Chapter 5: Congruent Triangles

Addressed or Prepped VA SOL:

G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
   a) ordering the sides by length, given angle measures;
   b) ordering the angles by degree measure, given side lengths;
   c) determining whether a triangle exists; and
   d) determining the range in which the length of the third side must lie.

G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent.

SOL Progression

Middle School:
- Understand that translations and reflections maintain congruence between the pre-image and the image, but change location
- Draw triangles with given conditions
- Find the distance between points with the same x- or y-coordinate

Algebra I:
- Write equations and use them to solve problems
- Solve multi-step linear equations
- Graph in the coordinate plane
- Simplify numerical expressions containing square or cube roots

Geometry:
- Identify and use corresponding parts
- Use theorems about the angles of a triangle
- Use SAS, SSS, HL, ASA, and AAS to prove two triangles congruent
- Prove constructions
- Write coordinate proofs
Chapter 5: Congruent Triangles

Section 5-1: Angles of Triangles

SOL: G.4 and G.5

Objectives:
- Classify triangles by sides and angles
- Find interior and exterior angles of triangles

Vocabulary:
- Corollary to a Theorem – a statement that can be proved easily using the theorem
- Equilateral – all sides of a triangle are equal; equilateral ↔ equiangular
- Equiangular – all angles of a triangle are equal; equiangular ↔ equilateral
- Exterior angles – angles formed outside the triangle (or polygon) by extending one side
- Interior angles – angles inside the triangle (or polygon)
- Isosceles – two sides of a triangle are equal
- Scalene – no sides of a triangle are equal; all sides have different lengths

Core Concept:

Note: all triangles have at least 2 acute angles!!

Note:
- The 3 interior angles of a triangle add to 180°.
- The 3 exterior angles of a triangle add to 360°.
- Interior and Exterior angles form a linear pair.
Chapter 5: Congruent Triangles

Theorem

**Theorem 5.1 Triangle Sum Theorem**
The sum of the measures of the interior angles of a triangle is 180°.

\[ m\angle A + m\angle B + m\angle C = 180° \]

*Proof* p. 210; Ex. 53, p. 214

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Theorem

**Theorem 5.2 Exterior Angle Theorem**
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

\[ m\angle 1 = m\angle A + m\angle B \]

*Proof* Ex. 42, p. 213

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**Corollary**

**Corollary 5.1 Corollary to the Triangle Sum Theorem**
The acute angles of a right triangle are complementary.

\[ m\angle A + m\angle B = 90° \]

*Proof* Ex. 41, p. 213

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**Examples:**

**Example 1:**

Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.
Example 2:
Classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle.

Example 3:
Find $m\angle PQS$

Example 4:
The measure of one acute angle of a right triangle is 1.5 times the measure of the other acute angle. Find the measure of each acute angle.

Concept Summary:
– Triangles can be classified by their angles as acute, obtuse or right
– Triangles can be classified by their sides as scalene, isosceles or equilateral
– Exterior angle = sum of remote interiors
– Interior angles sum to 180
– Exterior angles sum to 360

Khan Academy Videos:
1. Angles in a triangle sum to 180° proof

Homework: Triangle Classification WS

Reading Assignment: Section 5-2
Chapter 5: Congruent Triangles

Section 5-2: Congruent Polygons

SOL: G.5

Objectives:
- Identify and use corresponding parts
- Use the Third Angles Theorem

Vocabulary:
- Corresponding parts – corresponding parts map onto each other from a rigid motion mapping or from a statement of congruence or similarity by order

Core Concept:

Order Rules!!! – When matching congruent statements: \( \triangle DEF \) is the image of \( \triangle ABC \) or \( \triangle DEF \cong \triangle ABC \), order of appearance tells you which parts are corresponding.

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Corresponding sides</th>
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<tr>
<td>( \angle A \cong \angle D ), ( \angle B \cong \angle E ), ( \angle C \cong \angle F )</td>
<td>( AB \cong DE ), ( BC \cong EF ), ( AC \cong DF )</td>
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Note:
1. If two triangles are congruent, then all their corresponding parts are congruent.
2. If all the corresponding parts of two triangles are congruent, then the triangles are congruent.

Theorem

Theorem 5.3  Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

- Reflexive: For any triangle \( \triangle ABC \), \( \triangle ABC \cong \triangle ABC \).
- Symmetric: If \( \triangle ABC \cong \triangle DEF \), then \( \triangle DEF \cong \triangle ABC \).
- Transitive: If \( \triangle ABC \cong \triangle DEF \) and \( \triangle DEF \cong \triangle JKL \), then \( \triangle ABC \cong \triangle JKL \).

Proof: BigIdeasMath.com

Note: Since all three angles in any triangle always add to 180, the next theorem is really another corollary to the Angle Sum Theorem from lesson 5-1.
Chapter 5: Congruent Triangles

**Theorem**

*Theorem 5.4 Third Angles Theorem*
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

*Proof* Ex. 19, p. 220

![Diagram of triangles with angles and sides labeled](image)

**Examples:**

**Example 1:**
Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts (sides and angles)

**Example 2:**
In the diagram, \(\triangle DEFG \cong \triangle QMNP\)

a. Find the value of \(x\).

b. Find the value of \(y\).

**Example 3:**
Show that \(\triangle ABD \cong \triangle CDB\). Explain your reasoning

![Diagram of triangles with angles and sides labeled](image)
Chapter 5: Congruent Triangles

Example 4:
Find $m\angle P$.

Example 5:
Use the information in the figure to prove that $\triangle WXY \cong \triangle ZVY$.

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Concept Summary:
- Two triangles are congruent when all of their corresponding parts are congruent.
- CPCTC – corresponding parts of congruent triangles are congruent
- Order is important!
- Shared sides are Reflexive
- “Bowties” have Vertical Angles

Khan Academy Videos: none relate

Homework: Triangle Congruence WS 1

Reading Assignment: section 5.3
Chapter 5: Congruent Triangles

Section 5.3: Proving Triangle Congruence by SAS

SOL: G.5

Objectives:
- Use the Side-Angle-Side (SAS) Congruence Theorem
- Solve real-life problems

Vocabulary:
- Included angle – the angle formed by the two sides (angle between the two connected sides)

Core Concept:

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

Proof p. 222

Examples:

Example 1:
Write a proof.

Given: B is the midpoint of $\overline{AD}$. $\angle ABC$ and $\angle DBC$ are right angles
Prove: $\triangle ABC \cong \triangle DBC$

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Chapter 5: Congruent Triangles

Example 2:

What can you conclude about $\triangle PTS$ and $\triangle RTQ$? Explain.

Example 3:

The wings of a paper airplane have two congruent sides and two congruent angles. Use the SAS Congruence Theorem to show that $\triangle IPA \cong \triangle IPR$.

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Concept Summary:
- SAS is the first of several “short-cut” theorems for triangle congruence
- Angle must be between the two sides – “included”
  - Angle will be the letter the two sides have in common
  - Must be the same in both triangles

Homework: Triangle Congruence WS 2

Reading Assignment: read section 5.4
Chapter 5: Congruent Triangles

Section 5.4: Equilateral and Isosceles Triangles

SOL: G.5

Objectives:
Use the Base Angles Theorem
Use isosceles and equilateral triangles

Vocabulary:
Base – the non-equal side of an isosceles triangle
Base angles – angles adjacent to the base
Legs – the two equal sides of an isosceles triangle
Vertex angle – angle formed by the legs of an isosceles triangle

Core Concept:

Theorems

Theorem 5.6 Base Angles Theorem
If two sides of a triangle are congruent, then the angles opposite them are congruent.
If \( AB = AC \), then \( \angle B = \angle C \).
Proof p. 228

Theorem 5.7 Converse of the Base Angles Theorem
If two angles of a triangle are congruent, then the sides opposite them are congruent.
If \( \angle B = \angle C \), then \( AB = AC \).
Proof Ex. 27, p. 249

Corollaries

Corollary 5.2 Corollary to the Base Angles Theorem
If a triangle is equilateral, then it is equiangular.
Proof Ex. 37, p. 234

Corollary 5.3 Corollary to the Converse of the Base Angles Theorem
If a triangle is equiangular, then it is equilateral.
Proof Ex. 39, p. 234

Note: Angles opposite equal sides are equal and sides opposite equal angles are equal!!
Chapter 5: Congruent Triangles

Examples:

Example 1:
In the figure, \( \angle A \cong \angle B \). Name two congruent sides.

Example 2:
Find the value of \( x \).

Example 3:
Find the values of \( x \) and \( y \).

Example 4:
In the diagram, \( \overline{PT} \cong \overline{ST} \) and \( \overline{PQ} \cong \overline{SR} \).

a. Prove \( \triangle PQT \cong \triangle SRT \)

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b. Explain why $\Delta QRT$ is isosceles.

**Khan Academy Videos:**
1. Isosceles and equilateral triangles problems
2. Find angles in isosceles triangles
3. Proofs concerning isosceles triangles
4. Proofs concerning equilateral triangles

**Concept Summary:**
- Two sides of a triangle are congruent if, and only if, the angles opposite those sides are congruent
- A triangle is equilateral if, and only if, it is equiangular

**Homework:** Isosceles Triangle WS 1

**Reading Assignment:** read section 5.5
Chapter 5: Congruent Triangles

Section 5-5: Proving Triangle Congruence by SSS

SOL: G.5

Objectives:
Use the Side-Side-Side (SSS) Congruence Theorem
Use the Hypotenuse-Leg (HL) Congruence Theorem

Vocabulary:
Hypotenuse – side opposite of the right angle in a right triangle
Legs – sides that form the right angle in a right triangle
Stable – angles in the figure are fixed cannot change

Core Concept:

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $AB \cong DE$, $BC \cong EF$, and $AC \cong DF$, then $\Delta ABC \cong \Delta DEF$.

Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90^\circ$, then $\Delta ABC \cong \Delta DEF$.

Proof Ex. 38, p. 430; BigIdeasMath.com

Note: Because of the Pythagorean Theorem, HL is really a special case of SSS.
### Chapter 5: Congruent Triangles

**Examples:**

**Example 1:**

Write a proof

**Given:** $\overline{PQ} \cong \overline{RQ}, \overline{PS} \cong \overline{RS}$

**Prove:** $\Delta PQT \cong \Delta SRT$

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<th>Statement</th>
<th>Reason</th>
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<tbody>
<tr>
<td>$\overline{PQ} \cong \overline{RQ}$</td>
<td>Given</td>
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<td>$\overline{PS} \cong \overline{RS}$</td>
<td>Given</td>
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<td>$\angle Q \cong \angle S$</td>
<td>Right angles</td>
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**Example 2:**

Determine whether the hexagon is stable. Explain your reasoning.

**Example 3:**

Write a proof.

**Given:** $\overline{PQ} \cong \overline{RS}$, $\angle Q \cong \angle S$ are right angles.

**Prove:** $\Delta PQR \cong \Delta RSP$

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<td>$\overline{PQ} \cong \overline{RS}$</td>
<td>Given</td>
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<td>$\angle Q \cong \angle S$</td>
<td>Right angles</td>
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<tr>
<td>$\angle PQR \cong \angle RSP$</td>
<td>Given</td>
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<tr>
<td>$\overline{RP} \cong \overline{PR}$</td>
<td>Reflexive property</td>
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Example 4:

Two windows in a building are right triangles, with \( WX \cong XY \).
Prove that the triangles are congruent.

Given: \( \triangle WXYZ \) and \( \triangle YXZ \) are right triangles; \( WX \cong XY \)
Prove: \( \triangle WXYZ \cong \triangle YXZ \)

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Concept Summary:

- SSS is the second of several “short-cut” theorems for triangle congruence
- HL is a special case of SSS
  - Only works in right triangles

Khan Academy Videos:
1. Congruent triangles and the SSS postulate/criterion
2. Triangle congruence postulates/criteria
3. Why SSA isn’t a congruence postulate/criterion
4. Determining congruent triangles

Homework: Triangle Isosceles WS 2

Reading Assignment: read section 5.6
Section 5-6: Proving Triangle Congruence by ASA and AAS

SOL: G.5

Objectives:
Use the ASA and AAS Congruence Theorems

Vocabulary:
*Included side – the side in common between two angles (the end points are the vertexes)*

Core Concepts:

**Theorem 5.10** Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If \( \angle A = \angle D, \overline{AC} = \overline{DF}, \) and \( \angle C = \angle F, \)
then \( \triangle ABC \cong \triangle DEF. \)

*Proof* p. 244

**Theorem 5.11** Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D, \angle C \cong \angle F, \) and \( \overline{BC} = \overline{EF}, \) then \( \triangle ABC \cong \triangle DEF. \)

*Proof* p. 245

Note: AAS is listed as a corollary to ASA in some books because of the third angle theorem.
Chapter 5: Congruent Triangles

Examples:

Example 1:

Can the triangles be proven congruent with the information given in the diagram? If so, state the theorem you would use.
   a. \( \triangle EFG \cong \triangle HDG \)

   b. \( \triangle PQM \cong \triangle RQM \)

   c. \( \triangle LMP \cong \triangle NMP \)

Example 2:

Write a proof.

Given: \( DH \parallel FG, DE \cong EG \)
Prove: \( \triangle LMP \cong \triangle NMP \)

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Example 3:

Write a proof.

Given: \( \overline{AD} \parallel \overline{EC} \), \( \overline{BD} \cong \overline{BC} \)

Prove: \( \triangle ABD \cong \triangle EBC \) using AAS Congruence Theorem

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<tr>
<td>Given: ( \overline{AD} \parallel \overline{EC} ) ( \overline{BD} \cong \overline{BC} )</td>
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<tr>
<td>Prove: ( \triangle ABD \cong \triangle EBC ) using AAS Congruence Theorem</td>
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Concept Summary:

SSS, SAS, HL, ASA, AAS are the triangle congruence theorems

AAA, SSA or ASS are not possible

Khan Academy Videos: all related list in previous section

Homework: Triangle Congruence WS 3

Reading Assignment: read section 5.7
Section 5-7: Using Congruent Triangles

SOL: G.5

Objectives:
Use congruent triangles
Prove constructions

Vocabulary: No new vocabulary words or symbols.

Examples:

Example 1:
Write a proof.
Given: \( \angle QRT \cong \angle SRT \), \( \angle RTQ \cong \angle RTS \)
Prove: \( QT \cong ST \)

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Example 2:
Explain how to use the measurements in the diagram to find the distance across the pond.
**Chapter 5: Congruent Triangles**

**Example 3:**

Write a plan to prove $\triangle ADE \cong \triangle ABE$

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**Example 4:**

Write a proof to verify that the construction of the midpoint of a segment is valid.

Given: $\angle QRT \cong \angle SRT$, $\angle RTQ \cong \angle RTS$

Prove: $\overline{QT} \cong \overline{ST}$

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**Concept Summary:**

- Remember to look for hidden features
- Isosceles triangles make up many constructions

**Khan Academy Videos:** none related

**Homework:** Triangle Congruence WS 4

**Reading Assignment:** read section 5.8
Chapter 5: Congruent Triangles

Section 5-8: Coordinate Proofs

SOL: G.5

Objectives:
- Place figures in a coordinate plane
- Write coordinate proofs

Vocabulary:
*Coordinate proof* -- involves placing geometric figures in a coordinate plane.

Examples:

*Example 1:*
Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. Scalene right triangle

b. Isosceles trapezoid
Chapter 5: Congruent Triangles

Example 2:
Write a plan for a proof that used the SAS Congruence Theorem.

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Example 3:
Place an isosceles triangle on a coordinate plane with vertices $P(-2a, 0)$, $Q(0, a)$, and $R(2a, 0)$. Find the length of each side.

Example 4:
Write a coordinate proof. Prove that $\angle TOU \cong \angle VUO$.

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Example 5:

As part of a graphic design, you draw a rectangle and then connect the midpoints of the sides. Prove that the quadrilateral $MNPQ$ has four congruent sides.

![Diagram of rectangle and quadrilateral MNPQ with coordinates]

Concept Summary:

- Coordinate plane proofs use HL, SAS and SSS with distance formulas

Khan Academy Videos: semi-related
  1. Geometry proof problem: midpoint
  2. Geometry proof problem: congruent segments

Homework: none

Reading Assignment: read chapter 5 review
Chapter 5: Congruent Triangles

Section 5-R: Chapter Review

SOL: G.5

Objectives:
Review Chapter 5 knowledge

Vocabulary: none new.

Concept Summary:

Triangle Congruence has short-cut theorems based on sides (S) and angles (A):

- Sides are marked with congruent marks in the middle
- Angles are marked with congruent marks in the corners (vertices)

- SSS, SAS, ASA, AAS, and Hyp-Leg (HL)

Many triangles have hidden features:
- Vertical angles (in triangles that look like bow-ties)
- Alternate Interior Angles (in triangles with parallel sides marked – with triangular shapes)
- Shared sides (the most common)
- Shared angles (the rarest)

Opposite equal sides in triangles are equal angles
Opposite equal angles in triangles are equal sides

Since all angles in a triangle add to 180, an exterior angle = remote angle 1 + remote angle 2

Homework: Quiz Review Worksheet
**Chapter 5: Congruent Triangles**

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<tr>
<th>Known Parts</th>
<th>Definition</th>
<th>(\Delta \cong \Delta)?</th>
<th>Reason</th>
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<tbody>
<tr>
<td>SSS</td>
<td>You know that all three sides of one triangle are equal to the corresponding three sides of another triangle</td>
<td>Yes</td>
<td>SSS Theorem</td>
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<tr>
<td>SAS</td>
<td>You know that two sides and the angle between those two sides are equal to the corresponding two sides and included angle of another triangle</td>
<td>Yes</td>
<td>SAS Theorem</td>
</tr>
<tr>
<td>ASA</td>
<td>You know that two angles and the side included between those two angles are equal to the corresponding two angles and included side of another triangle</td>
<td>Yes</td>
<td>ASA Theorem</td>
</tr>
<tr>
<td>AAS</td>
<td>You know that two angles and one side not included between those angles are equal to the corresponding two angles and non-included side of another triangle</td>
<td>Yes</td>
<td>AAS Theorem</td>
</tr>
<tr>
<td>SAA</td>
<td>You know that one side and two angles which do not include that side are equal to the corresponding side and non-included angles of another triangle</td>
<td>Yes</td>
<td>Can get it into form of ASA, since other angle must also be congruent (sum of triangle’s angles = 180)</td>
</tr>
<tr>
<td>HL</td>
<td>You know the hypotenuse and a leg of one right triangle are congruent to the same in another right triangle</td>
<td>Yes</td>
<td>HL Theorem</td>
</tr>
<tr>
<td>ASS Or SSA</td>
<td>You know that one angle and two sides which do not include that angle are equal to the corresponding angle and two non-included sides of another triangle</td>
<td>No</td>
<td>Might be congruent, but cannot prove with given information</td>
</tr>
<tr>
<td>AAA</td>
<td>You know that all three angles of one triangle are equal to the corresponding three angles of another triangle</td>
<td>No</td>
<td>Sides are proportional (think concentric triangles)</td>
</tr>
</tbody>
</table>

**Hidden Features to look for to find the missing letter:**
- Shared Sides (reflexive property of congruence)
- Alternate Interior Angles (from parallel sides – sides with triangle shapes on them)
- Shared Angles (rare, but has been on the SOL)
- Vertical Angles (“bowties” shaped figure)
Chapter 5: Congruent Triangles

Constructions:

**CONSTRUCTION** Copying a Triangle Using SAS

Construct a triangle that is congruent to $\triangle ABC$ using the SAS Congruence Theorem. Use a compass and straightedge.

**SOLUTION**

**Step 1**
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

**Step 2**
Construct an angle
Construct $\angle D$ with vertex $D$ and side $\overline{DE}$ so that it is congruent to $\angle A$.

**Step 3**
Construct a side
Construct $\overline{DF}$ so that it is congruent to $\overline{AC}$.

**Step 4**
Draw a triangle
Draw $\triangle DEF$. By the SAS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

**CONSTRUCTION** Constructing an Equilateral Triangle

Construct an equilateral triangle that has side lengths congruent to $\overline{AB}$. Use a compass and straightedge.

**SOLUTION**

**Step 1**
Copy a segment
Copy $\overline{AB}$.

**Step 2**
Draw an arc
Draw an arc with center $A$ and radius $\overline{AB}$.

**Step 3**
Draw an arc
Draw an arc with center $B$ and radius $\overline{AB}$. Label the intersection of the arcs from Steps 2 and 3 as $C$.

**Step 4**
Draw a triangle
Draw $\triangle ABC$. Because $\overline{AB}$ and $\overline{AC}$ are radii of the same circle, $\overline{AB} = \overline{AC}$. Because $\overline{AB}$ and $\overline{BC}$ are radii of the same circle, $\overline{AB} = \overline{BC}$. By the Transitive Property of Congruence (Theorem 2.1), $\overline{AC} = \overline{BC}$. So, $\triangle ABC$ is equilateral.
Chapter 5: Congruent Triangles

**CONSTRUCTION**

**Copying a Triangle Using SSS**

Construct a triangle that is congruent to \(\triangle ABC\) using the SSS Congruence Theorem. Use a compass and straightedge.

**SOLUTION**

**Step 1**

**Construct a side**

Construct \(DE\) so that it is congruent to \(AB\).

**Step 2**

**Draw an arc**

Draw an arc with radius \(BC\) and center \(E\) that intersects the arc from Step 1. Label the intersection point \(F\).

**Step 3**

**Draw an arc**

Draw an arc with radius \(AC\) and center \(D\). Use this length to draw an arc with center \(D\).

**Step 4**

**Draw a triangle**

Draw \(\triangle DEF\). By the SSS Congruence Theorem, \(\triangle ABC \cong \triangle DEF\).

---

Classify Triangles by Angles: (by largest angle only)

- **Acute**: All 3 angles acute
- **Equiangular**: All angles are equal (60°)
- **Right**: Right angle in triangle
- **Obtuse**: One angle obtuse in triangle
Chapter 5: Congruent Triangles

Classify Triangles by Sides: (how many sides equal)

- **Scalene:** no sides equal
- **Isosceles:** 2 sides equal
- **Equilateral:** 3 sides equal

Equilateral means Equiangular and vice versa

Exterior Angle in Triangles

Exterior angle = Remote angle + Other Remote angle

Triangle Congruence

If proving triangles congruent, always mark *only one triangle* up with S (for sides) and A (for angles) based on the items you have congruent
### Chapter 5: Congruent Triangles

<table>
<thead>
<tr>
<th>Post/Thrm</th>
<th>Picture</th>
<th>Hidden Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSS</td>
<td><img src="image" alt="SSS Diagram" /></td>
<td>Shared Side</td>
</tr>
<tr>
<td>SAS</td>
<td><img src="image" alt="SAS Diagram" /></td>
<td>Shared Side, Vertical Angles, Parallel Lines $\Rightarrow$ Alternate Interior Angles</td>
</tr>
<tr>
<td>ASA</td>
<td><img src="image" alt="ASA Diagram" /></td>
<td>Shared Side, Vertical Angles, Parallel Lines $\Rightarrow$ Alternate Interior Angles</td>
</tr>
<tr>
<td>AAS</td>
<td><img src="image" alt="AAS Diagram" /></td>
<td>Shared Side, Vertical Angles, Parallel Lines $\Rightarrow$ Alternate Interior Angles</td>
</tr>
<tr>
<td>HL</td>
<td><img src="image" alt="HL Diagram" /></td>
<td>Shared Side</td>
</tr>
</tbody>
</table>

**HL** – Hypotenuse Leg is a short cut for SSS because of the Pythagorean Theorem
Chapter 5: Congruent Triangles

NEVER AAA or ASS or SSA
No Cars, donkeys, profanity or Social Security Administration

$\Delta ABC \cong \Delta LMN$  Order Rules!!! (first to first, second to second, etc)
Angles match by one letter  Sides match by two letter groups

“CPCTC” – corresponding parts of congruent triangles are congruent

Triangle Congruence Hidden Features

<table>
<thead>
<tr>
<th>Feature</th>
<th>Picture</th>
<th>Triangles $\cong$ by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shared Side (Reflexive Prop)</td>
<td><img src="Diagram1.png" alt="Diagram" /></td>
<td>SSS</td>
</tr>
<tr>
<td>Shared Angle (Reflexive Prop)</td>
<td><img src="Diagram2.png" alt="Diagram" /></td>
<td>ASA</td>
</tr>
<tr>
<td>Vertical Angles (&quot;Bow Tie&quot;)</td>
<td><img src="Diagram3.png" alt="Diagram" /></td>
<td>SAS</td>
</tr>
<tr>
<td>Parallel Sides ([\text{lines}])</td>
<td><img src="Diagram4.png" alt="Diagram" /></td>
<td>AAS</td>
</tr>
<tr>
<td>Alternate Interior Angles</td>
<td><img src="Diagram5.png" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>
### 9 Common Properties, Definitions & Theorems for Triangles

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1. Reflexive Property $AB = AB$</td>
<td>6. Midpoint Definition</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>When triangles share a side or an angle</td>
<td>Results in 2 segments being congruent</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>2. Vertical Angles Congruent</td>
<td>7. Angle Bisector Definition</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td>When two lines intersect (&quot;bowtie&quot;)</td>
<td>Results in 2 angles being congruent</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>3. Right Angles Congruent</td>
<td>8. Perpendicular Bisector Definition</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
</tr>
<tr>
<td>When you are given right triangles and/or a square/rectangle</td>
<td>Results in 2 segments being congruent and two right angles</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
</tr>
<tr>
<td>When two sides are parallel in given</td>
<td>If 2 angles of a triangle are congruent to 2 angles in another triangle, then the 3rd angles are congruent ((\Delta)'s (\angle)s sum to 180)</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Segment Bisector Definition</td>
<td>Note: DO NOT ASSUME ANYTHING, IF IT IS NOT GIVEN</td>
</tr>
<tr>
<td><img src="image9.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>Results in 2 segments being congruent</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 5: Congruent Triangles

Triangle Classifications:
By Angle: acute, obtuse or right (base on largest angle in triangle)
By Sides: scalene (no sides the same), isosceles (two sides equal) or equilateral (all sides equal)

Triangle Congruence:
Side-Side-Side (SSS)
Hypotenuse – Leg [HL]
Side-Angle-Side (SAS)
Angle-Side-Angle (ASA)
Angle-Angle-Side (AAS)

Common Features to look for:
Midpoint or segment bisector
(½ segment ≅ ½ segment)
All right angles congruent
Angle bisector (½ angle ≅ ½ angle)

Hidden Features to look for:
Shared Side
(AB ≅ AB Reflexive Prop)
Vertical Angles
(bowtie figure)
Parallel Sides (triangles on sides)
(Alternate Interior Angles)
Shared Angle
(∠A ≅ ∠A Reflexive Prop)

Exterior Angle Theorem:
Exterior angle = Remote angle + remote angle

Test Taking Tips:
Mark each congruence on one triangle with an S or an A to help identify Triangle Congruence
Eliminate answers with wrong number of A’s and S’s from what is given