Chapter 11 Circumference, Area and Surface Area

Addressed or Prepped VA SOL:

G.11 The student will solve problems, including practical problems, by applying properties of circles. This will include determining
- c) arc length; and
- d) area of a sector.

G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
- a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;
- b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;
- c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and
- d) solving problems, including practical problems, about similar geometric figures.

SOL Progression

Middle School:
- Find the area and circumference of a circle
- Find the area of triangles, special quadrilaterals, and polygons
- Solve real-life problems involving area of composite figures
- Construct three-dimensional models, given the top or bottom, side and front views
- Determine the surface areas of rectangular prisms, cylinders, cones and square-based pyramids

Algebra I:
- Rewrite and use literal equations and formulas of area
- Write and solve linear equations in one variable
- Use multi-step linear equations to solve real-life problems
- Use unit analysis to model real-life problems
- Solve quadratic equations in one variable

Geometry:
- Measure angles in radians
- Find arc lengths and areas of sectors of circles
- Find areas of rhombuses, kites, and regular polygons
- Find and use surface areas of prisms, cylinders, pyramids, cones and spheres
- Describe cross-sections and solids of revolution
- Describe how changes in one or more measures affect the measures of a figure

Chapter 11 Circumference, Area and Surface Area

Section 11-1: Circumference and Arc Length

SOL: G.11.c

Objective:

Use the formula for circumference
Use arc length to find measures
Solve real-life problems
Measure angles in radian

Vocabulary:

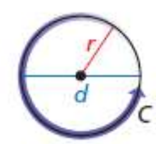
Arc length – a portion of the circumference of a circle
Circumference – the distance (perimeter) around the circle
Radian – a measure in π units of central angles (180° = 1 radian and 360° = 2 radians)

Core Concepts:

Core Concept

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

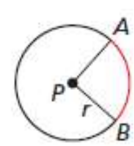
Core Concept

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

$$\frac{\text{Arc length of } \overparen{AB}}{2\pi r} = \frac{m\overparen{AB}}{360^\circ}, \text{ or}$$

$$\text{Arc length of } \overparen{AB} = \frac{m\overparen{AB}}{360^\circ} \cdot 2\pi r$$



Core Concept

Converting between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}.$$

Radians to degrees

Multiply radian measure by

$$\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}.$$

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Examples:

Example 1:

Find each indicated measure.

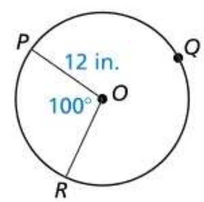
a. Circumference of a circle with a radius of 11 inches.

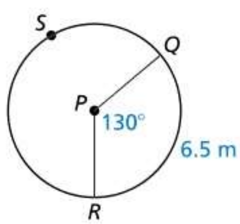
b. Radius of a circle with a circumference of 4 millimeters.

Example 2:

Find each indicated measure.

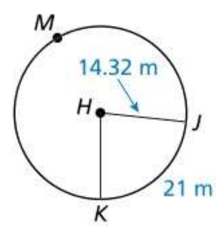
a. Arc length of $\widehat{PR}$





b. Circumference of $\odot P$

c. $m\widehat{JK}$



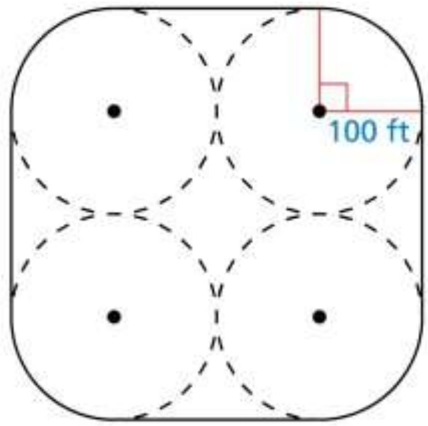
Example 3:

The radius of a wheel on a toy truck is 4 inches. To the nearest foot, how far does the wheel travel when it makes 7 revolutions?

Vocabulary, Objectives, Concepts and Other Important Information

Example 4:

A path is built around four congruent circular fields. The radius of each field is 100 feet. How long is the path? Round to the nearest hundred feet.



Example 5:

a. Convert 30° to radians

b. Convert $\frac{3\pi}{8}$ radians to degrees.

Concept Summary:

- Circumference of a circle (perimeter), $C = 2\pi r = d\pi$
- Arc length is a proportion of the circumference
- 180 degrees is π radians

Khan Academy Videos:

1. Arc Length from subtended (central) angle
2. Introduction to radians
3. Radians and degrees
4. Degrees to radians
5. Radians to degrees

Homework: none

Reading: student notes section 11.2

Section 11-2: Areas of Circles and Sectors

SOL: G.11.d.

Objective:

Use the formula for the area of a circle
Use the formula population density
Find areas of sectors
Use areas of sectors

Vocabulary:

Population density –a measure of how many people live within a given area
Sector of a circle – region of area bounded by two radii and their intercepted arc

Core Concepts:

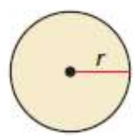
Core Concept

Area of a Circle

The area of a circle is

$$A = \pi r^2$$

where r is the radius of the circle.



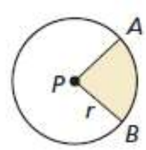
Core Concept

Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$



Chapter 11 Circumference, Area and Surface Area

Examples:

Example 1:

Find each indicated measure.

a. Area of a circle with a radius of 8.5 inches

b. Diameter of a circle with an area of 153.94 square feet

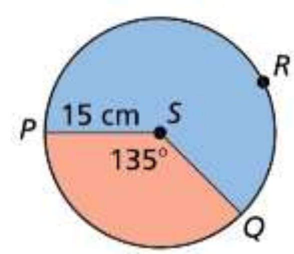
Example 2:

a. About 124,000 people live in a 2-mile radius of a city's post office. Find the population density in people per square mile.

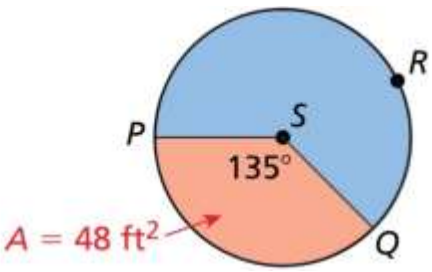
b. A region with a 10-mile radius has a population density of about 869 people per square mile. Find the number of people who live in the region.

Example 3:

Find the areas of the sectors formed by $\angle PSQ$



Vocabulary, Objectives, Concepts and Other Important Information

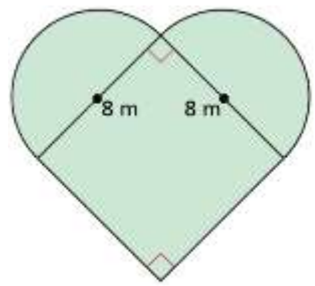


Example 4:

Find the area of ⊙*S*

Example 5:

A farmer has a field with the shape shown. Find the area of the shaded region to the nearest square meter.



Concept Summary:

- Area of a circle is $A = \pi r^2$
- A sector is a portion (like a piece of pie) of a circle
- The area of a sector is proportional to its central angle, $Sec\ Area\ = \left(\frac{central\ angle}{360}\right) circumference$

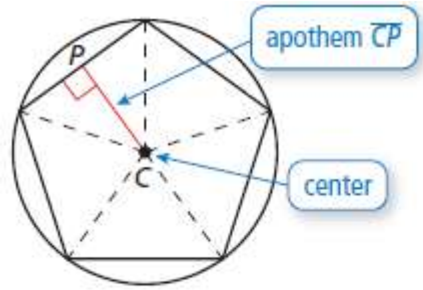
Khan Academy Videos:

1. Area of a circle
2. Area of a sector

Homework: Sector Worksheet

Reading: student notes section 11.3

Section 11-3: Areas of Polygons



SOL: G.13

Objective:

Find areas of rhombuses and kites
Find angle measures in regular polygons
Find areas of regular polygons

Vocabulary:

Apothem of a regular polygon – the distance from the center to any side of the polygon
Center of a regular polygon – the center of the circle circumscribed around the polygon
Central angle of a regular polygon – the angle formed by two radii drawn to consecutive vertices of the polygon; (also equal to the exterior angle of the polygon!!)
Radius of a regular polygon – the radius of the circle circumscribed around the polygon

Core Concepts:

Core Concept

Area of a Rhombus or Kite

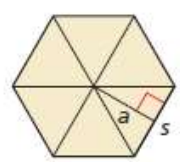
The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

Core Concept

Area of a Regular Polygon

The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

$A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$



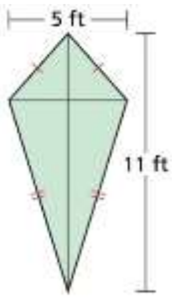
Chapter 11 Circumference, Area and Surface Area

Examples:

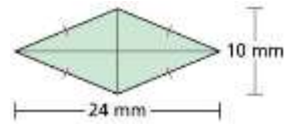
Example 1:

Find the area of each rhombus or kite.

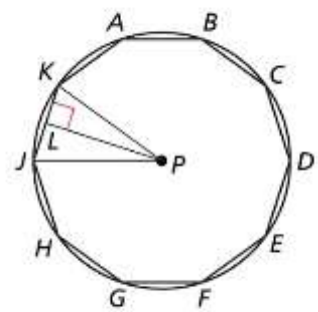
a.



b.



Example 2:

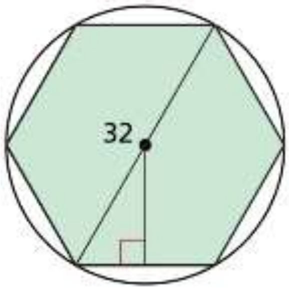


In the diagram, polygon *ABCDEFGHJK* is a regular decagon inscribed in ⊙P. Find each angle measure.

a. $m\angle KPJ$

b. $m\angle LPK$

c. $m\angle LJP$

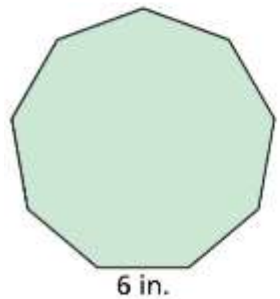


Example 3:

A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.

Example 4:

A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?



Concept Summary:

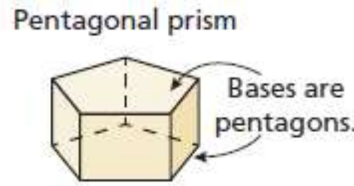
- Area formulas of most figures on the formula sheet
- Area of a polygon, $A = 1/2pa$, a = apothem and p = perimeter is not on it
- Area of a rhombus, $A = 1/2d_1d_1$, d is the whole length of diagonal is not on it
- Area of composite figures is the area of each of its parts added up

Khan Academy Videos:

1. Area of triangles,
2. Area of parallelograms
3. Area of trapezoids
4. Area of kites
5. Area of composite shapes

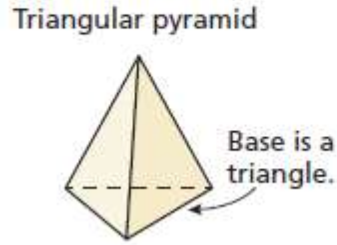
Homework: Area Worksheet 1 and 2

Reading: student notes section 11.4



Section 11-4: Three-Dimensional Figures

SOL: G.13



Objective:

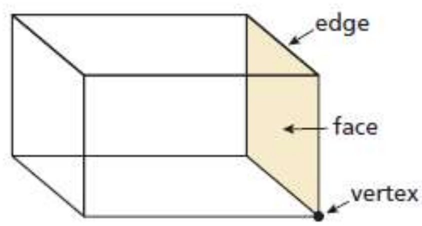
Classify solids
Describe cross sections
Sketch and describe solids of revolutions

Vocabulary:

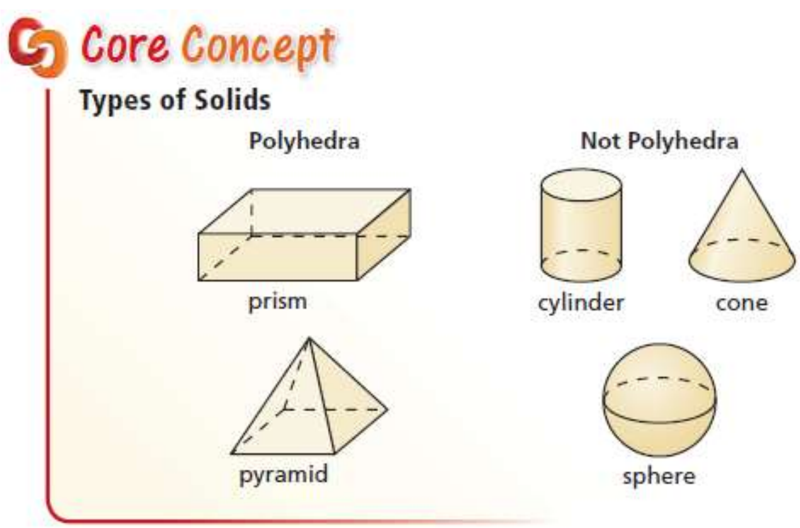
Axis of revolution – the line around which a shape is rotated
Cross section – the intersection of a plane and a solid
Edge – line segment formed by the intersection of two faces
Face – the sides of a polyhedron
Polyhedron – a solid that is bounded by polygons
Solid of revolution – a three dimensional figure that is formed by rotating a two-dimensional shape around an axis
Vertex – a point where three or more edges meet

A **polyhedron** is a solid that is bounded by polygons, called **faces**.

- Each *vertex* is a point.
- Each *edge* is a segment of a line.
- Each *face* is a portion of a plane.



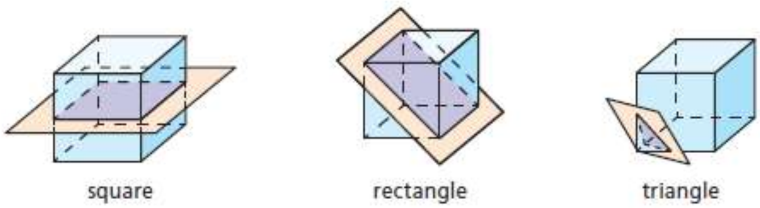
Core Concepts:



Chapter 11 Circumference, Area and Surface Area

Describing Cross Sections

Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

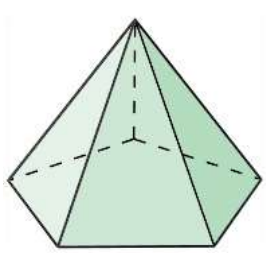
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.

Examples:

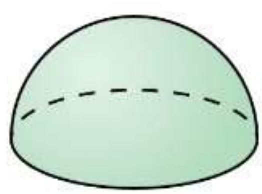
Example 1:

Tell whether each solid is a polyhedron. If it is, name the polyhedron.

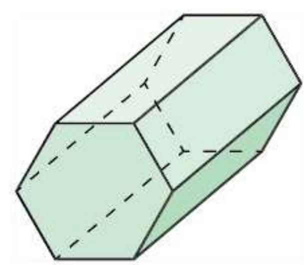
a.



b.



c.

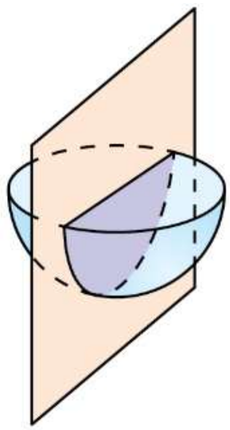


Vocabulary, Objectives, Concepts and Other Important Information

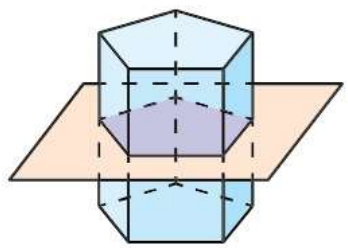
Example 2:

Describe the shape formed by the intersection of the plane and the solid.

a.



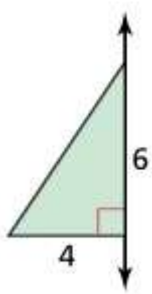
b.



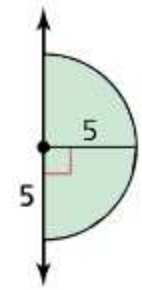
Example 3:

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid (defining parameters needed for surface area or volume).

a.



b.



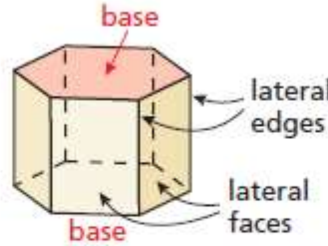
Concept Summary:
- A revolutionary solid is formed by rotating a 2-d figure around an axis or revolution
- Revolutionary solids are talked about in calculus courses a lot
- Conic sections in Algebra 2 are the cross-sections of the intersections of a plane and a cone
- Faces are sides, vertexes are corner points and edges are line segments connecting sides

Khan Academy Videos:
1. Rotating 2D shapes in 3D

Homework: None

Reading: student notes section 11.5

Section 11-5: Surface Areas of Prisms and Cylinders



SOL: G.13 and G.14

Objective:

Find lateral area and surface areas of right prisms
Find lateral area and surface areas of right cylinders
Use surface areas of right prisms and right cylinders
Find surface area of similar solids

Vocabulary:

Lateral area – the sum of the area of a figure's lateral faces
Lateral edges – segments connecting the bases
Lateral faces – faces of a polyhedron other than the bases
Net – a two-dimensional representation of the faces
Oblique cylinder – segment joining the centers of the bases is *not* perpendicular to the bases
Oblique prism – each lateral edge is *not* perpendicular to both bases
Right cylinder – segment joining the centers of the bases is perpendicular to the bases
Right prism – each lateral edge is perpendicular to both bases
Similar solids – two solids of the same type with equal ratios of corresponding linear measures (heights, radii, etc)
Surface area – the sum of the area of a figure's faces (lateral and base)

Key Concepts:

Core Concept

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their surface areas is equal to k^2.

Core Concept

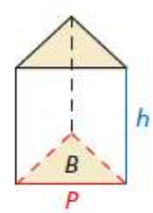
Lateral Area and Surface Area of a Right Prism

For a right prism with base perimeter P, base apothem a, height h, and base area B, the lateral area L and surface area S are as follows.

Lateral area $L = Ph$

Surface area $S = 2B + L$

$= aP + Ph$



Core Concept

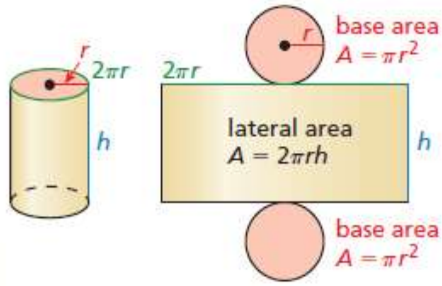
Lateral Area and Surface Area of a Right Cylinder

For a right cylinder with radius r, height h, and base area B, the lateral area L and surface area S are as follows.

Lateral area $L = 2\pi rh$

Surface area $S = 2B + L$

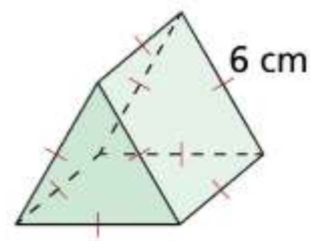
$= 2\pi r^2 + 2\pi rh$



Examples:

Example 1:

Find the lateral area and the surface area of the triangular prism.

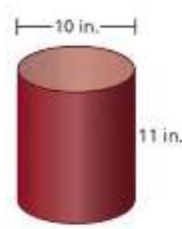


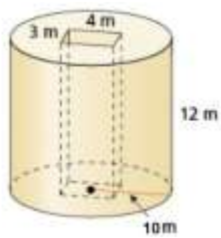
Example 2:

Find the lateral area and the surface area of a right cylinder with a radius of 2 inches and a height of 6 inches.

Example 3:

You are covering the lateral area of a wastebasket with paper. Find the minimum amount of paper you need.



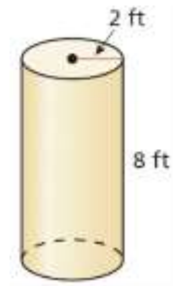


Example 4:

Find the lateral area and the surface area of the composite figure.

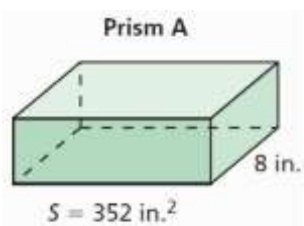
Example 5:

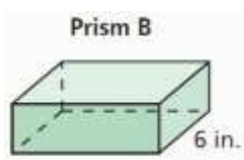
Describe how quadrupling all the linear dimensions of the cylinder affects the surface area of the right cylinder.



Example 6:

Prism A and Prism B are similar. Find the surface area of the Prism B.





Concept Summary:

- Lateral (means sides) area can be found, when it exists, on the formula sheet
- Surface areas of cylinders, rectangular and triangular prism are on the formula sheet
- Rectangular prisms have 6 surfaces (top/bottom, front/back, right/left sides)

Khan Academy Videos: None related

Homework: none

Reading: student notes section 11.6

Chapter 11 Circumference, Area and Surface Area

Section 11-6: Surface Areas of Pyramids and Cones

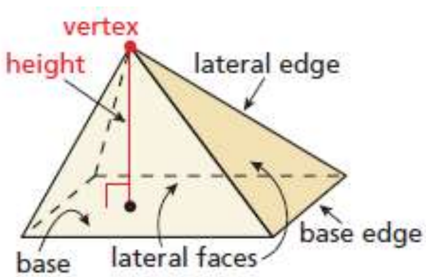
SOL: G.13 and G.14

Objective:

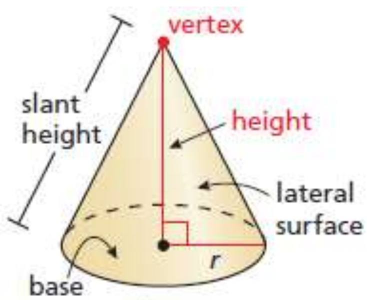
Find lateral areas and surface areas of regular pyramids and right cones
Find surface areas of similar pyramids and cones
Use surface areas of regular pyramids and right cones

Vocabulary:

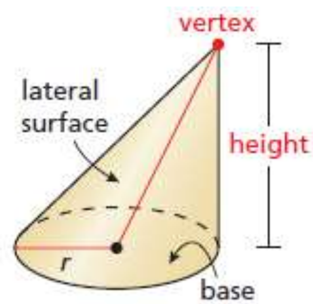
Lateral surface of a cone – consists of all segments that connect the vertex with points on the edge of the base
Oblique cone – segment joining the vertex and the center of the base is *not* perpendicular to the base
Regular pyramid – has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base
Right cone – segment joining the vertex and the center of the base is perpendicular to the base
Slant height of a regular pyramid – is the height of a lateral face of the regular pyramid
Slant height of a right cone – the distance between the vertex and a point on the edge of the base
Vertex of a cone – is not in the same plane as the base
Vertex of a pyramid – the common vertex of the triangular faces of a pyramid



Pyramid



Right cone



Oblique cone

Core Concepts:

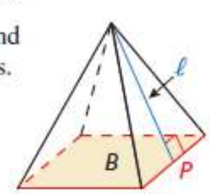
Core Concept

Lateral Area and Surface Area of a Regular Pyramid

For a regular pyramid with base perimeter P, slant height ℓ, and base area B, the lateral area L and surface area S are as follows.

Lateral area $L = \frac{1}{2}P\ell$

Surface area $S = B + L = B + \frac{1}{2}P\ell$



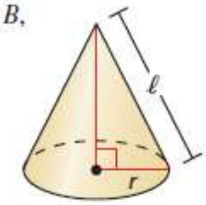
Core Concept

Lateral Area and Surface Area of a Right Cone

For a right cone with radius r, slant height ℓ, and base area B, the lateral area L and surface area S are as follows.

Lateral area $L = \pi r \ell$

Surface area $S = B + L = \pi r^2 + \pi r \ell$

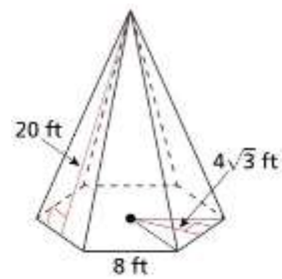


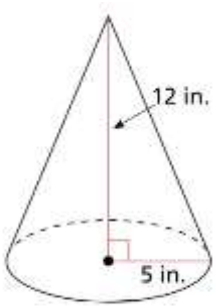
Note: If two similar solids have a scale factor of k, then the ratio of their surface areas is equal to k^2.

Examples:

Example 1:

Find the lateral area and the surface area of the regular hexagonal pyramid.



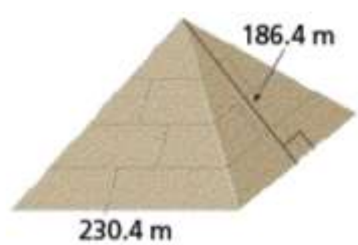


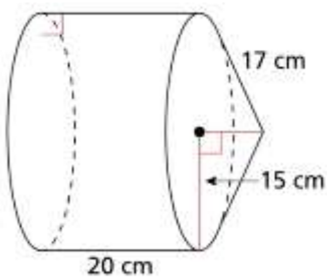
Example 2:

Find the lateral area and the surface area of the right cone.

Example 3:

The Great Pyramid of Giza is a regular square pyramid. It is estimated that when the pyramid was first built, each base edge was 230.4 meters long and the slant height was 186.4 meters long. Find the lateral area of a square pyramid with those dimensions.



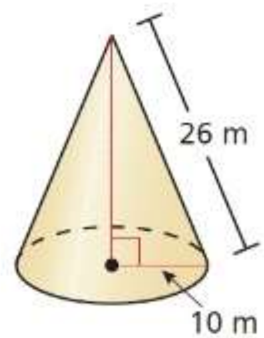


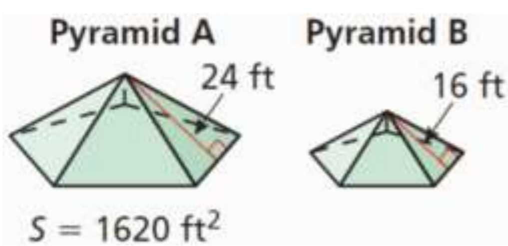
Example 4:

Find the lateral area and the surface area of the composite solid.

Example5:

Describe how doubling all the linear dimensions of the right cone affects the surface area of the solid.





Example 6:

Pyramids A and B are similar regular pyramids. Find the surface area of pyramid B.

Concept Summary:

- Surface area formulas for pyramids and cones are found on the formula sheet
- Surface area is a squared relationship so with similar figures you need to check the variables carefully to determine changes

Khan Academy Videos: None related

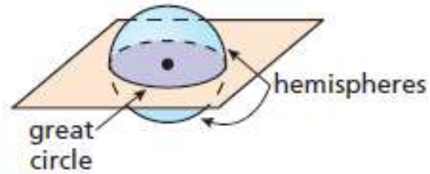
Homework: pg

Reading: student notes section 11.7

Chapter 11 Circumference, Area and Surface Area

Section 11-7: Surface Areas of Spheres

SOL: G.13 and G.14



Objective:

Find surface areas of spheres and hemi-spheres
Find surface areas of similar spheres

Vocabulary:

Chord of a sphere – a segment whose endpoints are on the sphere
Great circle – the intersection of a sphere and a plane that goes through its center

Note: in global navigation routes, the shortest distance between two points on the surface of the sphere (like the earth) is along the great circle containing those two points.

Key Concepts:

Core Concept

Surface Area of a Sphere

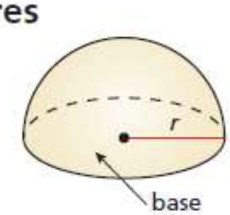
The surface area S of a sphere is

$$S = 4\pi r^2$$

where r is the radius of the sphere.

Finding Surface Areas of Hemispheres

The *curved surface area* of a hemisphere does not include the surface area of the circular base. The *total surface area* of a hemisphere includes the surface area of the circular base.



Core Concept

Surface Area of a Hemisphere

The curved surface area S of a hemisphere is $S = 2\pi r^2$ where r is the radius of the hemisphere. The total surface area S of a hemisphere is $S = 2\pi r^2 + \pi r^2 = 3\pi r^2$ where r is the radius of the hemisphere.

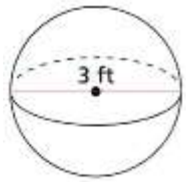
Note: The surface area of a hemisphere is half the surface area of the sphere (the sides of the hemisphere) plus the area of a circle (the cross-section of the sphere cut in half).

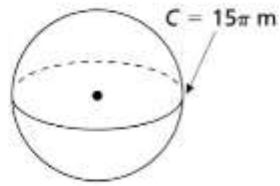
Chapter 11 Circumference, Area and Surface Area

Examples:

Example 1:

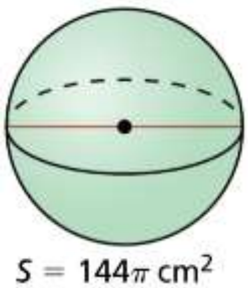
Find the surface area of each sphere.





a.

b.

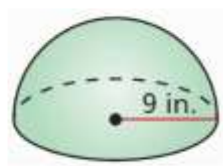


Example 2:

Find the diameter of the sphere.

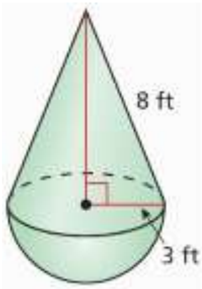
Example 3:

Find the indicated surface area of the hemisphere



a. Curved surface area

b. Total surface area

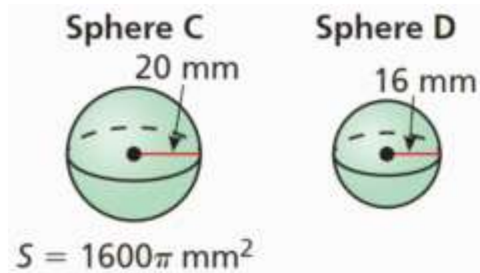


Example 4:

Find the surface area of the composite solid.

Example 5:

Spheres C and D are similar. Find the surface area of Sphere D.



Concept Summary:

- Surface area of a sphere: $SA = 4\pi r^2$ and is on the formula sheet
- Area is a squared relationship, so similar sphere's surface area is proportional to the squares of their radii
- Surface area of a hemi-sphere: $SA = 3\pi r^2$ is not on the SOL formula sheet. It is the sum of half the surface area of the sphere plus the area of the newly exposed circle

Khan Academy Videos: None related

Homework: none

Reading: student notes section 11.R

Chapter 11 Circumference, Area and Surface Area

Section 11-R: Chapter Review

SOL: G.13 and G.14

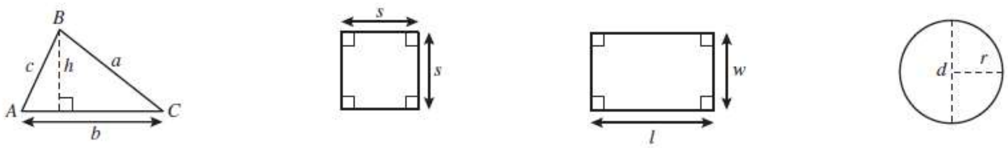
Objective:

Review Chapter 11 Material on Area

Vocabulary: none new

Key Concepts:

Geometric Formulas

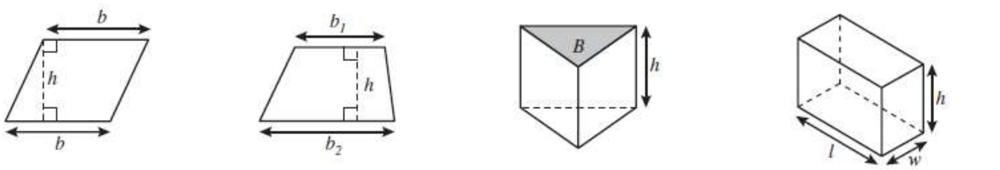


Triangle: $A = \frac{1}{2}bh$, $A = \frac{1}{2}ab \sin C$

Square: $p = 4s$, $A = s^2$

Rectangle: $p = 2l + 2w$, $A = lw$

Circle: $C = 2\pi r$, $C = \pi d$, $A = \pi r^2$



Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Prism: $V = Bh$, $L.A. = hp$, $S.A. = hp + 2B$

Rectangular prism: $V = lwh$, $S.A. = 2lw + 2lh + 2wh$

Cylinder: $V = \pi r^2 h$, $L.A. = 2\pi rh$, $S.A. = 2\pi r^2 + 2\pi rh$

Sphere: $V = \frac{4}{3}\pi r^3$, $S.A. = 4\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h$, $L.A. = \pi rl$, $S.A. = \pi r^2 + \pi rl$

Pyramid: $V = \frac{1}{3}Bh$, $L.A. = \frac{1}{2}lp$, $S.A. = \frac{1}{2}lp + B$

Homework: pg

Reading: none