Chapter 12 Volume

Addressed or Prepped VA SOL:

G.13 The student will use surface area and volume of three-dimensional objects to solve practical problems.

G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include

a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;
b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;
c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and
d) solving problems, including practical problems, about similar geometric figures.

SOL Progression

Middle School:

- Find the area and circumference of a circle
- Find the area of triangles, special quadrilaterals, and polygons
- Apply the formulas for volumes of rectangular prisms, cones and cylinders
- Solve real-life problems involving area of composite figures

Algebra I:

- Rewrite and use literal equations and formulas of area
- Write and solve linear equations in one variable
- Use multi-step linear equations to solve real-life problems
- Use unit analysis to model real-life problems
- Solve quadratic equations in one variable

Geometry:

- Find and use volumes of prisms, cylinders, pyramids, cones, and spheres
- Describe cross-sections and solids of revolution

Chapter 12 Volume

Section 12-1: Volumes of Prisms and Cylinders

"What if someone hits the mute button?"

SOL: G.13, G.14

Objective:

Find volumes of prisms and cylinders
Use the formula for density
Use volumes of prisms and cylinders

Vocabulary:

Cavalieri's Principle – states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Density – the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

Volume – the number of cubic units contained in a solid's interior

Core Concept:

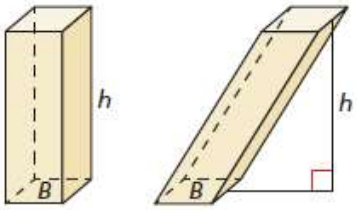
Core Concept

Volume of a Prism

The volume V of a prism is

$$V = Bh$$

where B is the area of a base and h is the height.



Note: like in two-dimensional figures, the height, h, is always perpendicular to the area of the Base, B

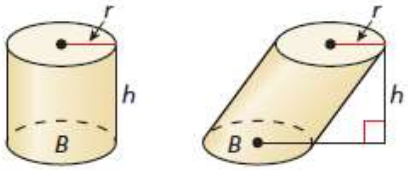
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.



Note: $Density = \frac{Mass}{Volume}$

Core Concept

Similar Solids

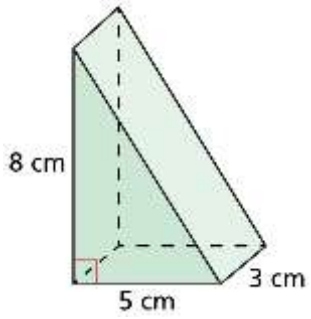
Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called *similar solids*. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their volumes is equal to k^3.

Examples:

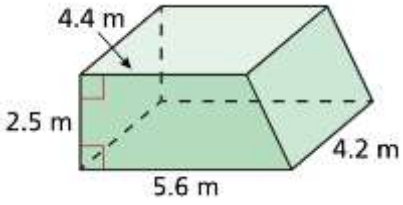
Example 1:

Find the volume of each prism.

a.



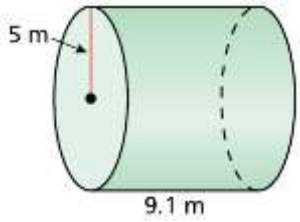
b.



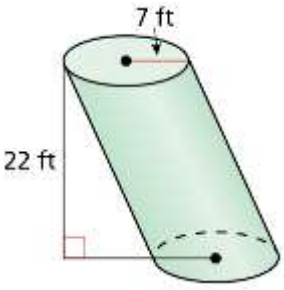
Example 2:

Find the volume of each cylinder.

a.



b.



Example 3:

The density of water is 1000 kilograms per cubic meter. Find the mass of 1 cubic foot of water. Use the fact that 1 foot = 0.3048 meters.

Example 4:

You are building a cylindrical packing tube. You want the length of the tube to be 30 inches and the volume to be 589 cubic inches. What should the radius of the base be?

Example 5:

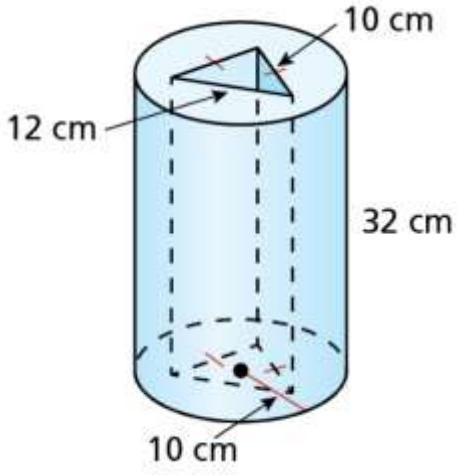
You are building a 3-foot tall dresser. You want the volume to be 42 cubic feet. What should the area of the base be? Give a possible length and width.

Example 6:

Square prism A and square prism B are similar. Each base edge of prism A is 4 inches, and each base edge of prism B is 6 inches. The volume of prism B is 135 cubic inches. Find the volume of prism A.

Example 7:

Find the volume of the composite solid.



Concept Summary:

Volume of Prism and Cylinders is Base Area times Height
Volume is a cubic relationship so similar figures will have their volumes found by using the cube of the scaling factor

Homework:

Khan Academy Videos:

1. Cylinder volume and surface area
2. Volume of triangular prism and cube

Reading: student notes section 12-2

Section 12-2: Volumes of Pyramids

SOL: G.13, G.14

Objective:

Find volumes of pyramids
Use volumes of pyramids

Vocabulary: none new

Core Concept:

Core Concept

Volume of a Pyramid

The volume V of a pyramid is

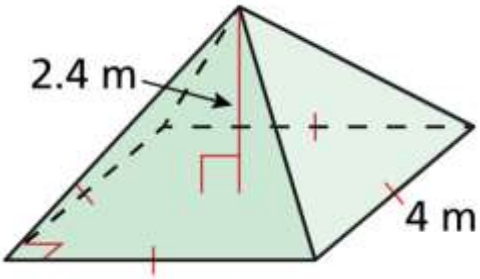
$$V = \frac{1}{3}Bh$$

where B is the area of the base and h is the height.

Examples:

Example 1:

Find the volume of the pyramid.

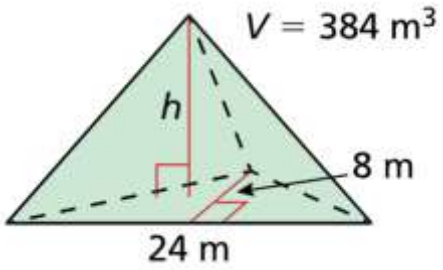


Example 2:

A square pyramid has a height of 12 centimeters and a volume of 64 cubic centimeters. Find the side length of the square base.

Example 3:

Find the height of the triangular pyramid.

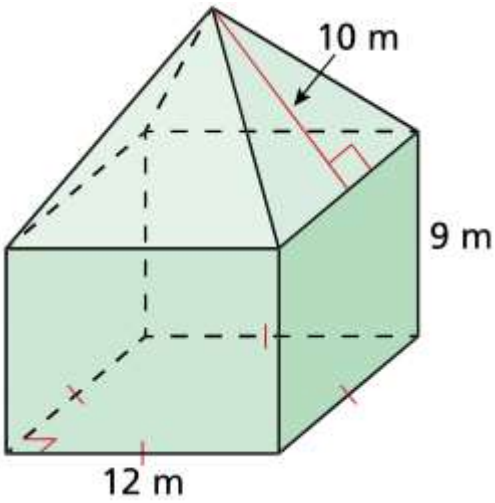


Example 4:

Square pyramid A and square pyramid B are similar. The height of pyramid A is 6 inches and the height of pyramid B is 15 inches. The volume of pyramid B is 312.5 cubic inches. Find the volume of pyramid A.

Example 5:

Find the volume of the composite solid.



Concept Summary:

- Volume of Pyramid is 1/3 the Base Area times the height
- SOL usually only has square-based pyramids

Homework:

Reading: student notes section 12-3

Section 12-3: Volume of Cones

SOL: G.13, G.14

Objective:

Find volumes of cones
Use volumes of cones

Vocabulary: none new

Core Concept:

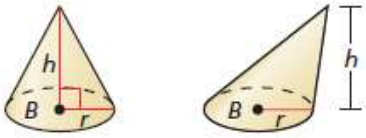
Core Concept

Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

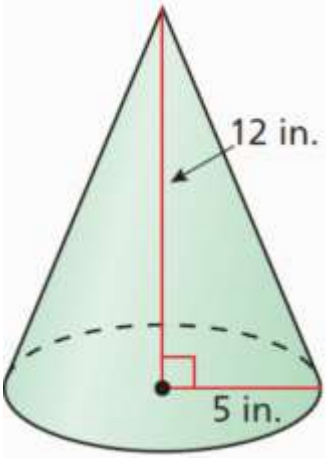
where B is the area of the base, h is the height, and r is the radius of the base.



Examples:

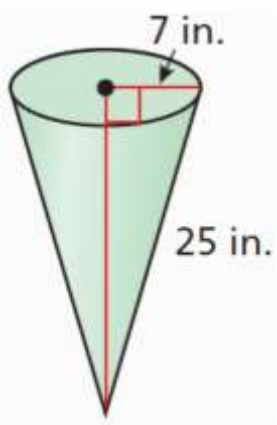
Example 1:

Find the volume of the cone.



Example 2:

Find the volume of the cone.



Example 3:

A waffle ice cream cone has a height of 6 inches and a diameter of 3 inches. Find the amount of ice cream (in cubic inches) the cone can hold.

Example 4:

Cone A and cone B are similar. The height of cone A is 6 inches and the height of cone B is 2 inches. The volume of cone B is 18π cubic inches. Find the volume of cone A.

Concept Summary:

- Volume of a Cone is 1/3 the Base Area times the height
- A Pythagorean relationship exists between the slant height, the radius and the height in a cone
- Volume is a cubic relationship so similar figures will have their volumes found by using the cube of the scaling factor

Khan Academy Videos:

1. Volume of a cone

Homework: none

Reading: student notes section 12-4

Section 12-4: Volumes of Spheres

SOL: G.13

Objective:

Find volumes of spheres
Find volumes of hemi-spheres
Solve real-life problems

Vocabulary: none new

Core Concept:

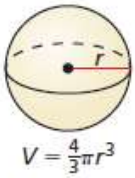
Core Concept

Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.



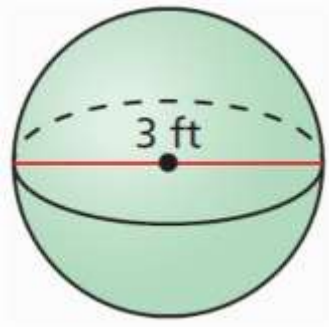
Examples:

Example 1:

Find the volume of the ball.

Example 2:

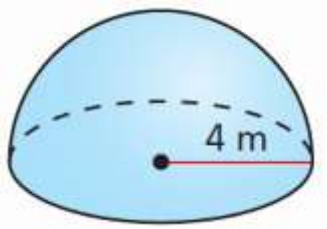
Find the volume of the sphere.



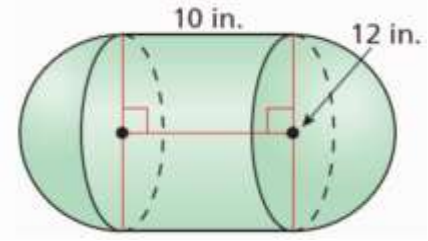
Example 3:

The surface area of a sphere is 144π square meters. Find the volume of the sphere.

Example 4: Find the volume of the hemi-sphere.



Example 5: Find the volume of the composite solid.



Example 6:

Sphere A and sphere B are similar. The radius of sphere A is 4 inches, and the radius of prism B is 6 inches. The volume of prism B is 288π cubic inches. Find the volume of sphere A.

Concept Summary:

- Volume of Sphere is 4/3 times pi times the radius cubed
- Volume is a cubic relationship so similar figures will have their volumes found by using the cube of the scaling factor
- Spheres are the most common SOL 3D figures used in similarities

Khan Academy Videos:

1. Volume of a sphere

Homework: SA/Vol Worksheet 3

Reading: student notes section 12-R

Chapter 12 Volume

Section 12-R: Chapter Review

SOL: G.13

Objective:
Review chapter 12 material

Vocabulary: none new

Core Concept:

Volume:

Volume in Similar Figures:

Volume in Composite Figures:

Examples:

Concept Summary:
xxx

Homework: pg

Reading: none