Big Ideas Chapter 2: Reasoning and Direct Proofs

Addressed or Prepped VA SOL:

G.1 The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include

a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form; and
c) determining the validity of a logical argument.

SOL Progression

Middle School:

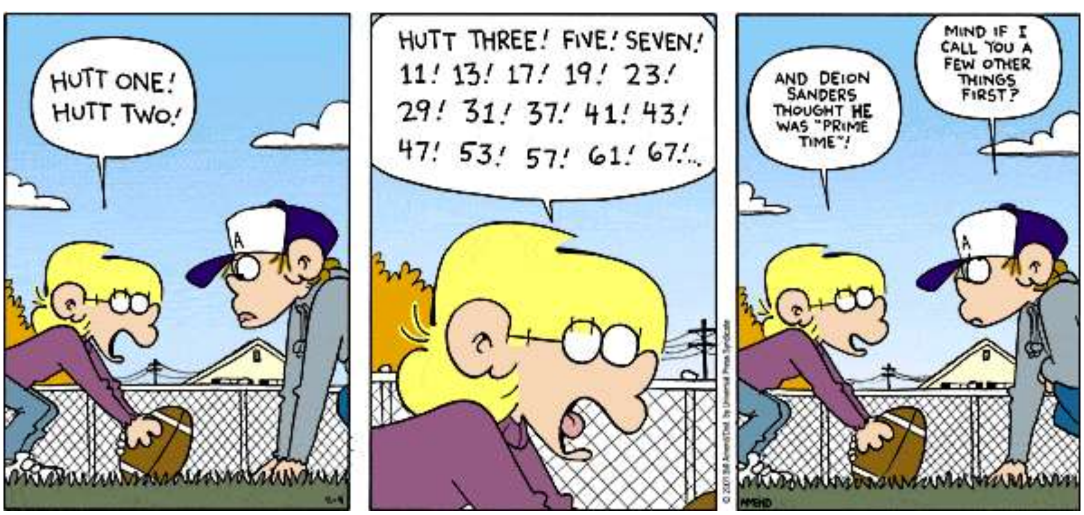
- Solve linear equations in one variable
- Use the Distributive Property
- Determine congruence of segments, angles, polygons

Algebra I:

- Represent verbal quantitative situations algebraically
- Solve literal equations
- Solve multistep linear equations
- Identify and extend arithmetic and geometric sequences

Geometry:

- Write conditional and biconditional statements
- Use inductive and deductive reasoning
- Use properties of equality to justify steps in solving equations and to find segment lengths and angle measures
- Write two-column proofs, flowchart proofs, and paragraph proofs



Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-1: Conditional Statements

SOL: G.1.a and b

Opening:

1. A ______________________ has six sides.
2. If two lines form a __________________ angle, they are perpendicular.
3. Two angles that form a right angle are ____________________________ angles.
4. A ____________________ angle has measure of 180°.

Objectives: Students will be able to:

Write conditional statements
Use definitions written as conditional statements
Write biconditional statements
Make truth tables

Vocabulary:

Biconditional statement – when a conditional statement and its converse are both true, then it is biconditional; (in symbols $p \leftrightarrow q$ read *p if and only if q*)
Conclusion – the "then" part of a conditional statement (in symbols, *q*)
Conditional statement – a logical statement that has two parts, a hypothesis and a conclusion (in symbols $p \rightarrow q$ read *p implies q)*
Contrapositive – a new conditional statement exchanging the hypothesis and conclusion and negating them
Converse – a new conditional statement exchanging the hypothesis and conclusion
Equivalent statements – statements that have the same logic values (true or false)
Hypothesis – the "if" part of a conditional statement (in symbols, *p*)
If-then form – a conditional statement in traditional form
Inverse – a new conditional statement negating the hypothesis and conclusion
Negation – the opposite of the original statement
Perpendicular lines – two line that intersect to form a right angle
Truth table – determines the conditions under which a statement is true or false
Truth value – whether a statement is true or false

Core Concepts:

Core Concept

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If *p*, then *q*. **Symbols** $p \rightarrow q$ (read as "*p* implies *q*")

Core Concept

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

Words not p **Symbols** $\sim p$

Core Concept

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \rightarrow \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

Note: Just because a conditional statement and its contrapositive are both true does not mean that its converse and inverse are both false. The converse and inverse could also both be true.

Core Concept

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

Big Ideas Chapter 2: Reasoning and Direct Proofs

Example: **If** two segments have the same measure, **then** they are congruent			
	Hypothesis	p	two segments have the same measure
	Conclusion	q	they are congruent
Statement	Formed by	Symbols	Examples
Conditional	Given hypothesis and conclusion	$p \rightarrow q$	If two segments have the same measure, then they are congruent
Converse	Co – *changing the order*	$q \rightarrow p$	If two segments are congruent, then they have the same measure
Inverse	In – *insert nots into both parts*	$\sim p \rightarrow \sim q$	If two segments do not have the same measure, then they are not congruent
Contrapositive	Cont – *change order and add nots*	$\sim q \rightarrow \sim p$	If two segments are not congruent, then they do not have the same measure

Biconditional: a biconditional statement is the conjunction of a conditional and its converse or in symbols $(p \rightarrow q) \wedge (q \rightarrow p)$ is written $(p \leftrightarrow q)$ & read p ***if and only if*** q;
All definitions are biconditional statements

Examples:

Example 0:

Use (H) to identify the hypothesis and (C) to identify the conclusion.
If we get a blizzard, then we miss school.

Example 1

Use (H) to identify the hypothesis and (C) to identify the conclusion.
Then write each conditional in if-then form.

a. $x > 5$ if $x > 3$.

b. All members of the soccer team have practice today.

Example 2

Write the negation of each statement.

a. The car is white.

b. It is not snowing.

Example 3

Let p be "you are in New York City" and let q be "you are in the United States."
Write each statement using symbols and decide whether it is true or false.

a. If you are in New York City, then you are in the United States.

b. If you are in the United States, then you are in New York City.

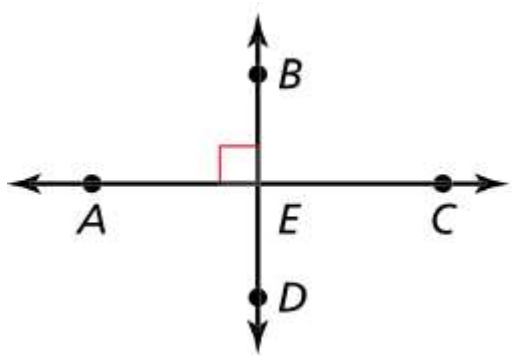
c. If you are not in New York City, then you are not in the United States.

d. If you are not in the United States, then you are not in New York City.

Example 4

Decide whether each statement about the diagram is true.

Explain your answer using the definitions you have learned.



a. $m\angle AEB = 90°$

b. Points A, C, and D are collinear.

c. $\overrightarrow{AC}$ and $\overrightarrow{CA}$ are opposite rays.

Example 5

Rewrite the definition of complementary angles as a biconditional statement.

Dfn: If two angles are complementary, then the sum of the measures of the angles is 90°.

Example 6

Make a truth table for the conditional statement $\sim(\sim p \rightarrow q)$.

Big Ideas Chapter 2: Reasoning and Direct Proofs

Concept Summary:

Conditional statements are written in *if-then* form

Form the converse, inverse and contrapositive of an if-then statement by using negations and by exchanging the hypothesis and conclusion (let the word help you with what to do)

Converse – CO change order
Inverse – IN insert nots
Contrapositive – CONT change order add nots

Truth tables

"Ands" require both parts to be true for the combined statement to be true
"Ors" require just one part to be true for the combined statement to be true
"Nots" will flip or do the opposite

Homework:
<conditional statement worksheet>
<truth-table worksheet>

Reading: student notes section 2-2

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-2: Inductive and Deductive Reasoning

SOL: G.1.b and c

Opening: Find the common difference of the arithmetic sequence. Find the next two terms.

1. 0.009, 0.15, 0.21,
2. 3.36, 1.14, -1.08,
3. 8, 3, -2,
4. 2.4, 2.9, 3.4,
5. 2, 4, 6,
6. 16, 9, 2,

Objectives:

Use inductive reasoning
Use deductive reasoning

Vocabulary:

Conjecture – an unproven statement based on observations
Counterexample – a specific case for which the conjecture is false
Deductive reasoning – uses facts, definitions, accepted properties, and the laws of logic to form a logical argument
Inductive reasoning – uses specific examples and patterns to make a conjecture for the general case

Core Concepts:

Core Concept

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Core Concept

Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

Core Concept

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

Laws of Logic

Consider statements p, q, and r.

Law of Detachment

If $p \rightarrow q$ is a true statement and statement p is true, then statement q is also true.

Law of Syllogism

$p \rightarrow q$
$q \rightarrow r$ — If these statements are true,

$p \rightarrow r$ — then this statement is true.

Law of Contrapositive

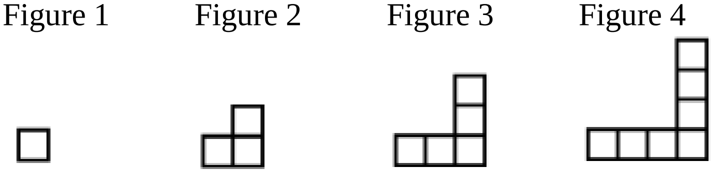
If $p \rightarrow q$ is a true statement and $\sim q$ is a true statement, then $\sim p$ is also a true statement.

Examples:

Example 1

Describe how to sketch the fifth figure in the pattern. Then sketch the fifth figure.

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5



Example 2

Make and test a conjecture about the product of a negative integer and a positive integer.

Example 3

A student makes a conjecture about absolute values.
Find a counterexample to disprove the student's conjecture.
Conjecture: The absolute value of the sum of two numbers is equal to the sum of the two numbers.

Example 4

Write each logical argument symbolically.
Then use deductive reasoning to determine whether each argument is valid.

a. If two rectangles both have side lengths of 3 inches and 4 inches, then the two rectangles are congruent. If two rectangles are congruent, then they have the same area. Therefore, if two rectangles both have side lengths of 3 inches and 4 inches, then they have the same area.

b. If two rectangles are congruent, then the two rectangles have the same area. Two rectangles do not have the same area. Therefore, the two rectangles are congruent.

c. If two rectangles both have side lengths of 3 inches and 4 inches, then the two rectangles are congruent. Two rectangles both have side lengths of 3 inches and 4 inches. Therefore, the two rectangles are congruent.

Example 5

The table shows the sum of the measures of the interior angles in various polygons.

Polygon	**Number of sides**	**Sum of interior angles**
Triangle	3	$180°$
Quadrilateral	4	$360°$
Pentagon	5	$540°$
Hexagon	6	$720°$

What conclusion can you make about the sum of the interior angles in an n-sided polygon?

Example 6

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain.

a. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3. The sum of the digits of the number 147 is 12. So the number 147 is divisible by 3.

b. Each time you forget to do your math homework, your parents take away your phone privileges for a day. So, the next time you forget to do your math homework, you will lose your phone privileges.

Venn Diagrams: (Fall 2018 test only)

Example 7: DANCING: The Venn diagram shows the number of students enrolled in Monique's Dance School for tap, jazz, and ballet classes.

a. How many students are enrolled in *all three classes*?

b. How many students are enrolled in *tap or ballet*?

c. How many students are enrolled in *jazz and ballet, but not tap*?

Concept Summary:

Venn Diagrams:

Overlaps of the circles have the key word "some"

Circle that have no overlaps have the key word "none"

Conjectures are guesses

Counter examples are examples that go against the conjectures

Inductive reasoning uses examples and looks for patterns

Deductive reasoning uses facts, theorems and postulates to prove things

Homework:

<Venn diagram worksheet>

Reading: Student notes section 2-3

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-3: Postulates and Diagrams

SOL: G.1

Opening:

1. $\angle 1$ is a supplement of $\angle 2$ and $m\angle 1 = 32°$. Find $m\angle 2$.

2. $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 155°$. Find $m\angle 4$.

3. $\angle 5$ is a complement of $\angle 6$ and $m\angle 5 = 59°$. Find $m\angle 6$.

4. $\angle 7$ is a supplement of $\angle 8$ and $m\angle 7 = 18°$. Find $m\angle 8$.

Objectives:

Identify postulates using diagrams
Sketch and interpret diagrams

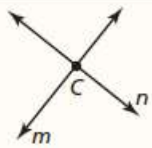
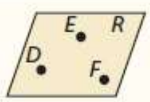
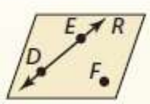
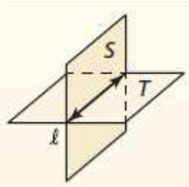
Vocabulary:

Line perpendicular to a plane – if and only if it is perpendicular to every line in the plane that intersects it

Core Concepts:

Postulates

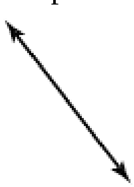
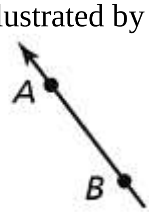
Point, Line, and Plane Postulates

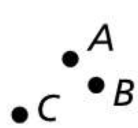
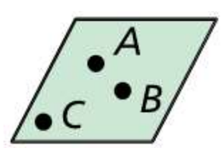
Postulate	Example	
2.1 Two Point Postulate Through any two points, there exists exactly one line. **2.2 Line-Point Postulate** A line contains at least two points.		Through points *A* and *B*, there is exactly one line ℓ. Line ℓ contains at least two points.
2.3 Line Intersection Postulate If two lines intersect, then their intersection is exactly one point.		The intersection of line *m* and line *n* is point *C*.
2.4 Three Point Postulate Through any three noncollinear points, there exists exactly one plane. **2.5 Plane-Point Postulate** A plane contains at least three noncollinear points.		Through points *D*, *E*, and *F*, there is exactly one plane, plane *R*. Plane *R* contains at least three noncollinear points.
2.6 Plane-Line Postulate If two points lie in a plane, then the line containing them lies in the plane.		Points *D* and *E* lie in plane *R*, so $\overleftrightarrow{DE}$ lies in plane *R*.
2.7 Plane Intersection Postulate If two planes intersect, then their intersection is a line.		The intersection of plane *S* and plane *T* is line ℓ.

Examples:

Example 1

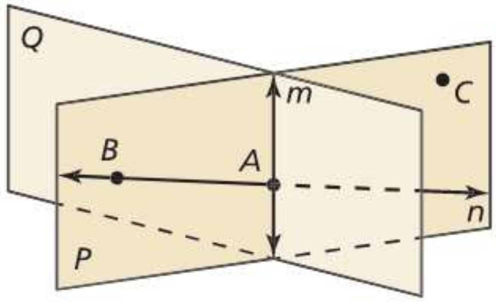
State the postulate illustrated by the diagram.

a. If  then 

b. If  then 

Example 2

Use the diagram to write an example of the Three Point Postulate.



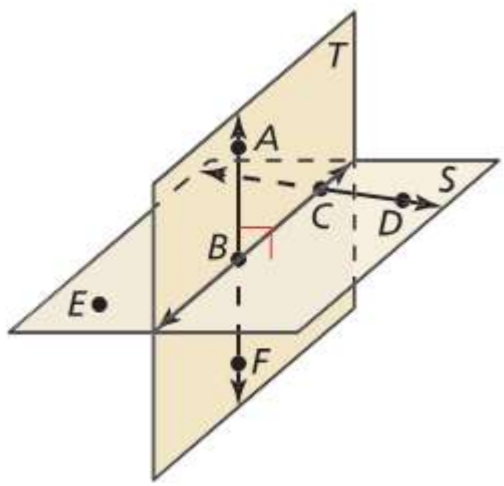
Example 3

Sketch a diagram showing $\overrightarrow{VX}$ intersecting $\overleftrightarrow{UW}$ at V so that $\overrightarrow{VX}$ is perpendicular to $\overleftrightarrow{UW}$ and U, V, and W are collinear.

Example 4

Use the diagram. Which statements *cannot* be assumed from the diagram?

- There exists a plane that contains points A, D, and E.
- $AB = BF$.



Concept Summary:

Postulates are things that we accept to be true

Theorems are things that we can prove to be true

Homework: <none>

Reading: student notes section 2-4

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-4: Algebraic Reasoning

SOL: G.1

Opening: State the mistake made in solving the equation. Rewrite the solution so it is correct.

1. $f - 23 = -17$

   $f - 23 - 23 = -17 - 23$

   $f = -40$

3. $\left(\frac{4}{7}\right)m = 22$

   $\left(\frac{7}{4}\right)\left(\frac{4}{7}\right)m = \left(\frac{4}{7}\right)22$

   $m = \frac{88}{7}$

2. $8r = 4$

   $\frac{8r}{-8} = \frac{4}{-8}$

   $r = \frac{-1}{2}$

4. $-\frac{n}{6} = 3$

   $\frac{6}{1}\left(\frac{n}{6}\right) = \frac{6}{1}(3)$

   $n = 18$

Objectives:

Use Algebraic Properties of Equality to justify the steps in solving an equation
Use the Distributive Property to justify the steps in solving an equation
Use properties of equality involving segments lengths and angle measures

Vocabulary:

Equation – two mathematical expressions connected by the "=" sign
Formula – an equation for a specific quantity
Solve an equation – to solve for the unknown variable

Core Concepts:

Core Concept

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c, c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}, c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

Core Concept

Distributive Property

Let a, b, and c be real numbers.

Sum $a(b + c) = ab + ac$ **Difference** $a(b - c) = ab - ac$

Core Concept

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Examples:

Example 1

Solve $2x - 5 = 13$. Justify each step.

Statement	Reason

Example 2

Solve $2(x + 1) = -4$. Justify each step.

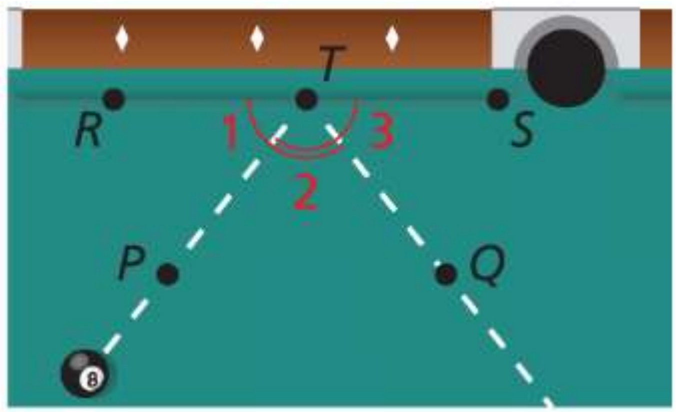
Statement	Reason

Example 3

Use the formula $p(1 + rt) = a$ to find the value a of an investment, where p is the original principal invested, r is the rate of simple interest (as a decimal), and t is the time in years the money is invested. Solve the formula for t. How many years will it take until a principal of $250 grows to a value of $285 when the simple interest rate is 2%?

Example 4

You bounce a pool ball off the wall of a pool table, as shown. Determine whether $m\angle RTP = m\angle STQ$.



Example 5

There are two exits from Theater 10 at the local cinema. The cinema manager wants to put a trash can along the wall, the same distance from each of the two exits. Create a diagram to model this problem. Show that the distance from the trash can to the left exit is half the distance between the two exits.

Concept Summary:

Algebraic Statements must be justified by properties of equality
Start with the givens; end with what you are trying to prove
Reflexive, Symmetric and Transitive (alpha order) matches number of equal signs (1, 2, 3)

Homework: <algebraic proof worksheet>

Reading: student notes section 2.5

Section 2-5: Proving Statements about Segments and Angles

SOL: G.1

Opening: Find the complement and supplement of the angle measurement.

1. 59°
2. 20°
3. 53°
4. 22.6°
5. 28°
6. 74°

Objectives:

Write two-column proofs
Name and prove properties of congruence

Vocabulary:

Axiom – or a postulate, is a statement that describes a fundamental relationship between the basic terms of geometry
Postulate – accepted as true
Proof – a logical argument in which each statement you make is supported by a statement that is accepted as true
Theorem – is a statement or conjecture that can be shown to be true
Two-column proof – has numbered statements and corresponding reasons that show an argument in a logical order

Core Concepts:

Theorems

Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs Ex. 11, p. 95; Example 3, p. 93; Chapter Review 2.5 Example, p. 109

Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs Ex. 25, p. 109; 2.5 Concept Summary, p. 94; Ex. 12, p. 95

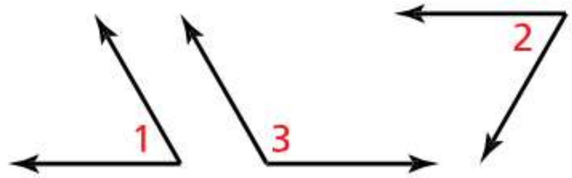
Big Ideas Chapter 2: Reasoning and Direct Proofs

Definition of Congruence: If $\overline{AB} \cong \overline{CD}$, then $AB = CD$
And since all definitions are biconditional: If $AB = CD$, then $\overline{AB} \cong \overline{CD}$

So if you need to change from a congruent to an equal or vise-versa, then you make use of the definition of Congruence.

Example:

Example 1
Write a two-column proof.
Given: $\angle 1$ is supplementary to $\angle 3$.
$\angle 2$ is supplementary to $\angle 3$
Prove: $\angle 1 \cong \angle 2$



Statements	Reasons

Example 2
Name the property that the statement illustrates.
a. $\angle A \cong \angle A$

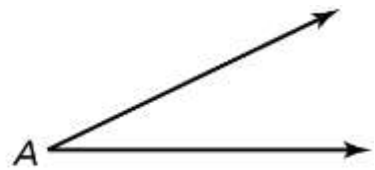
b. If $\overline{PQ} \cong \overline{JG}$ and $\overline{JG} \cong \overline{XY}$, then $\overline{PQ} \cong \overline{XY}$

Example 3

Write a two-column proof for the Reflexive Property of Angle Congruence.

Given: $\angle A$

Prove: $\angle A \cong \angle A$



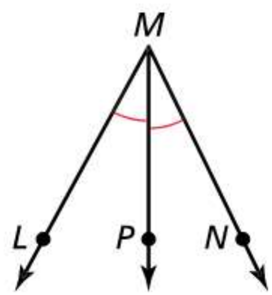
Statements	Reasons

Example 4

Write a two-column proof.

Given: $\overrightarrow{MP}$ bisects $\angle LMN$.

Prove: $2(m\angle LMP) = m\angle LMN$



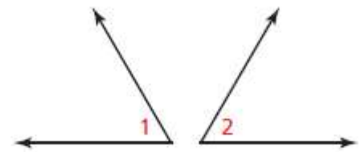
Statements	Reasons

Concept Summary:

Concept Summary

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 = m\angle 2$	**2.** Definition of congruent angles
3. $m\angle 2 = m\angle 1$	**3.** Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	**4.** Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

Homework:
<Geometric Proof worksheet>

Reading: student notes section 2.6

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-6: Proving Geometric Relationships

"Do I get partial credit for simply having the courage to get out of bed and face the world again today?"

SOL: G.1

Opening: Solve

1. 9x + 6 = 10x – 3

2. 6y = 5y + 35

3. 9x + 5 = 5(x – 3)

4. 17 y + 18 = 15y

5. 14x – 44 = 20x – 2

6. 7x – 1 = 13x + 41

Objective:

Write flow-chart proofs to prove geometric relationships
Write paragraph proofs to prove geometric relationships

Vocabulary:

Flow or flowchart proof – uses boxes and arrows to show the flow of a logical argument
Paragraph Proof – statements and reasons of a proof are presented as sentences in a paragraph

Core Concepts:

Theorem

Theorem 2.3 Right Angles Congruence Theorem

All right angles are congruent.

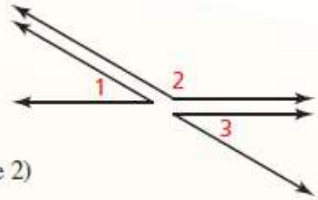
Proof Example 1, p. 98

Theorems

Theorem 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

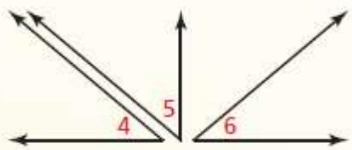


Proof Example 2, p. 99 (case 1); Ex. 20, p. 105 (case 2)

Theorem 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.



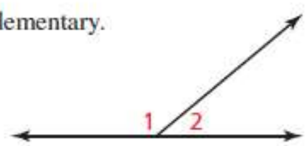
Proof Ex. 19, p. 104 (case 1); Ex. 22, p. 105 (case 2)

Postulate and Theorem

Postulate 2.8 Linear Pair Postulate

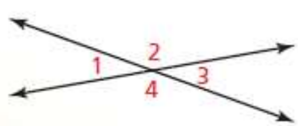
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



Theorem 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



Proof Example 3, p. 100

$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

Big Ideas Chapter 2: Reasoning and Direct Proofs

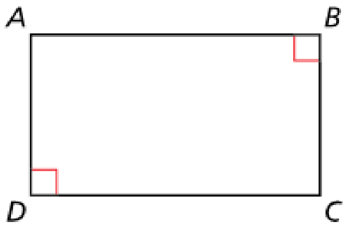
Examples:

Example 1

Use the given flowchart proof to write a two-column proof.

Given: $\overline{AB} \perp \overline{BC}, \overline{AD} \perp \overline{DC}$

Prove: $\angle B \cong \angle D$



$\overline{AB} \perp \overline{BC}, \overline{AD} \perp \overline{DC}$

Given

↓

$\angle B$ and $\angle D$ are right angles

Definition of ⊥ lines

↓

$\angle B \cong \angle D$

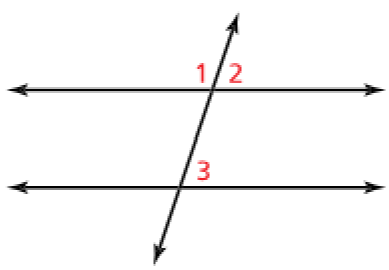
All right angles are congruent.

Example 2

Write a flowchart proof.

Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 1$ and $\angle 3$ are supplementary.

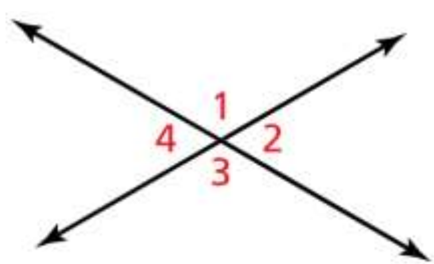
Prove: $\angle 2 \cong \angle 3$



Example 3

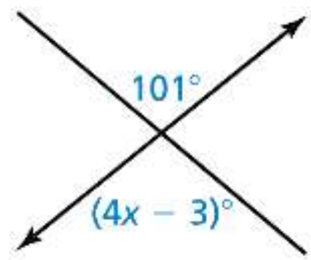
Use the diagram and the given angle measure to find the other three angle measures.

$m\angle 3 = 128°$



Example 4

Find the value of x.

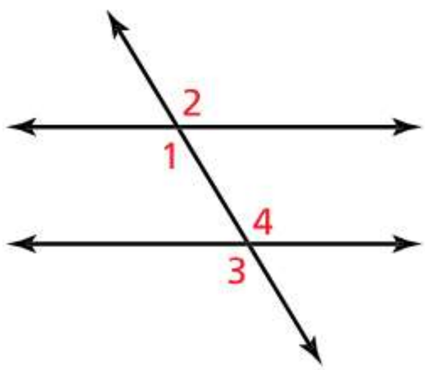


Example 5

Write a paragraph proof.

Given: $\angle 1 \cong \angle 4$

Proof: $\angle 2 \cong \angle 3$



Concept Summary:

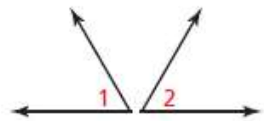
Concept Summary

Types of Proofs

Symmetric Property of Angle Congruence (Theorem 2.2)

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Flowchart Proof

$\angle 1 \cong \angle 2$ → $m\angle 1 = m\angle 2$ → $m\angle 2 = m\angle 1$ → $\angle 2 \cong \angle 1$

Given → Definition of congruent angles → Symmetric Property of Equality → Definition of congruent angles

Paragraph Proof

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

Homework:
<Geometric Proof worksheet>

Reading: student notes chapter review section

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 2-R: Chapter Review

SOL: G-1

Objectives:

Review Key Concepts of the chapter

Key Concepts:

- Conditional Statements:
  - An if-then statement is written in the form if *p* then *q*; where *p* is the hypothesis and *q* is the conclusion

Statement	Symbolically	Memory Jogger
○ Conditional	$p \to q$	If … , then …
○ Converse	$q \to p$	Co – change order
○ Inverse	$\sim p \to \sim q$	In – insert nots
○ Contrapositive	$\sim q \to \sim p$	Cont – change order insert nots

- Reasoning:
  - Inductive Reasoning
    - Inductive Reasoning: a conjecture is reached based on observations or patterns
    - Counterexample: an example that proves a conjecture is false
  - Deductive Reasoning:
    - Deductive Reasoning: a conclusion is reached using laws and theorems
    - Law of Detachment: If $p \to q$ is true and *p* is true, then *q* is also true
    - Law of Syllogism: If $p \to q$ and $q \to r$ are true, then $p \to r$ is also true
    - Law of Contrapositive: If $p \to q$ is true and $\sim q$ is true, then $\sim p$ is also true

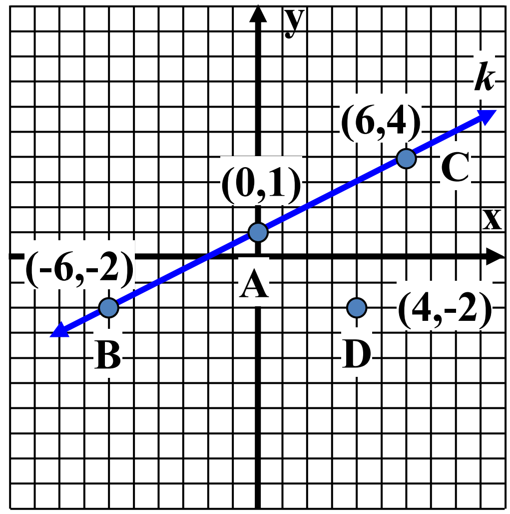
- Symbols
  - Not: negation of a statement (truth hint word opposite) (symbolically: ~)
  - And: joins two statements (truth hint word both) (symbolically: $\wedge$)
  - Or: joins two statements (truth hint word either) (symbolically: $\vee$)
  - Therefore: a wrapping-up word (symbolically: $\therefore$)
  - If …, then: conditional statement (symbolically: $\to$)
  - If and only if: biconditional statement (symbolically: $\leftrightarrow$)

- Proofs:
  - Step 1: List the given information and draw a diagram if possible
  - Step 2: State what is to be proven
  - Step 3: Create a deductive argument
  - Step 4: Justify each statement with a reason
  - Step 5: State what you have proven

Big Ideas Chapter 2: Reasoning and Direct Proofs

5 Minute Reviews

Chapter 1:



1. Identify the line
2. Find the distance between A and C
3. Name three collinear points
4. Find the midpoint between C and D
5. If A is a midpoint and C is the endpoint, find the other endpoint
6. Name an obtuse angle with a vertex of D

Section 1:

Given the conditional statement: "If It's Tuesday, This Must Be Belgium"

1. Find the *hypothesis* and *conclusion*
2. Find the *converse* in words and symbols
3. Find the *inverse* in words and symbols
4. Find the *contrapositive* in words and symbols
5. If p is true and q is false, find the truth value of $\sim p \vee q$
6. If p is true and q is false, find the truth value of $p \wedge \sim q$

Big Ideas Chapter 2: Reasoning and Direct Proofs

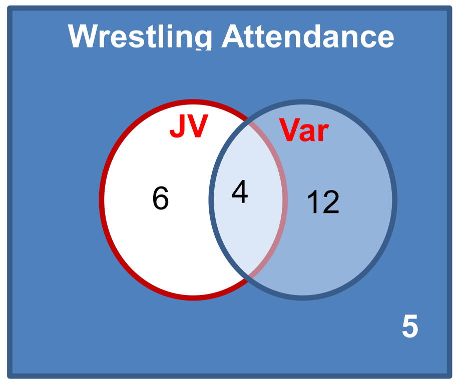
Section 2:

1. What is the type of reasoning that uses examples?

2. What is the type of reasoning that uses facts and theorems?

3. What are the three laws of logic from last section?

Use the Venn diagram to answer the following:



4. How many students were at the JV match?

5. How many students were at the varsity match?

6. How many were at both?

7. How many were at neither?

Section 3:

1. What do two lines intersect in?

2. How many points does it take to define a line?

3. How many points does it take to define a plane?

4. What has to be special about the points that define a plane?

5. What do two planes intersect in?

6. What is accepted as fact in Geometry?

Big Ideas Chapter 2: Reasoning and Direct Proofs

Section 4:

1. What algebraic steps are followed by a “simplify” step?

2. Match the following properties (to the equal signs involved):

Reflexive	A = B, B = C, A = C
Symmetric	A = B, B = A
Transitive	A = A

3. First step in a two-column proof:

4. Last step in a proof:

Section 5:

1. Why can we go from a ≅ to an = (or vice-versa)?

2. What are the first things usually listed in a proof?

3. What are used as reasons in proofs?

4. Match the following:

Complement	Equal
Congruent	Adds to 90
Supplement	Adds to 180

Section 6:

1. Which type of proof is used on SOLs?

2. What is the reason for $m\angle ABC + m\angle DBC = m\angle ABD$?

3. What could be the reason for $m\angle ABC = m\angle DBC$?

4. Match the following:

Linear Pairs	Equal
Right Angles	Is 90
Vertical Angles	Adds to 180

Logic Symbols

$\wedge$ --- and	$\vee$ --- or	$\therefore$ --- therefore
~ --- not	$\rightarrow$ --- if ..., then	$\leftrightarrow$ --- if and only if

English to Symbols

Let P = We have a test and let Q = It snows tomorrow
R = we will do makeup work and T = finish our test

If it snows tomorrow, then we don't have a test.

Q → ~P

If we don't have a test, then we will do makeup work or finish our test.

~P → R $\vee$ T

If it snows tomorrow, then we don't have a test and we will not do makeup work.

Q → ~P $\wedge$ ~ R

More proof stuff

	Equality	Congruence
Reflexive Property	x = x	$\overline{AD} \cong \overline{AD}$
Symmetric Property	7 = x so x = 7	$\overline{AD} \cong \overline{JK}$ so $\overline{JK} \cong \overline{AD}$
Transitive Property	a = b, b =c so a = c	$\overline{AD} \cong \overline{BC}$, $\overline{BC} \cong \overline{XY}$ so $\overline{AD} \cong \overline{XY}$

Alphabetical order (RST) increase in number of = or $\cong$ signs (1,2,3)

Logic Laws (help us draw correct conclusions)

Detachment: (like school rules)

$A \rightarrow B$ is true. A is true, therefore B is true

If you have more than 3 tardies, you get MIP. Jon has 3 tardies; so Jon will get MIP.

Syllogism: (like transitive property of equality – removes the middle man)

$A \rightarrow B$ and $B \rightarrow C$, so $A \rightarrow C$

If you have more than 10 absences, you have to take the final. If you have to take the final, then you don't get out early. So, if you have more than 10 absence, you don't get out early.

Special If … , then … statements

Converse: converse --- change order (flips)

If it snows, then we get out early.

If we get out early, then it snows.

Inverse: inverse --- insert nots (negate: ~ --- not)

If it snows, then we get out early.

If it does not snow, then we do not get out early.

Contrapositive: contrapositive --- change order and nots (both)

If it snows, then we get out early.

If we did not get out early, then it did not snow. 25

Symbols:

Symbol	∧	∨	~	∴	→	↔
Logical	And	Or	Not	Therefore	If ..., then	If and only if
Meaning	Both	Either	Opposite	conclusion	conditional	biconditional

Logic Laws:

Law of syllogism: p→q, q→r, so p→r

example:

If it snows, we miss school. If we miss school, you sleep in. If it snows, you sleep in.

Law of detachment: p→q is true statement; so if p is true, then q must be

example:

If you have 4 tardies, you get ISS. Tym has 4 tardies. Tym will get ISS

Statements:

Statement	Symbols	Hint	Example below
Conditional	P → Q	Hypothesis, Conclusion	
If it snows, we get out of school.			
Converse	Q → P	Flip	
If we get out of school, it snows.			
Inverse	~P → ~Q	Negate	
If it didn't snow, we don't get out of school.			
Contrapositive	~Q → ~P	Both	
If we don't get out of school, then it didn't snow			

Test Taking Tips:

Stop and think – don't hurry through;

Does the sentence make any sense

Do conclusions fit all your other knowledge (especially in geometry)