

Chapter 6 Relationships Within Triangles

Addressed or Prepped VA SOL:

- G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
- a) ordering the sides by length, given angle measures;
 - b) ordering the angles by degree measure, given side lengths;
 - c) determining whether a triangle exists; and
 - d) determining the range in which the length of the third side must lie.
- G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.

SOL Progression

Middle School:

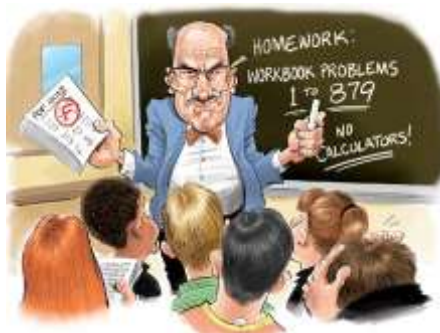
- Construct geometric figures with given coordinates
- Draw polygons in the coordinate plane given vertices and find lengths of sides
- Read, write, and evaluate algebraic expressions

Algebra I:

- Write and solve linear equations in one variable
- Use linear equations to solve real-life problems
- Graph in the coordinate plane
- Find the slope of a line

Geometry:

- Understand and use angle bisectors and perpendicular bisectors to find measures
- Find and use the circumcenter, incenter, centroid, and orthocenter of a triangle
- Use the Triangle Midsegment Theorem and the Triangle Inequality Theorem
- Write indirect proofs



Chapter 6 Relationships Within Triangles

Section 6-1: Perpendicular and Angle Bisectors

SOL: G.7

Objective:

Use perpendicular bisectors to find measures

Use angle bisectors to find measures and distance relationships

Write equation for perpendicular bisectors

Vocabulary:

Angle Bisector – a ray that cuts an angle in halves

Equidistant – same distance from both (if points, then it could be a midpoint)

Perpendicular Bisector – a segment, ray, line or plane that goes through the midpoint at a 90° angle

Core Concepts:

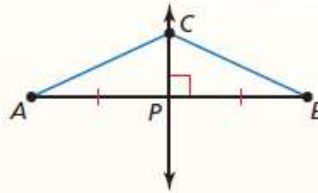
Theorems

Theorem 6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If \overline{CP} is the \perp bisector of \overline{AB} , then $CA = CB$.

Proof p. 272

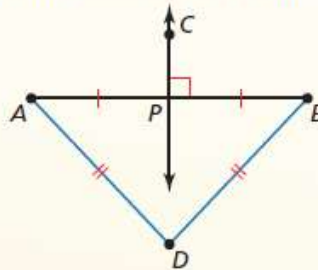


Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the \perp bisector of \overline{AB} .

Proof Ex. 32, p. 278



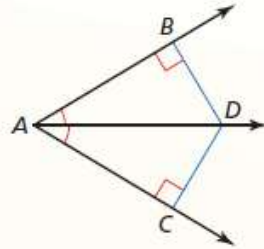
Theorems

Theorem 6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

If \overrightarrow{AD} bisects $\angle BAC$ and $\overline{DB} \perp \overline{AB}$ and $\overline{DC} \perp \overline{AC}$, then $DB = DC$.

Proof Ex. 33(a), p. 278

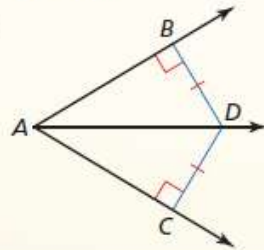


Theorem 6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overline{AB}$ and $\overline{DC} \perp \overline{AC}$ and $DB = DC$, then \overrightarrow{AD} bisects $\angle BAC$.

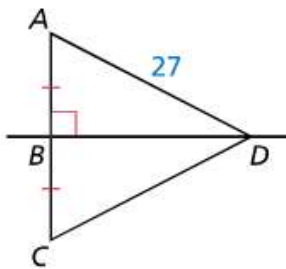
Proof Ex. 33(b), p. 278



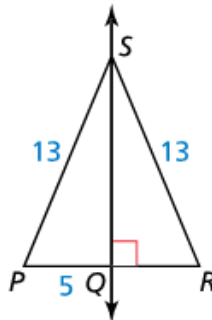
Examples:

Example 1

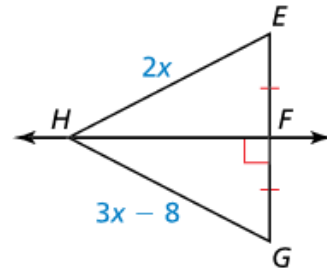
Find each measure.



a. CD



b. PR

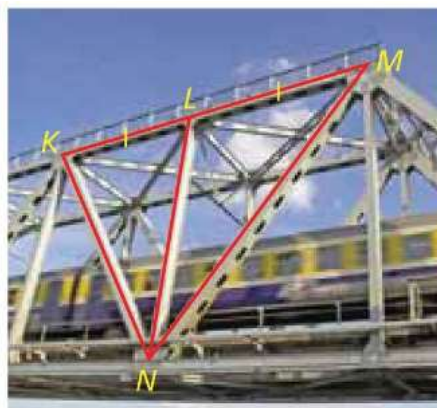


c. GH

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Example 2

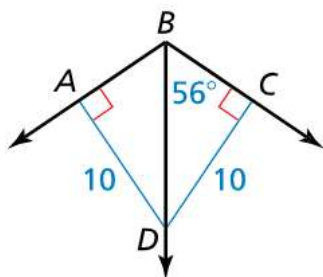
Is there enough information given in the diagram to conclude that point L lies on the perpendicular bisector of \overline{KM} ?



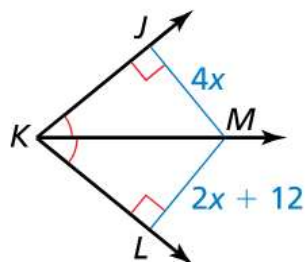
Example 3

Find each measure.

a. $m\angle ABC$

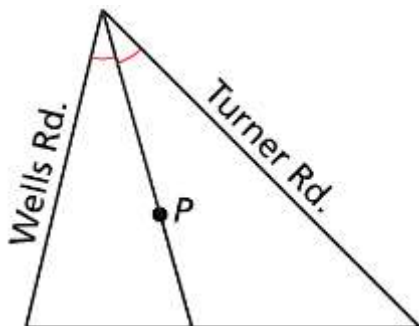


b. \overline{JM}



Example 4

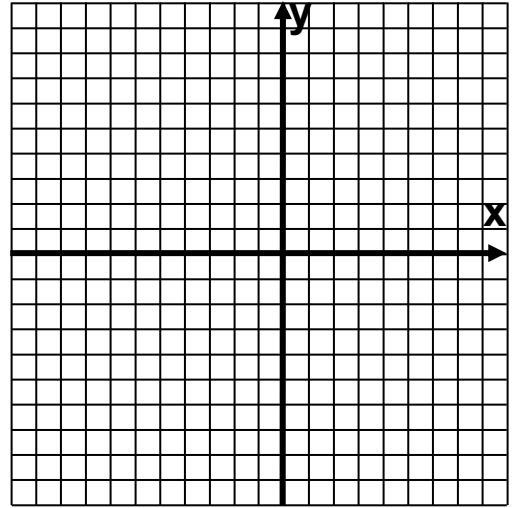
A parachutist lands in a triangular field at point P , and then walks to a road. Will he have to walk further to Wells Road or to Turner Road?



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Example 5

Write an equation of the perpendicular bisector of the segment with endpoints $D(5, -1)$ and $E(-11, 3)$.



Concept Summary:

- Perpendicular bisectors and angle bisectors of a triangle are all special segments in triangles
- Perpendicular bisectors:
 - form right angles
 - divide a segment in half – go through midpoints
 - equal distance from the vertices of the triangle
- Angle bisector:
 - cuts angle in half
 - equal distance from the sides of the triangle

In constructions of both bisectors, we build a rhombus because of the characteristics of its diagonals: perpendicular and bisect opposite angles.

Khan Academy Videos:

1. [Circumcenter](#),
2. [Incenter](#)

Homework: [Special Segments WS](#) part 1

Reading: student notes section 6-2

Chapter 6 Relationships Within Triangles

Section 6-2: Bisectors of Triangles

SOL: G.7

Objective:

Use and find the circumcenter of a triangle
Use and find the incenter of a triangle

Vocabulary:

Circumcenter – the point of concurrency of perpendicular bisectors in a triangle;
center of the circle drawn that contains the three vertices of a triangle;
equidistant from the vertices of the triangle
Concurrent – three or more lines, rays or segments that intersect (come together) at a single point
Incenter – the point of concurrency of the angle bisectors in a triangle;
center of the largest circle drawn within the triangle;
equidistant from the sides of the triangle
Point of Concurrency – the point of intersection of concurrent lines, rays or segments

Core Concept:

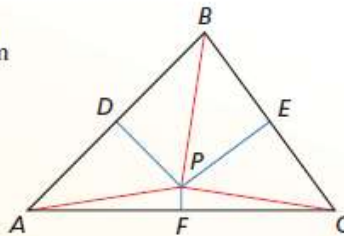
Theorems

Theorem 6.5 Circumcenter Theorem

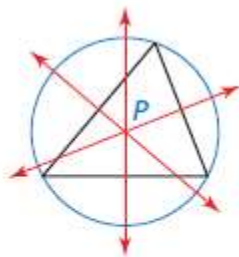
The circumcenter of a triangle is equidistant from the vertices of the triangle.

If \overline{PD} , \overline{PE} , and \overline{PF} are perpendicular bisectors, then $PA = PB = PC$.

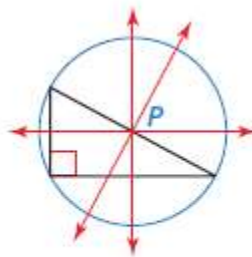
Proof p. 280



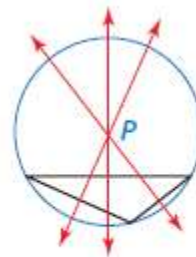
Location of circumcenter depends on the angle classification of the triangle



Acute triangle
 P is inside triangle.



Right triangle
 P is on triangle.



Obtuse triangle
 P is outside triangle.

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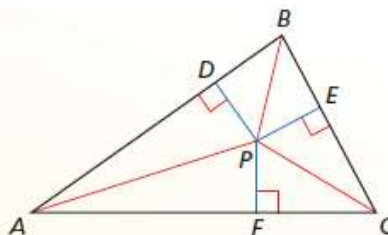
Theorem

Theorem 6.6 Incenter Theorem

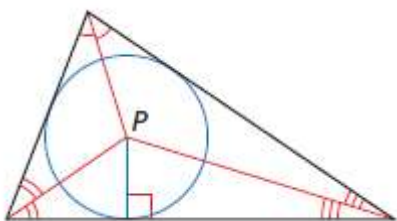
The incenter of a triangle is equidistant from the sides of the triangle.

If \overline{AP} , \overline{BP} , and \overline{CP} are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

Proof Ex. 38, p. 287



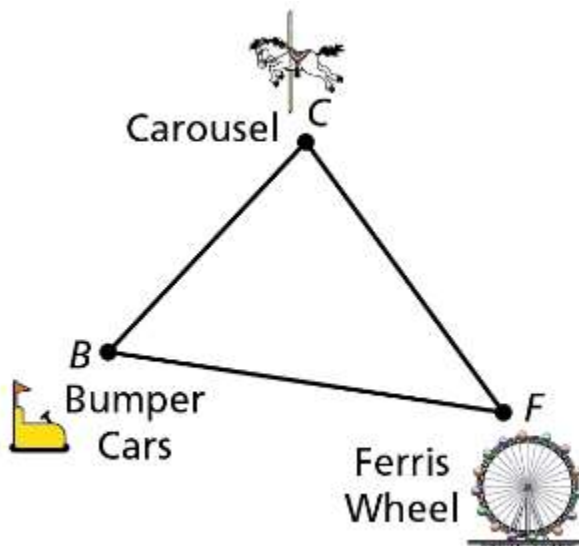
Incenter by its name is always inside a triangle, no matter what its angles are



Examples:

Example 1

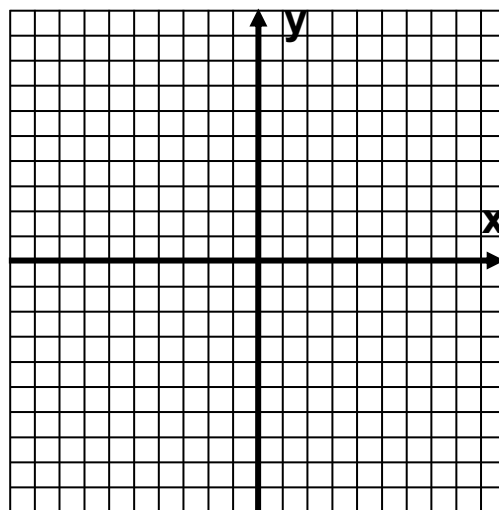
A carnival operator wants to locate a food stand so that it is the same distance from the carousel (C), the Ferris wheel (F), and the bumper cars (B). Find the location of the food stand (S).



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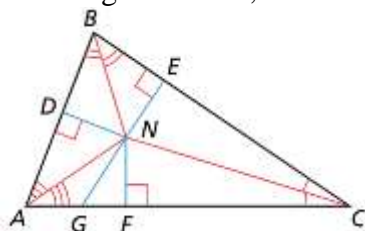
Example 2

Find the coordinates of the circumcenter of $\triangle DEF$ with vertices $D(6, 4)$, $E(-2, 4)$, and $F(-2, -2)$.



Example 3

In the figure shown, $NE = 6x + 1$ and $NF = 4x + 15$.



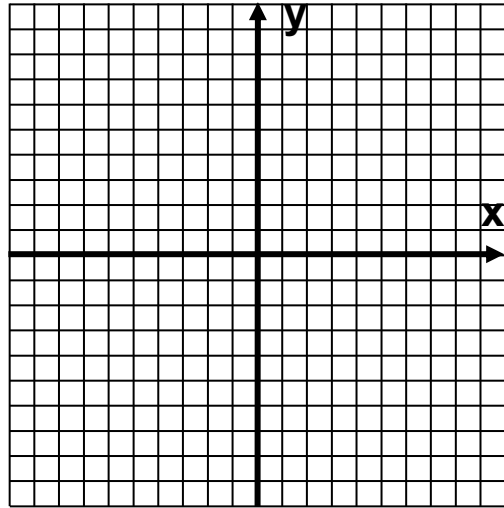
a. Find ND .

b. Can $NB = 40$? Explain your reasoning.

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Example 4

A school has fenced in an area in the shape of a scalene triangle to use for a new playground. The school wants to place a swing set where it will be the same distance from all three fences. Should the swing set be placed at the *circumcenter* or the *incenter* of the triangular playground? Explain.



Concept Summary:

Circumcenter is equidistant from vertices of triangle

It is inside the circle for acute triangles

It is on the hypotenuse for right triangles

It is outside the triangle for obtuse triangles

Incenter is equidistant from the sides of triangle

It is always inside the circle (from its name)

Khan Academy Videos: [Circumcenter](#), [Incenter](#)

Homework: [Special Segments WS](#) part 2

Reading: student notes section 6.3

Chapter 6 Relationships Within Triangles

Section 6-3: Medians and Altitudes of Triangles

SOL: G.7

Objective:

- Use medians and find the centroid of triangles
- Use altitudes and find the orthocenter of triangles

Vocabulary:

- Altitude of a triangle – the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side
- Centroid – the point of concurrency of the medians of a triangle; also known as the center of gravity
- Median of a triangle – the segment from a vertex to the midpoint on the opposite side.
- Orthocenter – the point of concurrency of the altitudes of a triangle

Core Concept:

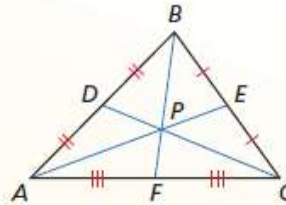
Theorem

Theorem 6.7 Centroid Theorem

The centroid of a triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of $\triangle ABC$ meet at point P , and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.

Proof BigIdeasMath.com



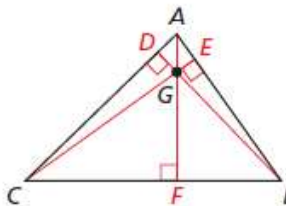
Note: The centroid is also known as the center of mass; and must be inside any triangle. It is located $\frac{1}{3}$ the way from the midpoint to the vertex.

Core Concept

Orthocenter

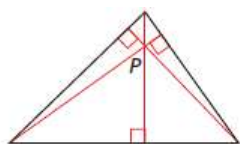
The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the **orthocenter** of the triangle.

The lines containing \overline{AF} , \overline{BD} , and \overline{CE} meet at the orthocenter G of $\triangle ABC$.

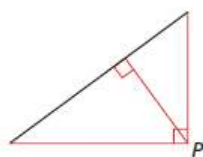


Note: The location of the orthocenter depends on the classification of the triangle's angles:

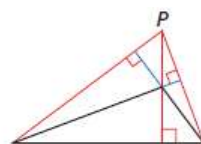
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Acute triangle
 P is inside triangle.



Right triangle
 P is on triangle.

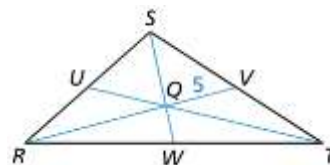


Obtuse triangle
 P is outside triangle.

Examples:

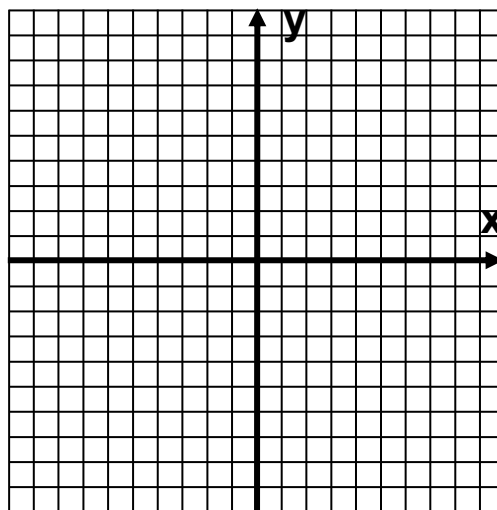
Example 1

In $\triangle RST$, point Q is the centroid, and $VQ = 5$. Find RQ and RV .



Example 2

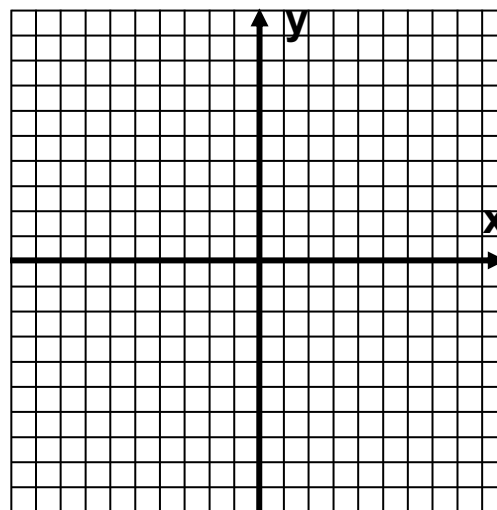
Find the coordinates of the centroid of $\triangle ABC$ with vertices $A(0, 4)$, $B(-4, -2)$, and $C(7, 1)$.



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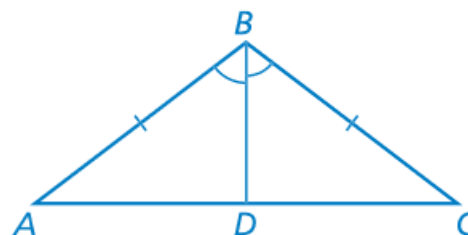
Example 3

Find the coordinates of the orthocenter of $\triangle DEF$ with vertices $D(0, 6)$, $E(-4, -2)$, and $F(4, 6)$.

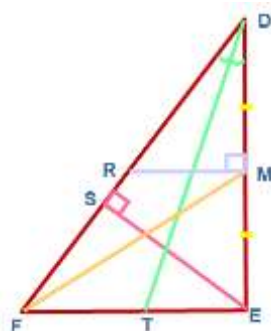


Example 4

Prove that the bisector of the vertex angle of an isosceles triangle is an altitude.



Example 5: Identify the



Altitude:

Angle Bisector:

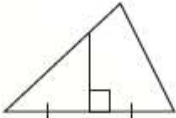
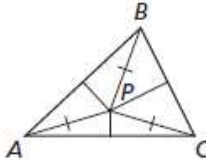
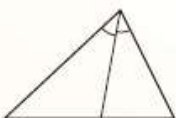
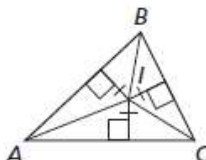

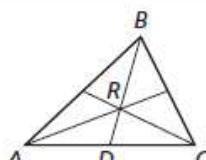
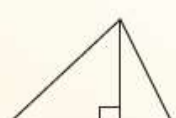
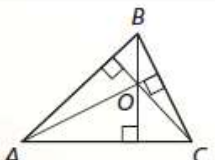
Median:

Perpendicular Bisector:

Chapter 6 Relationships Within Triangles

Concept Summary:

Concept Summary

Segments, Lines, Rays, and Points in Triangles				
	Example	Point of Concurrence	Property	Example
perpendicular bisector		circumcenter	The circumcenter P of a triangle is equidistant from the vertices of the triangle.	
angle bisector		incenter	The incenter I of a triangle is equidistant from the sides of the triangle.	
median		centroid	The centroid R of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude		orthocenter	The lines containing the altitudes of a triangle are concurrent at the orthocenter O .	

Khan Academy Videos:

1. [Triangle Medians and Centroids](#),
2. [Dividing triangles with medians](#)

Homework: [Special Segments WS](#) part 3

Reading: section 6.4 of student notes

Chapter 6 Relationships Within Triangles

Section 6-4: Using Triangle Midsegment Theorem

SOL: G.7

Objective:

Use midsegments of triangles in the coordinate plane
Use the Triangle Midsegment Theorem to find distances

Vocabulary:

Midsegment of a triangle – a segment that connects the midpoints of two sides of the triangle

Core Concept:

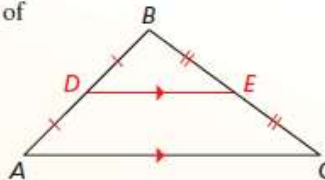
Theorem

Theorem 6.8 Triangle Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

\overline{DE} is a midsegment of $\triangle ABC$, $\overline{DE} \parallel \overline{AC}$,
and $DE = \frac{1}{2}AC$.

Proof Example 2, p. 299; Monitoring Progress Question 3, p. 299; Ex. 22, p. 302

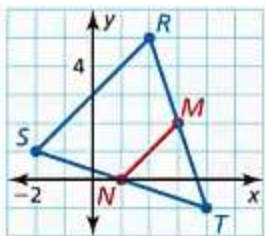


Note: Midsegments are also in trapezoids (as seen the quadrilaterals chapter)

Examples:

Example 1

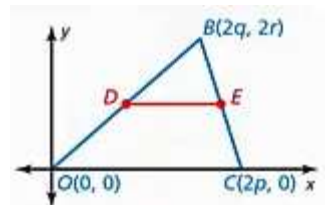
In $\triangle RST$, show that midsegment \overline{MN} is parallel to \overline{RS} and that $MN = \frac{1}{2}RS$.



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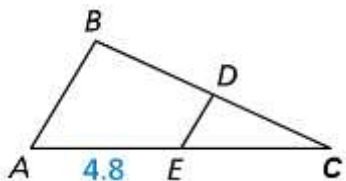
Example 2

Use the diagram to find the coordinates of F , the midpoint of \overline{OC} . Show that $\overline{FD} \parallel \overline{BC}$ and $FD = \frac{1}{2}BC$.



Example 3

\overline{DE} is a midsegment of $\triangle ABC$. Find AC .



Example 4

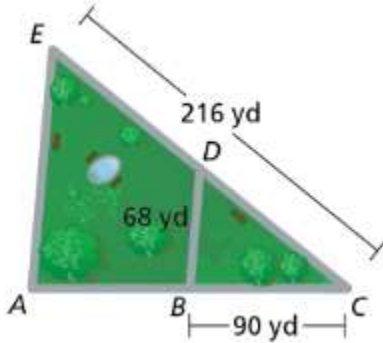
In the figure, $CF = FB$ and $CD = DA$. Which segments must be parallel?



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Example 5

A walking path \overline{BD} in a park intersects the sides of the park at their midpoints. You walk from park corner A to walking path \overline{BD} , over the path to the other side of the park, up to corner E , and then back down to your starting point. How many yards do you walk?



Concept Summary:

- Midsegments are parallel to the side that they don't intersect
- Midsegments are half the side that they are parallel to
- Divide triangle into a triangle half the size of the original triangle (and a trapezoid)

Khan Academy Videos: none relate

Homework: TBD

Reading: student notes section 6.5

Chapter 6 Relationships Within Triangles

Section 6-5: Indirect Proof and Inequalities in One Triangle

SOL: G.5

Objective:

Write indirect proofs

List sides and angles of a triangle in order by size

Use the Triangle Inequality Theorem to find possible side lengths of triangle

Vocabulary:

Indirect proof – make a temporary assumption that the desired conclusion is false

Core Concept:

Core Concept

How to Write an Indirect Proof (Proof by Contradiction)

- Step 1** Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.
- Step 2** Reason logically until you reach a contradiction.
- Step 3** Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

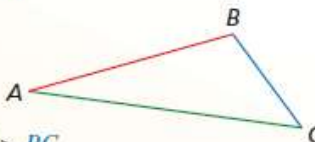
Theorem

Theorem 6.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AB + BC > AC \quad AC + BC > AB \quad AB + AC > BC$$

Proof Ex. 47, p. 310



Note:

When given three sides, to see if it makes a triangle, make sure
small + medium > large

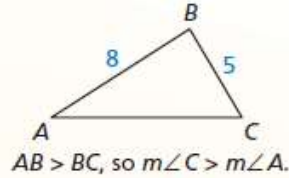
When given two sides, then the third side must be between
large – small < 3rd side < large + small

Theorems

Theorem 6.9 Triangle Longer Side Theorem

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

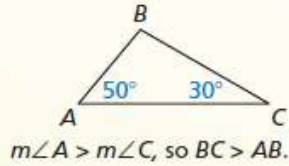
Proof Ex. 43, p. 310



Theorem 6.10 Triangle Larger Angle Theorem

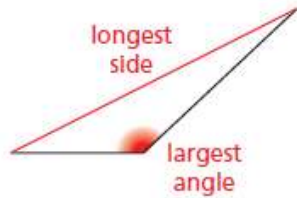
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Proof p. 305

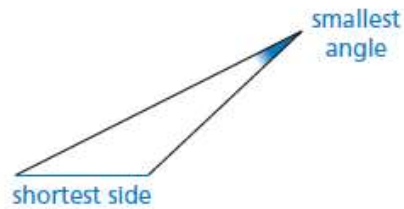


Note: Largest side is opposite the largest angle (and vice versa)

SOLUTION



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

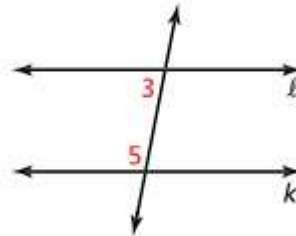
Examples:

Example 1:

Write an indirect proof.

Given Line l is not parallel to line k .

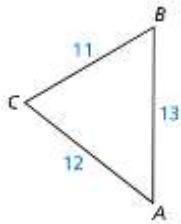
Prove $\angle 3$ and $\angle 5$ are not supplementary.



Chapter 6 Relationships Within Triangles

Example 2:

Draw an obtuse triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

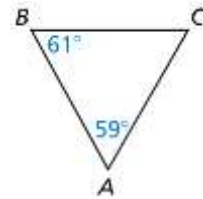


Example 3:

List the angles of $\triangle ABC$ in order from smallest to largest.

Example 4:

List the sides of $\triangle ABC$ in order from shortest to longest.



Example 5:

A triangle has one side length 6 and another side of length 15. Describe the possible lengths of the third side.

Concept Summary:

- When given three sides, to see if it makes a triangle, make sure: small + medium > large
- When given two sides, then the third side must be between
large – small < 3rd side < large + small

Khan Academy Video:

1. [Triangle inequality theorem](#)

Homework: SOL Worksheet

Reading: student notes section 6-6

Chapter 6 Relationships Within Triangles

Section 6-6: Inequalities in Two Triangles

SOL: G.5

Objective:

- Compare measures in triangles
- Solve real-life problems using the Hinge Theorem

Vocabulary: none new

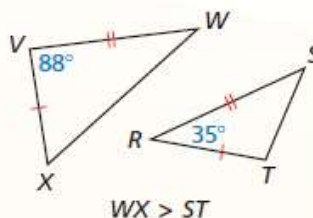
Core Concept:

Theorems

Theorem 6.12 Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

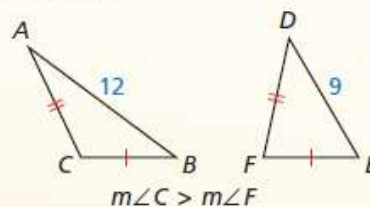
Proof BigIdeasMath.com



Theorem 6.13 Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

Proof Example 3, p. 313

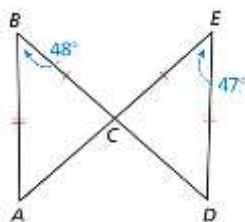
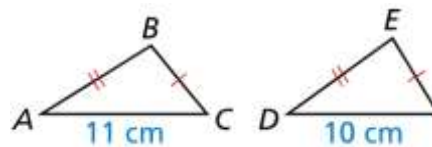


Hinge Theorem is also known as SAS and its converse as the SSS Triangle Inequality Theorem

Examples:

Example 1:

Given that $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$, how does $m\angle B$ compare to $m\angle E$?



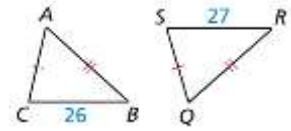
Example 2:

Given that $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EC}$, how does AC compare to DC ?

Chapter 6 Relationships Within Triangles

Example 3:

What can you conclude about the measures of $\angle A$ and $\angle Q$ in this figure? Explain.

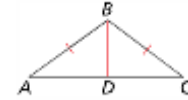


Example 4:

Write a paragraph proof.

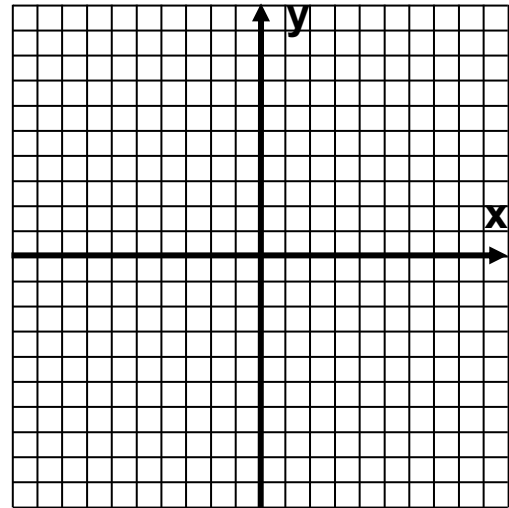
Given $\overline{AB} \cong \overline{BC}$, $AD > CD$

Prove $m\angle ABD > m\angle CBD$



Example 5:

Three groups of bikers leave the same camp heading in different directions. Group A travels 2 miles due east, then turns 45° toward north and travels 1.2 miles. Group B travels 2 miles due west, then turns 30° toward south and travels 1.2 miles. Group D travels 2 miles due south, then turns 25° toward east and travels 1.2 miles. Is Group D farther from camp than Group A, Group B, both groups, or neither group? Explain your reasoning.



Concept Summary:

- All things (sides of angles) being equal in two triangles, then opposite the larger angle is the larger side
- Converse of that is true as well (opposite the larger side must be the larger angle)

Homework: SOL Worksheet

Reading: student notes chapter review section

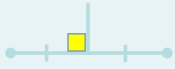



Chapter 6 Relationships Within Triangles

Section 6-R: Chapter Review

Special Segments

Segment	Point of Concurrency	Special Characteristic	Starts	Finishes
Perpendicular Bisector	Circumcenter	Equidistant from vertices	Nowhere Special	Midpoint
Angle Bisector	Incenter	Equidistant from sides	Vertex	Nowhere Special
Median	Centroid	Center of Gravity	Vertex	Midpoint
Altitude	Orthocenter	Nothing Special	Vertex	Nowhere Special

Special Segments

Segment	Picture	Problems
Perpendicular Bisector		Angle = 90° Sides = each other
Angle Bisector		Angles = each other Total = $2 (1/2 \text{ angle})$
Median		Sides = each other
Altitude		Angle = 90°

Sides and Angles

- *Largest Side is opposite the largest angle*
- *Middle Side is opposite the middle angle*
- *Smallest side is opposite the smallest angle*

Given: 3 sides or angles measurements

1. Arrange numbers in order requested
2. Replace numbers with side (2 Ltrs) or angle (1 Ltr)
3. Replace with missing letter(s)

$$97 > 51 > 32$$

$$\angle N > \angle M > \angle P$$

$$MP > NP > MN$$



Triangle Inequality Theorem

- Any two sides must be bigger than the third side
- Given three sides (can they make a triangle):
Add the smallest two sides together
If they are bigger than the largest side, then Yes
If they are equal or smaller, then No
- Given two sides (find the range of the third side)
Min value = Larger number – smaller number
Max value = Larger number + smaller number

$$\text{Min value} < \text{third side} < \text{Max Value}$$

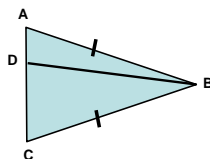
Triangle Relationship Theorems

- **SAS Inequality, or Hinge Theorem**

If $\angle ABD < \angle CBD$, then $AD < DC$

- **SSS Inequality**

If $AD < DC$, then $\angle ABD < \angle CBD$



This is our virtual alligator problem

Indirect Proof

- **Step 1:** Assume that the conclusion (what we are trying to prove) is false, so then the opposite is true.
- **Step 2:** Show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, corollary or given.
Statement Reason part of the proof
- **Step 3:** Point out that because the false conclusion leads to an incorrect statement, the original conclusion must be true (the opposite of what we assumed in step 1)

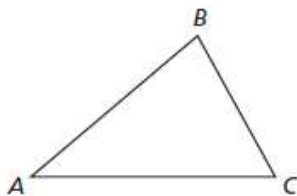
Chapter 6 Relationships Within Triangles

Constructions:

CONSTRUCTION

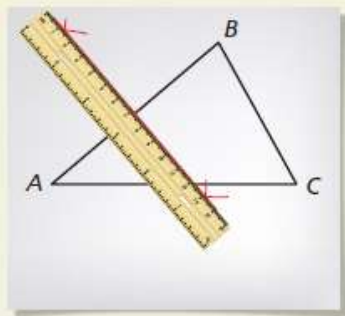
Circumscribing a Circle About a Triangle

Use a compass and straightedge to construct a circle that is circumscribed about $\triangle ABC$.



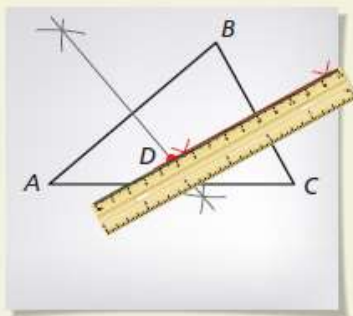
SOLUTION

Step 1



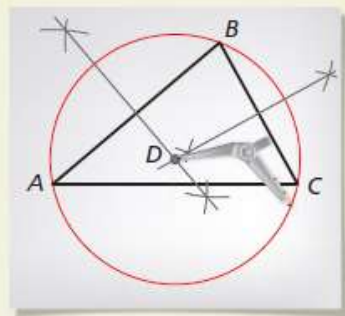
Draw a bisector Draw the perpendicular bisector of \overline{AB} .

Step 2



Draw a bisector Draw the perpendicular bisector of \overline{BC} . Label the intersection of the bisectors D . This is the circumcenter.

Step 3



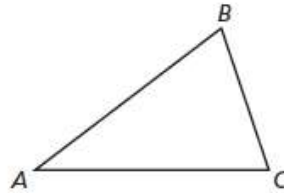
Draw a circle Place the compass at D . Set the width by using any vertex of the triangle. This is the radius of the *circumcircle*. Draw the circle. It should pass through all three vertices A , B , and C .

Chapter 6 Relationships Within Triangles

CONSTRUCTION

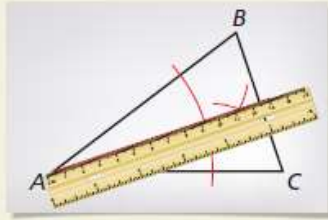
Inscribing a Circle Within a Triangle

Use a compass and straightedge to construct a circle that is inscribed within $\triangle ABC$.



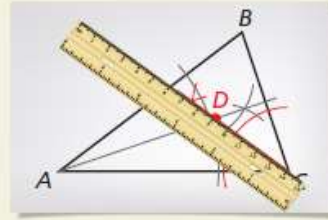
SOLUTION

Step 1



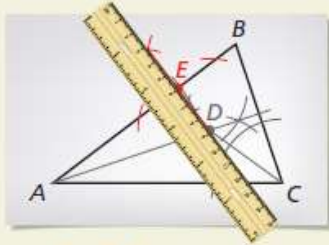
Draw a bisector Draw the angle bisector of $\angle A$.

Step 2



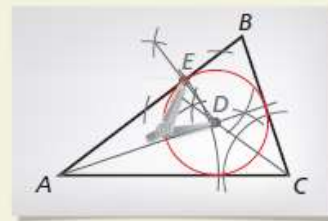
Draw a bisector Draw the angle bisector of $\angle C$. Label the intersection of the bisectors D . This is the incenter.

Step 3



Draw a perpendicular line Draw the perpendicular line from D to \overline{AB} . Label the point where it intersects \overline{AB} as E .

Step 4



Draw a circle Place the compass at D . Set the width to E . This is the radius of the *incircle*. Draw the circle. It should touch each side of the triangle.

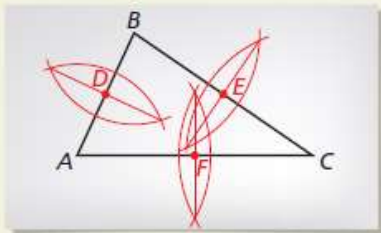
CONSTRUCTION

Finding the Centroid of a Triangle

Use a compass and straightedge to construct the medians of $\triangle ABC$.

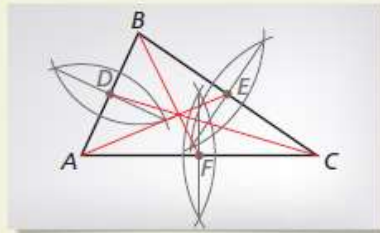
SOLUTION

Step 1



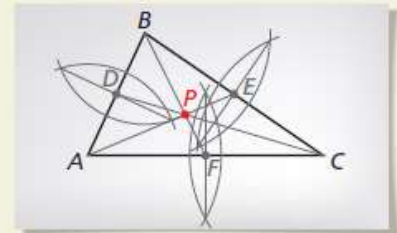
Find midpoints Draw $\triangle ABC$. Find the midpoints of \overline{AB} , \overline{BC} , and \overline{AC} . Label the midpoints of the sides D , E , and F , respectively.

Step 2



Draw medians Draw \overline{AE} , \overline{BF} , and \overline{CD} . These are the three medians of $\triangle ABC$.

Step 3



Label a point Label the point where \overline{AE} , \overline{BF} , and \overline{CD} intersect as P . This is the centroid.

Given 3 numbers, do they make a triangle?

1. Put in order from smallest to largest
2. Is **Small + Middle** > **Large**?
 - A. Yes, then it makes a triangle
 - B. No, then it does not make a triangle

Example A: 8, 11, 4

1. Ordered: 4, 8, 11

2. $4 + 8 = 12 > 11$ so triangle

Example B: 9, 13, 4

1. Ordered: 4, 9, 13

2. $4 + 9 = 13 = 13$ so no triangle

Given 2 numbers, find range of the third side

1. Put in order from smallest to largest
2. $(\text{Large} - \text{Small}) < \text{Third Side} < (\text{Large} + \text{Small})$

Example: Given 3 and 9 find third side range

$$9 - 3 = 6 < \text{Third Side} < 12 = 9 + 3$$

Angles versus Sides in Triangles

1. Largest side is opposite the largest angle
2. Middle side is opposite the middle angle
3. Smallest side is opposite the smallest angle

Steps to assure order

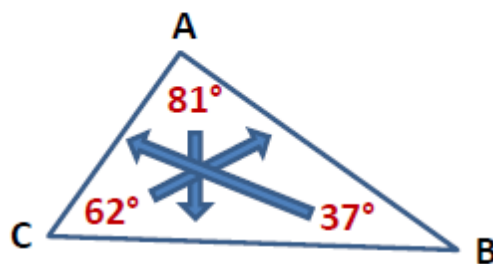
1. Put given information in order requested
(**smallest to largest** or **largest to smallest**)
2. Substitute the corresponding letters
(**1 Angles; 2 for Sides**)
3. Put in the missing letters
(**from the 3 in triangle**)

Chapter 6 Relationships Within Triangles

Sides Example: Find Sides largest to smallest

$$Lg > Md > Sm$$

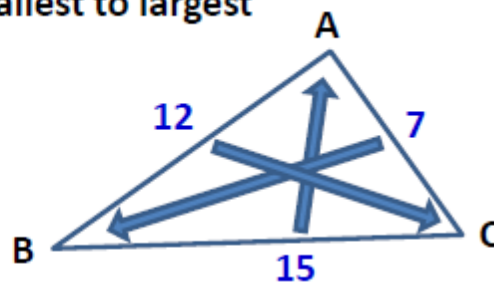
1. $81^\circ > 62^\circ > 37^\circ$
2. $\angle A > \angle C > \angle B$
3. $BC > AB > AC$



Angles Example: Find Angles smallest to largest

$$Sm < Md < Lg$$

1. $7 < 12 < 15$
2. $AC < AB < BC$
3. $\angle B < \angle C < \angle A$



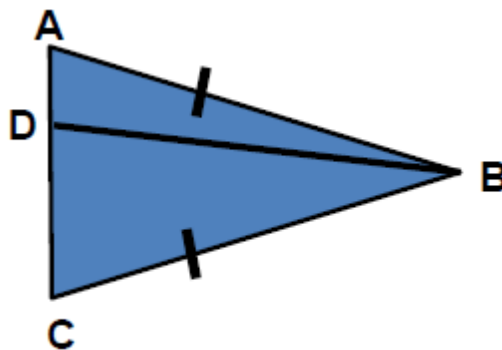
“Bigger angle opposite bigger side”

- **SAS Inequality, or Hinge Theorem**

If $\angle ABD < \angle CBD$, then $AD < DC$

- **SSS Inequality**

If $AD < DC$, then $\angle ABD < \angle CBD$

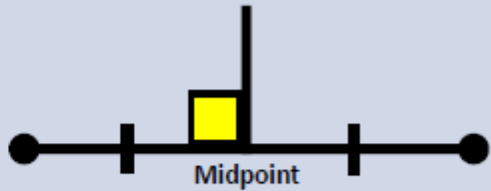

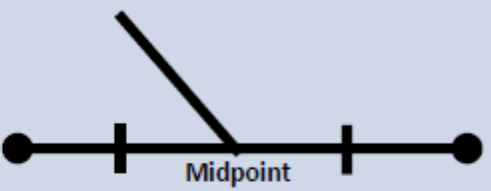



This is our virtual alligator problem

Special Segments in Triangles

Segment	Point of Concurrency	Special Characteristic	Starts	Finishes
Perpendicular Bisector	Circumcenter	Equidistant from vertices	Nowhere Special	Midpoint
Angle Bisector	Incenter	Equidistant from sides	Vertex	Nowhere Special
Median	Centriod	Center of Gravity	Vertex	Midpoint
Altitude	Orthocenter	Nothing Special	Vertex	Nowhere Special

Point of concurrency is where the three segments of the same type come together (3 Medians of a triangle cross at the Centroid)

Segment	Picture	Problems
Perpendicular Bisector		Angle = 90° $\frac{1}{2}$ Sides = each other
Angle Bisector		$\frac{1}{2}$ Angles = each other Total = $2 (1/2 \text{ angle})$
Median		$\frac{1}{2}$ Sides = each other
Altitude		Angle = 90°