radius, r, of $\odot O$.



Example 5:

$\overline{JH}$ is tangent to $\odot L$ at H, and $\overline{JK}$ is tangent to $\odot L$ at K. Find the value of x.



Concept Summary:

- A line that is tangent to a circle intersects the circle in exactly one point.
- A tangent is perpendicular to a radius (or diameter) of a circle
  - Pythagorean Theorem will apply
- Two segments tangent to a circle from the same exterior point are congruent

Khan Academy Videos:

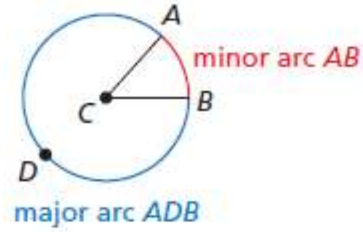
1. Glossary of Circles
2. Radius, diameter, circumference and π

Homework: Circle Items WS

Reading: Student notes section 10-2

Chapter 10 Circles

Section 10-2: Finding Arc Measures



SOL: G.11.a

Objective:

Find arc measures
Identify congruent arcs
Prove circles are similar

Vocabulary:

Adjacent arcs – two arcs of the same circle that intersect at exactly one point
Arc – edge of the circle defined by a central angle
Central Angle – an angle whose vertex is the center of the circle with two radii as sides
Congruent arcs – arcs that have the same measure
Congruent circles – circles with the same radius length
Minor Arc – an arc with the central angle less than 180° in measurement
Major Arc – an arc with the central angle greater than 180° in measurement
Semicircle – an arc with the central angle equal to 180° in measurement
Similar arcs – if and only if they have the same measure

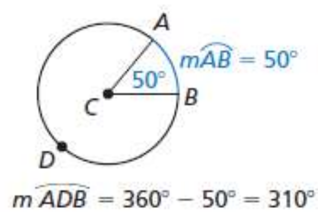
Core Concepts:

Core Concept

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\widehat{AB}$ is read as "the measure of arc AB."

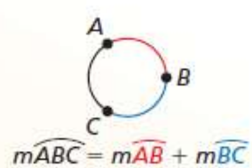
The measure of the entire circle is 360°. The **measure of a major arc** is the difference of 360° and the measure of the related minor arc. The measure of a semicircle is 180°.



Postulate

Postulate 10.1 Arc Addition Postulate

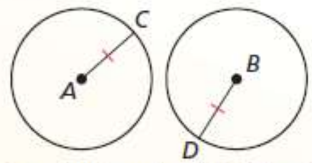
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



Theorem

Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



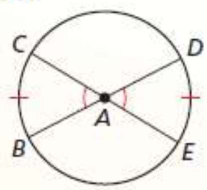
$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 35, p. 488

Theorem

Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



$\overparen{BC} \cong \overparen{DE}$ if and only if $\angle BAC \cong \angle DAE$.

Proof Ex. 37, p. 488

Theorem

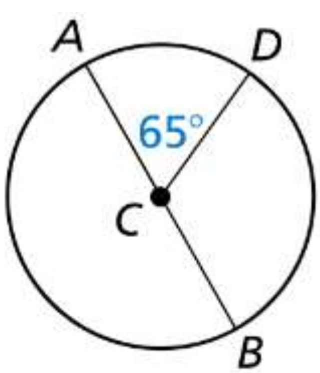
Theorem 10.5 Similar Circles Theorem

All circles are similar.

Proof p. 485; Ex. 33, p. 488

Examples:

Example 1:



Find the measure of each arc of $\odot C$, where $\overline{AB}$ is a diameter.

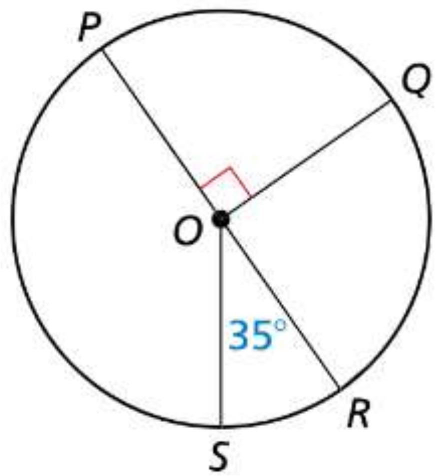
a. $\overparen{AD}$

b. $\overparen{DAB}$

c. BDA

Example 2:

Find the measure of each arc.



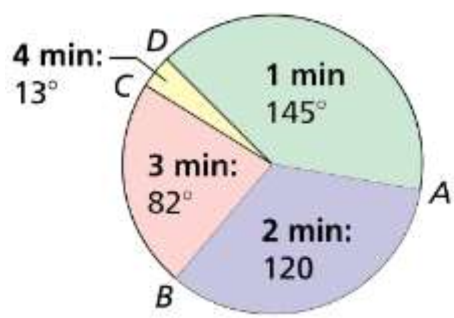
a. $\widehat{SRQ}$

b. $\widehat{RPQ}$

c. $\widehat{PRS}$

Example 3:

A survey asked people how many minutes they spend brushing their teeth each morning. The circle graph shows the results. Find the indicated arc measures.



a. m$\widehat{ABC}$

b. m$\widehat{ACB}$

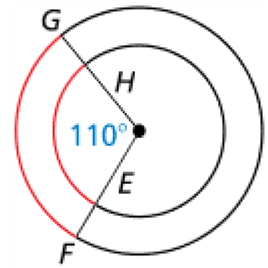
c. m$\widehat{BD}$

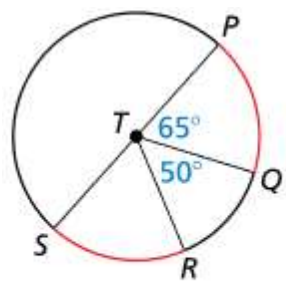
d. m$\widehat{CBD}$

Example 4:

Tell whether the red arcs are congruent. Explain why or why not.

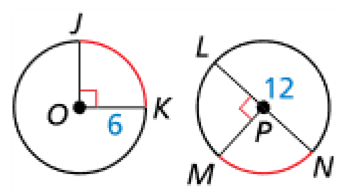
a. GF and HE





b. PQ and RS

c. JK and MN



Concept Summary:

- Sum of measures of central angles of a circle with no interior points in common is 360°
- Measure of each arc is related to the measure of its central angle
- Length of an arc is proportional to the length of the circumference

Khan Academy Videos:

1. Introduction to arc measures
2. Finding arc measures
3. Finding arc measure with equations

Homework: Circle Items WS

Reading: Student notes section 10-3

Chapter 10 Circles

Section 10-3: Using Chords

SOL: G.11.b

Objective:

Use chords of circles to find lengths and arc measures

Vocabulary:

Inscribed Polygon – all vertices lie on the circle

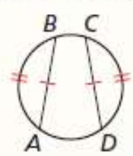
Circumscribed – circle contains all vertices of a polygon

Core Concept:

Theorems

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

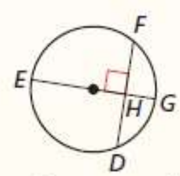


Proof Ex. 19, p. 494

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

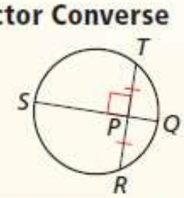


Proof Ex. 22, p. 494

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.



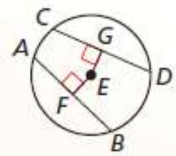
Proof Ex. 23, p. 494

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Theorem

Theorem 10.9 Equidistant Chords Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



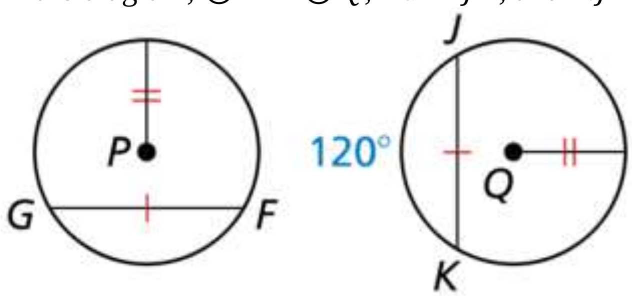
Proof Ex. 25, p. 494

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

Examples:

Example 1:

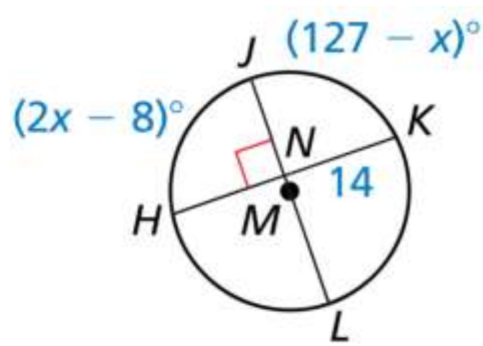
In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\widehat{JK} = 120°$. Find $m\widehat{FG}$.



Example 2:

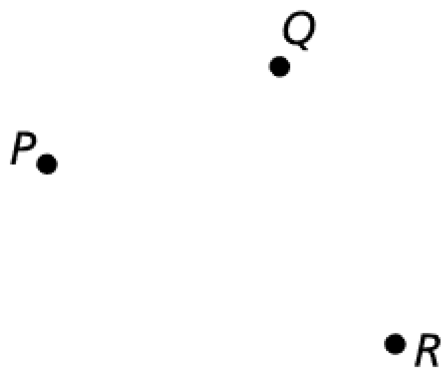
a. Find KH

b. Find mHLK



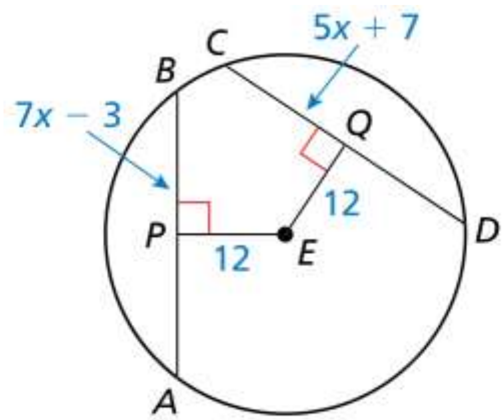
Example 3:

A telephone company plans to install a cell tower that is the same distance from the centers of three towns, labeled P, Q, and R. Where should the cell tower be placed?



Example 4:

In the diagram, $EP = EQ = 12$, $CD = 5x + 7$, and $AB = 7x - 3$. Find the radius of $\odot E$.



Concept Summary:
- The endpoints of a chord are also the endpoints of an arc
- Diameters and radii that are perpendicular to chords bisect chords and intercepted arcs

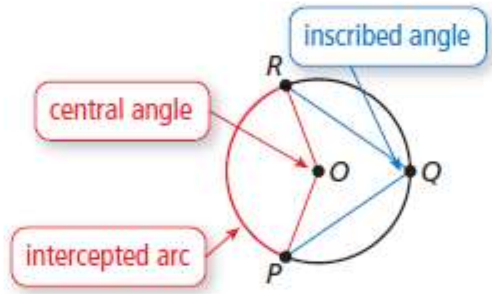
Khan Academy Videos: None relate

Homework: Circle Segments Worksheet

Reading: Student notes section 10-4

Chapter 10 Circles

Section 10-4: Inscribed Angles and Polygons



SOL: G.11.a and G.4.h

Objective:

Use inscribed angles
Use inscribed polygons

Vocabulary:

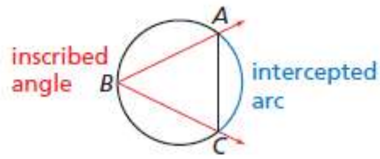
Circumscribed circle – the circle that contains the vertices of an inscribed polygon
Inscribed Angle – an angle with its vertex on the circle and whose sides contain chords of the circle
Inscribed Polygon – a polygon whose vertices lie on a circle
Intercepted arc – an arc that lies between two lines, rays or segments
Subtend – the sides or arc of an inscribed angle

Core Concept:

Core Concept

Inscribed Angle and Intercepted Arc

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.

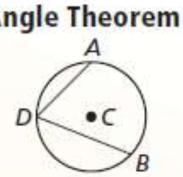


$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Theorem

Theorem 10.10 Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one-half the measure of its intercepted arc.



$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

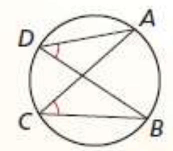
Proof Ex. 37, p. 502

Inscribed angles measure one-half of their arcs

Theorem

Theorem 10.11 Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong \angle ACB$

Proof Ex. 38, p. 502

Core Concept

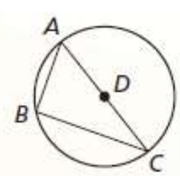
Inscribed Polygon

A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

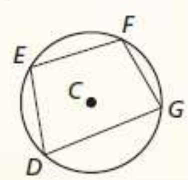


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 502

Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

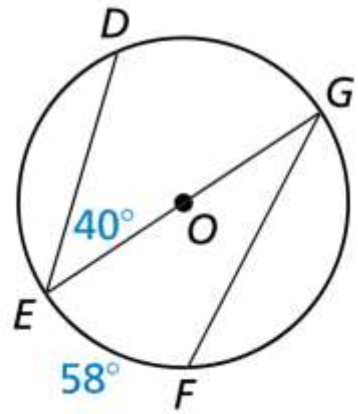


D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

Proof Ex. 40, p. 502; *BigIdeasMath.com*

Examples:

Example 1:



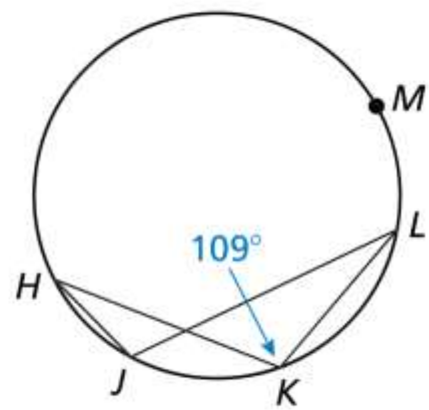
Find the indicated measure.

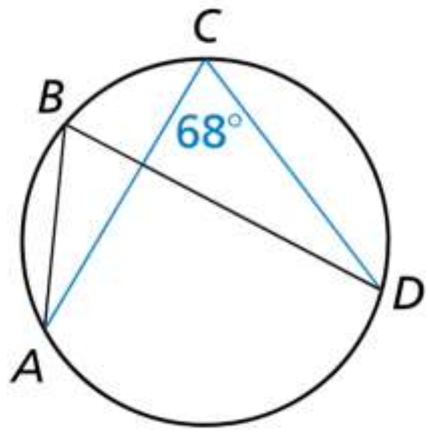
a. $m\widehat{DG}$

b. $m\measuredangle G$

Example 2:

Find $m\widehat{HML}$ and $m\widehat{HJL}$. What do you notice about $\measuredangle HKL$ and $\measuredangle LKH$?



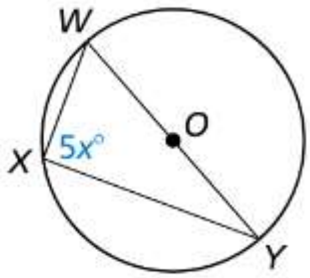


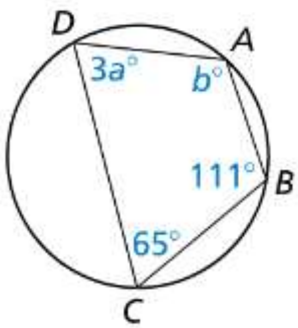
Example 3:

Given $m\measuredangle C = 68°$, find $m\measuredangle B$.

Example 4:

Find the value of each variable.

a. 

b. 

Example 5:

Explain how to find locations where the right side of the statue is all that is seen in your camera's field of vision.

Concept Summary:

- The measure of the inscribed angle is half the measure of its intercepted arc
- The angles of inscribed polygons can be found by using arc measures
- Opposite angles in inscribed quadrilaterals are supplementary

Khan Academy Videos:

1. Inscribed angles
2. Solving inscribed quadrilaterals

Homework: Circle Angles Worksheet

Reading: Student notes section 10-4

Section 10-5: Angle Relationships in Circles

SOL: G.11.a

Objective:

Find angle and arc measures
Use circumscribed angles

Vocabulary:

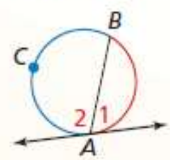
Tangent – a line that intersects a circle in exactly one point
Point of tangency – point where a tangent intersects a circle

Core Concept:

Theorem

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



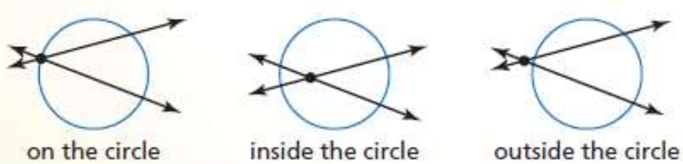
Proof Ex. 33, p. 510

$m\angle 1 = \frac{1}{2}m\widehat{AB}$ $m\angle 2 = \frac{1}{2}m\widehat{BCA}$

Core Concept

Intersecting Lines and Circles

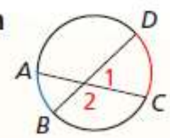
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



Theorems

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



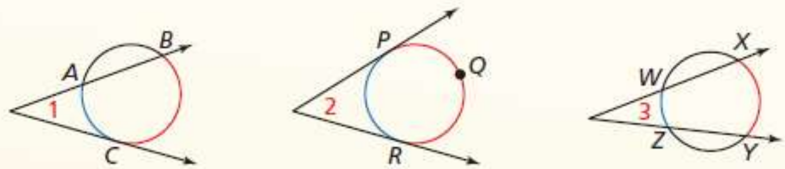
$m\angle 1 = \frac{1}{2}(m\overparen{DC} + m\overparen{AB})$,

$m\angle 2 = \frac{1}{2}(m\overparen{AD} + m\overparen{BC})$

Proof Ex. 35, p. 510

Theorem 10.16 Angles Outside the Circle Theorem

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overparen{BC} - m\overparen{AC})$ $m\angle 2 = \frac{1}{2}(m\overparen{PQR} - m\overparen{PR})$ $m\angle 3 = \frac{1}{2}(m\overparen{XY} - m\overparen{WZ})$

Proof Ex. 37, p. 510

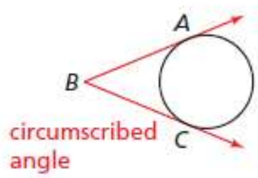
An inside angle's measure is one-half the sum of the front and back arcs.

An outside angle's measure is one-half the difference between the far and near arcs.

Core Concept

Circumscribed Angle

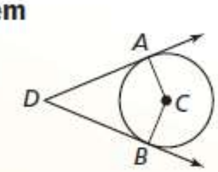
A **circumscribed angle** is an angle whose sides are tangent to a circle.



Theorem

Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



Proof Ex. 38, p. 510

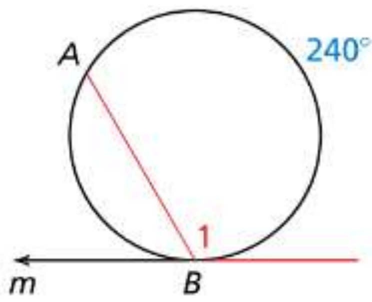
$m\angle ADB = 180° - m\angle ACB$

Examples:

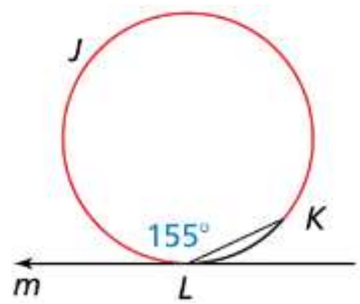
Example 1:

Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a. $\measuredangle 1$



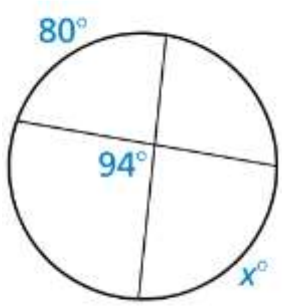
b. $m\widehat{LJK}$



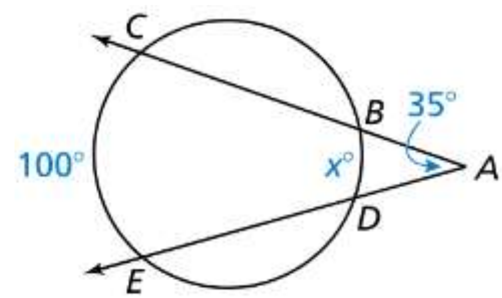
Example 2:

Find the value of x.

a.

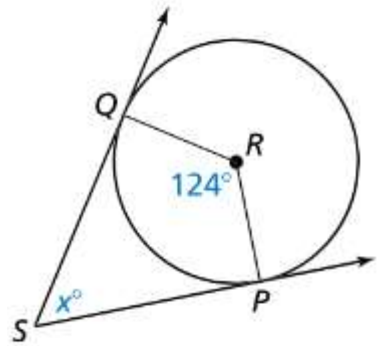


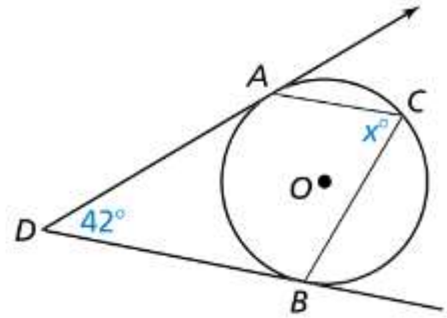
b.



Example 3:

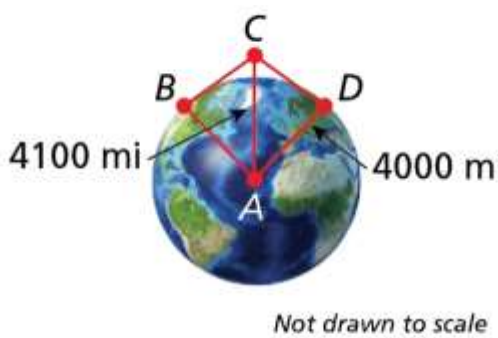
Find the value of x.

a. 

b. 

Example 4:

Use the information (radius of Earth is about 4000 miles). A flash occurs 100 miles above Earth at point C. Find the measure of $\widehat{BD}$, the potion of Earth from which the flash is visible.



Concept Summary:

- Central angle is equal to its arc
- Inscribed angle is equal to half of its arc
- Interior angle is equal to the average of the sum of its vertical angle pairs
- Exterior angle is equal to the average of the difference of far and near arcs

Khan Academy Videos: None relate

Homework: Circle Angles Worksheet

Reading: Student notes section 10-6

Section 10-6: Segment Relationships in Circles

SOL: G.11.a

Objective:

Use segments of chords, tangents, and secants

Vocabulary:

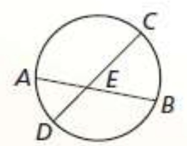
Secant – a line that intersects a circle in exactly two points

Core Concept:

Theorem

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot EB = EC \cdot ED$

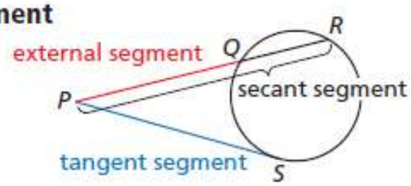
Proof Ex. 19, p. 516

Part of a chord times its other part = Part of the second chord time its other part

Core Concept

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.

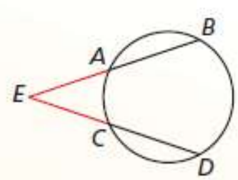


$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot EB = EC \cdot ED$

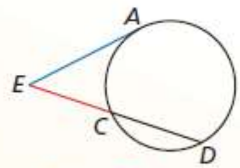
Proof Ex. 20, p. 516

Outside times the whole (outside + inside) = Outside times the whole

Theorem

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



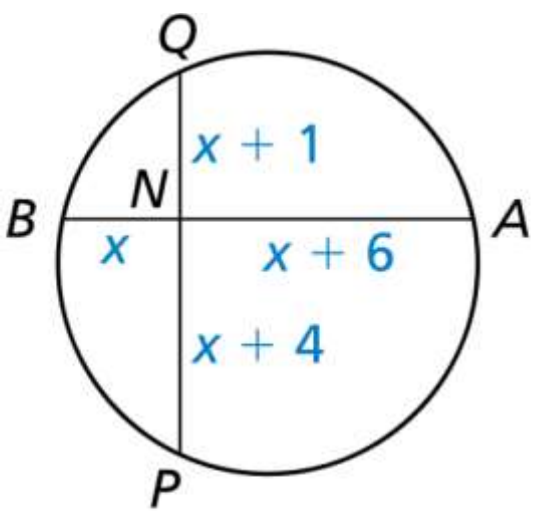
$EA^2 = EC \cdot ED$

Proof Exs. 21 and 22, p. 516

Examples:

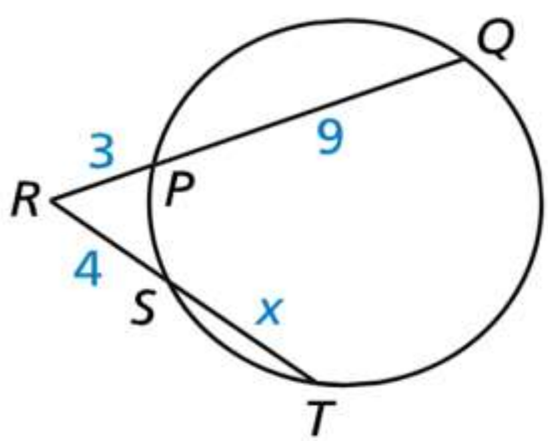
Example 1:

Find *AB* and *PQ*.



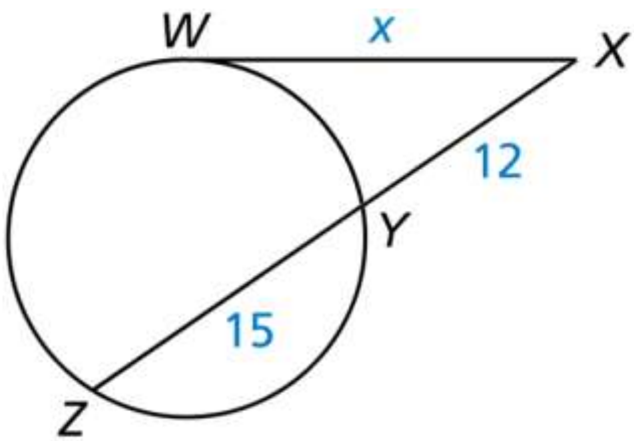
Example 2:

Find the value of x.



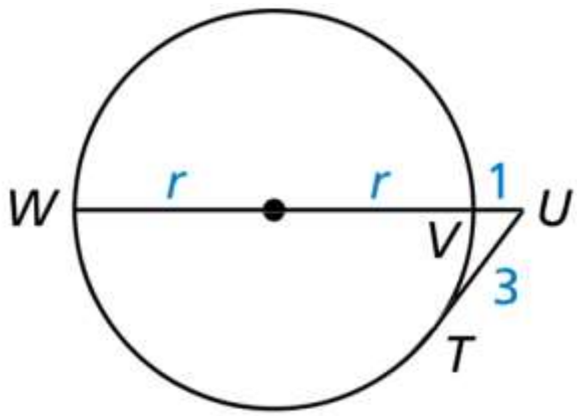
Example 3:

Find WX



Example 4:

Find the radius of the circle



Concept Summary:

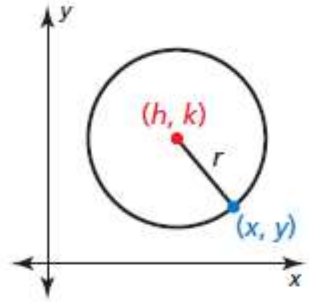
- The length of segments inside the circle are found using:
  - Part of segment x other part of segment = Part of the second segment x other part of second segment
- The length of segments outside the circle are found using:
  - Outside x Whole = Outside x Whole

Khan Academy Videos: None relate

Homework: Circle Segments Worksheet

Reading: Student notes section 10-7

Section 10-7: Circles in the Coordinate Plane



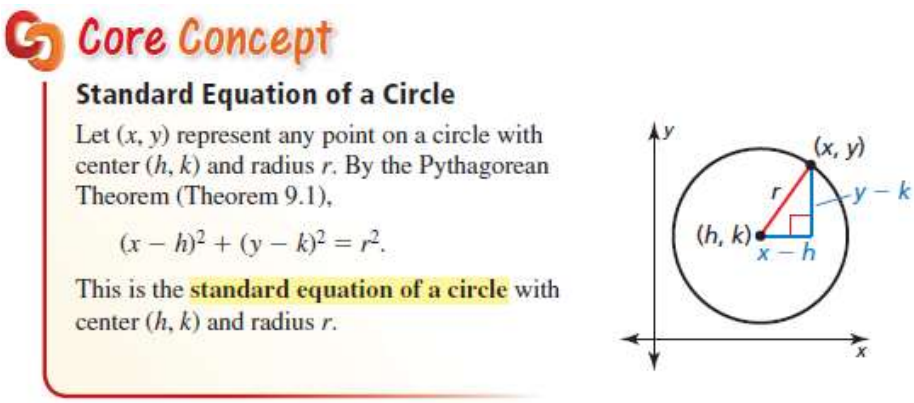
SOL: G.12

Objective:

Write and graph equations of circles
Write and coordinate proofs involving circles
Solve real-life problems using graphs of circles

Vocabulary: None New

Core Concept:



Core Concept

Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

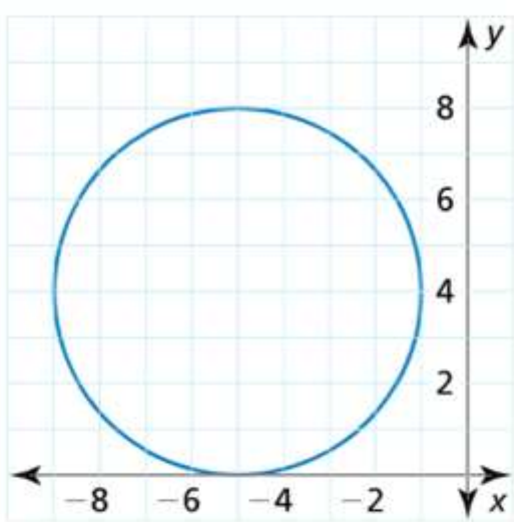
$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center (h, k) and radius r.

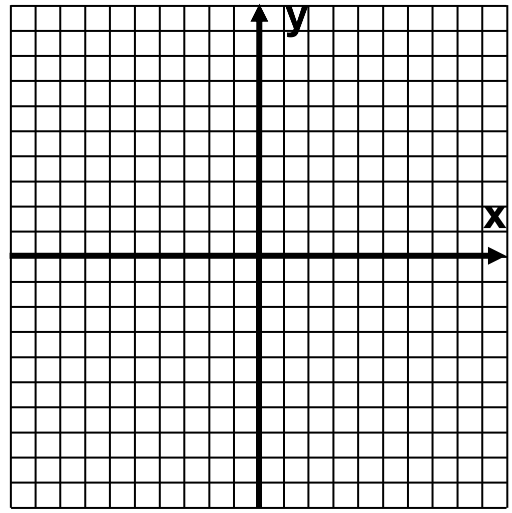
Examples:

Example 1:

Write the standard equation of the circle:

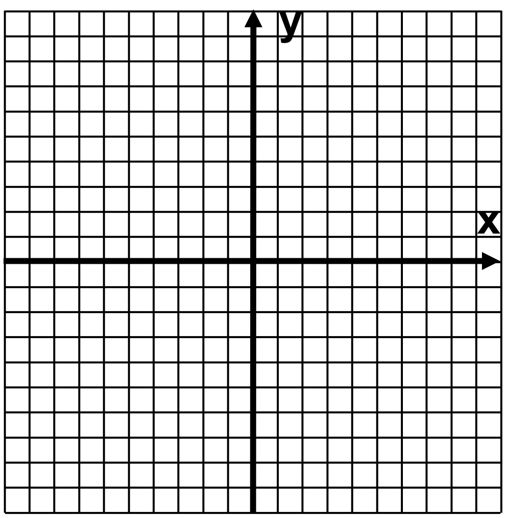
a. 

b. A circle with center at the origin and radius 3.5



Example 2:

The point (4, 1) is on a circle with center (1, 4). Write the standard equation of the circle.

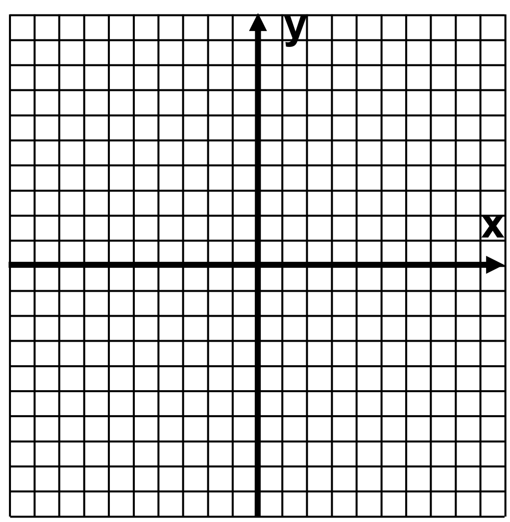


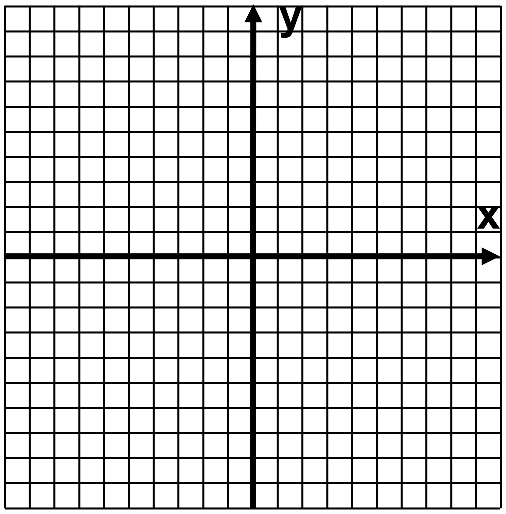
Example 3:

The equation of a circle is $x^2 + y^2 - 2x + 6y - 6 = 0$. Find the center and the radius of the circle. Then graph the circle.

Example 4:

Prove or disprove that the point $(3, \sqrt{7})$ lies on the circle centered at the origin and containing the point (1, 4).





Example 5:

The epicenter of an earthquake is 10 miles away from (-1,-3), 2 miles away from (5,3) and 5 miles away from (2,9). Find the coordinates of the epicenter.

Concept Summary:

- The coordinates of the center of a circle (h, k) and its radius r can be used to write an equation for the circle in the form $(x - h)^2 + (y - k)^2 = r^2$
  - Find the center and flip the signs (negatives in the equation)
- A circle can be graphed on a coordinate plane by using the equation written in standard form
- A circle can be graphed through any three noncollinear points on the coordinate plane
  - Using perpendicular bisectors of the sides of the triangle to find the center (the circumcenter from chapter 6)
- Use midpoint of diameter to find the center of a circle
- Use distance formula (from center to edge) to find the radius

Khan Academy Videos:

1. Features of a circle from its standard equation
2. Graphing a circle from its standard equation
3. Write standard equation of a circle
4. Points inside, outside or on a circle

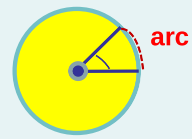
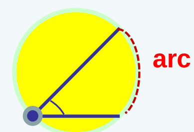
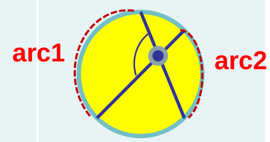
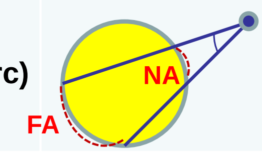
Homework: Circle Equations Worksheet

Reading: Student notes section 10-R

Section 10-R: Circles in the Coordinate Plane

Chapter 10 Review sheet

Circle Angles

Angle	Vertex Location	Sides	Formula (arcs)	Picture
Central	Center	Radii	= arc	
Inscribed	Edge	Chords	= ½ arc	
Interior	Inside (not at center)	Chords	= ½ (arc1+arc2)	
Exterior	Outside	Secants Tangents	= ½ (Far arc – Near arc)	

Remember: Vertex is the corner point (hinge point) of the angle.
Arcs are ***around the edge*** of the circle.
Circle's arcs always sum to 360°

Major arcs measure > 180
Minor arcs measure < 180
Semi-circles measure = 180 (formed by diameters)

Central angle is twice the inscribed angle with the same arc

Arc 1 and arc 2 in interior angles are formed by the vertical angle pair (follow the "X" out to the edge of the circle)

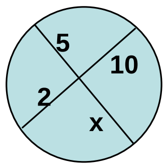
FA = Far Arc (or the big arc) NA = Near Arc (or the little arc)

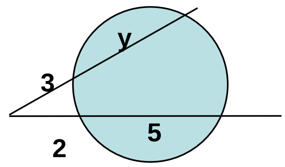
Remember Vertical Angles, Linear Pairs and 3 angle in a triangle rules!!!

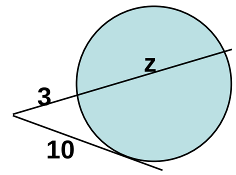
Chapter 10 Circles

Chapter 10 Review sheet

Chords, Secants and Tangents







"Parts * parts are equal"

to get "x"
$5*x = 2*10$
$5x = 20$
$x = 4$

"outside part times whole thing = outside part times whole thing"
$3(3 + y) = 2*(2 + 5)$
$9 + 3y = 14$
$3y = 5$
$y = 5/3$

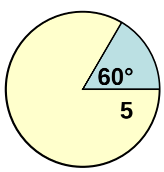
"outside part times whole thing = tangent squared"
$3(3 + z) = 10^2$
$9 + 3z = 100$
$3z = 91$
$z = 30.33$

Area of a sector (% of total area) and Arc Length (% of circumference)

$$A\ sec = \frac{\theta}{360°} * \pi r^2$$

$$Arc\ Len = \frac{\theta}{360°} * 2\pi r$$

where θ is central angle



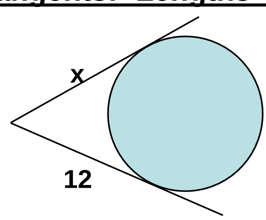
$$A = \frac{\theta}{360°} * \pi r^2$$

$$A = \frac{60°}{360°} * \pi 5^2 = \frac{25\pi}{6} \approx 13.083$$

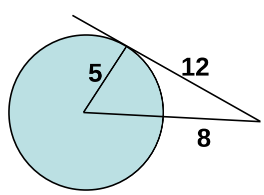
$$Arc\ Len = \frac{\theta}{360°} * \pi r^2$$

$$Arc\ Len = \frac{60°}{360°} * 2\pi 5 = \frac{10\pi}{6} \approx 5.236$$

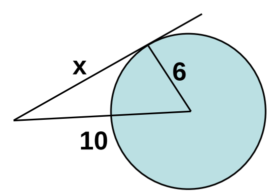
Tangents: Lengths outside the circle and relationship to radius or diameter



"tangents = from same point" have same length, so $x = 12$



Pythagorean Theorem to check if tangent
$(5+8)^2 = 5^2 + 12^2$
$169 = 25 + 144$
$169 = 169$
Yes, tangent



Tangents are perpendicular to radii or diameter; use the Pythagorean Theorem to check if tangent
$10^2 = 6^2 + x^2$
$100 = 36 + x^2$
$64 = x^2$
$8 = x$

Equation of a Circle: $(x – h)^2 + (y – k)^2 = r^2$

Where (h,k) is the center and r the radius
Center is the midpoint of the ends of the diameter;
point on the edge of the circle satisfies the equation

Homework: Chapter 10 SOL Gateway

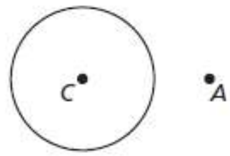
Reading: Student notes section 10-R

Chapter 10 Circles

Constructions:

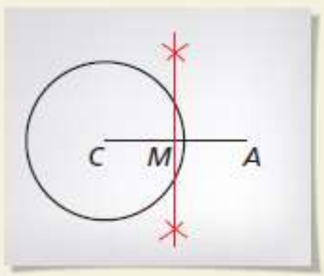
CONSTRUCTION Constructing a Tangent to a Circle

Given ⊙C and point A, construct a line tangent to ⊙C that passes through A. Use a compass and straightedge.



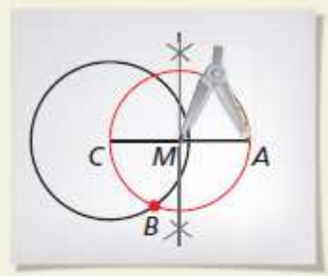
SOLUTION

Step 1



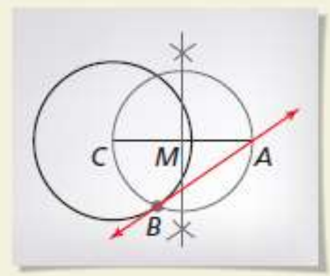
Find a midpoint
Draw $\overline{AC}$. Construct the bisector of the segment and label the midpoint M.

Step 2



Draw a circle
Construct ⊙M with radius MA. Label one of the points where ⊙M intersects ⊙C as point B.

Step 3



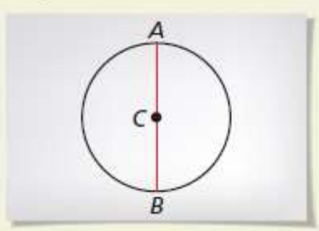
Construct a tangent line
Draw $\overleftrightarrow{AB}$. It is a tangent to ⊙C that passes through A.

CONSTRUCTION Constructing a Square Inscribed in a Circle

Given ⊙C, construct a square inscribed in a circle.

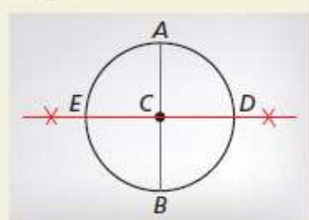
SOLUTION

Step 1



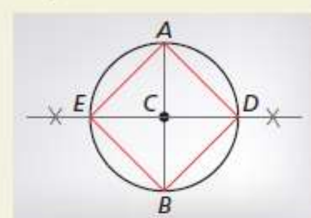
Draw a diameter
Draw any diameter. Label the endpoints A and B.

Step 2



Construct a perpendicular bisector
Construct the perpendicular bisector of the diameter. Label the points where it intersects ⊙C as points D and E.

Step 3



Form a square
Connect points A, D, B, and E to form a square.

Khan Academy Videos on Constructions:

1. Circle-inscribed square
2. Circle-inscribed equilateral triangle
3. Circle-inscribed regular hexagon
4. Triangle inscribing circle
5. Triangle circumscribing circle