Chapter 8: Similarity

Addressed or Prepped VA SOL:

G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar.

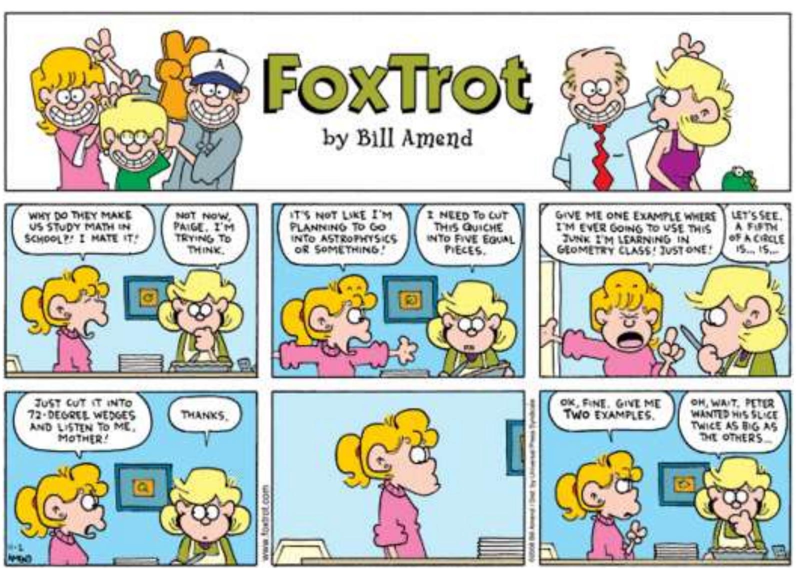
G.14 The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
- a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;
- d) solving problems, including practical problems, about similar geometric figures.

SOL Progression

Middle School:
- Understand ratios and describe ratio relationships
- Decide whether two quantities are proportional (ratio tables, graphs)
- Represent proportional relationships with equations
- Identify corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles

Algebra I:
- Solve linear equations in one variable
- Use linear equations to solve real-life problems
- Find the slope of a line
- Identify and use parallel and perpendicular lines in real-life problems

Geometry:
- Use the AA, SSS and SAS Similarity Theorems to prove triangles are similar
- Decide whether polygons are similar
- Use similarity criteria to solve problems about lengths, perimeters, and areas
- Prove the slope criteria using similar triangles
- Use the Triangle Proportionality Theorem and other proportionality theorems



Chapter 8: Similarity

Section 8-1: Similar Polygons

SOL: G.14.a, d and G.7

Objective:

Use similarity statements
Find corresponding lengths in similar polygons
Find perimeters and area of similar polygons
Decide whether polygons are similar

Vocabulary:

Corresponding parts – sides (in ratio equal to scaling factor) or angles (that are congruent) that line up in similar figures
Similar figures – a similarity transformation maps one of the figures onto the other
Similarity transformation – dilation or a composition of rigid motions and dilations

Core Concept:

Core Concept

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k. So, corresponding side lengths are proportional.

Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

Note: the scaling factor k is a number greater than 0

Core Concept

Corresponding Lengths in Similar Polygons

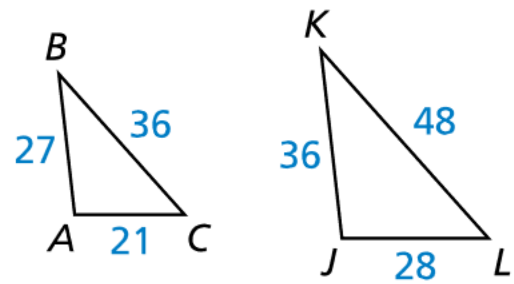
If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

Examples:

Example 1:

In the diagram, $\Delta ABC \sim \Delta JKL$.

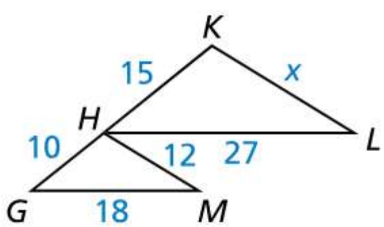
a. Find the scale factor from ΔABC to ΔJKL.



b. List all pairs of congruent angles.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

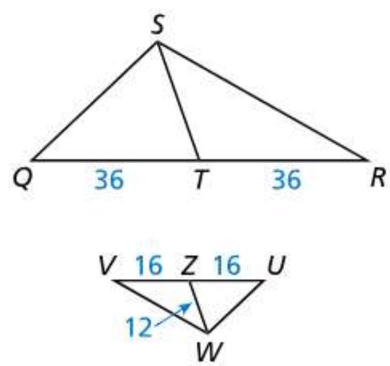
Example 2:



In the diagram, $\Delta GHM \sim \Delta HKL$. Find the value of x.

Example 3:

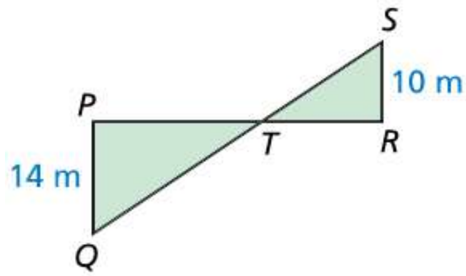
In the diagram, $\Delta UVW \sim \Delta QRS$. Find the length of the median of $\overline{ST}$.



Example 4:

Your neighbor has decided to enlarge his garden. The garden is rectangular with width 6 feet and length 15 feet. The new garden will be similar to the original one, but will have a length of 35 feet. Find the perimeter of the original garden and the enlarged garden.

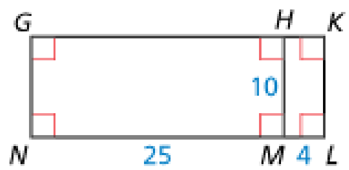
Example 5:



In the diagram, $\Delta PQT \sim \Delta RST$, and the area of ΔRST is 75 square meters. Find the area of ΔPQT.

Example 6:

Decide whether $GNMH$ and $MLKH$ are similar. Explain your reasoning.



Concept Summary:

- A ratio is a comparison of two quantities
- A proportion is an equation stating that two ratios are equal
- The scaling factor is the ratio of corresponding sides of similar figures
- Recipes are "scaled up" or "scaled down" to fit the amount required

Khan Academy Videos:

1. Similar shapes and transformations
2. Introduction to triangle similarity

Homework: Proportions WS 1 and WS 2

Reading: student notes section 8-2

Chapter 8: Similarity

Section 8-2: Proving Triangle Similarity by AA

SOL: G.7

Objective:

Use the Angle-Angle Similarity Theorem
Solve real-life problems

Vocabulary: None new

Core Concept:

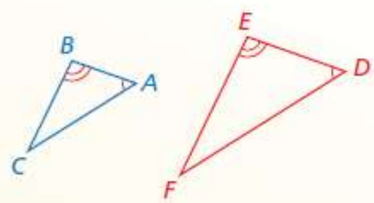
Theorem

Theorem 8.3 Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

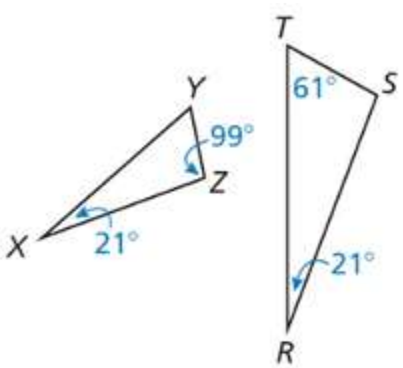
Proof p. 392



AA corresponds to the ASA and AAS triangle congruence Theorems

Examples:

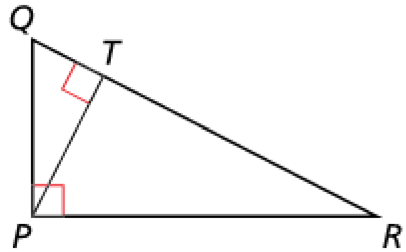
Example 1:



Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

Example 2:

Show that $\Delta QPR \sim \Delta QTP$.



Example 3:

A school flagpole casts a shadow that is 45 feet long. At the same time, a boy who is five feet eight inches tall casts a shadow that is 51 inches long. How tall is the flagpole to the nearest foot?

Concept Summary:

- AA, SSS and SAS Similarity can all be used to prove triangles similar
- Similarity of triangles is reflexive, symmetric, and transitive

Khan Academy Videos:

1. Triangle similarity postulates/criteria
2. Determining similar triangles

Homework: Similar Polygons Worksheet

Reading: student notes section 8-3

Chapter 8: Similarity

Section 8-3: Proving Triangle Similarity by SSS and SAS

SOL: G.7

Objective:

Use the Side-Side-Side Similarity Theorem
Use the Side-Angle-Side Similarity Theorem
Prove slope criteria using similar triangles

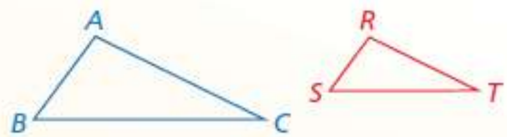
Vocabulary: None new

Core Concepts:

Theorem

Theorem 8.4 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.



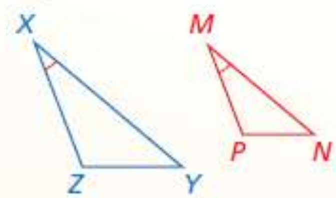
If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof p. 399

Theorem

Theorem 8.5 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

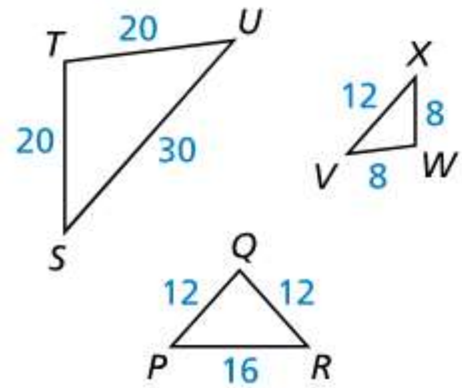
Proof Ex. 37, p. 406

Chapter 8: Similarity

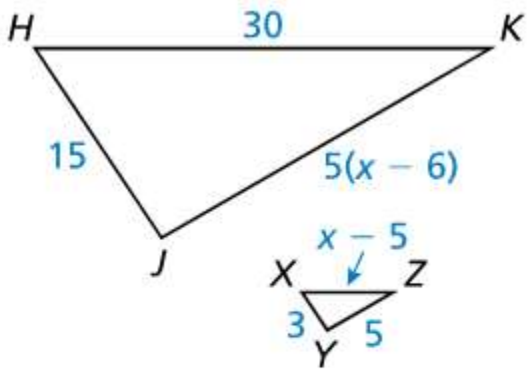
Examples:

Example 1:

Is either ΔPQR or ΔSTU similar to ΔVWX?



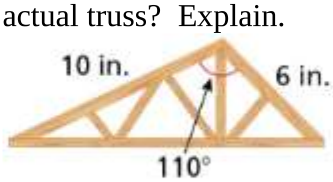
Example 2:



Find the value of x that makes $\Delta XYZ \sim \Delta HJK$.

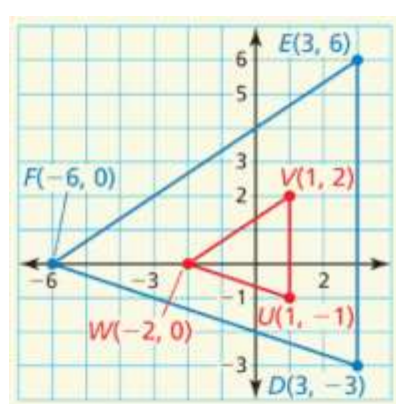
Example 3:

The diagram is a scale drawing of a triangular roof truss. The lengths of the two upper sides of the actual truss are 18 feet and 40 feet. The actual truss and the scale drawing both have an included angle of $110°$. Is the scale drawing of the truss similar to the actual truss? Explain.



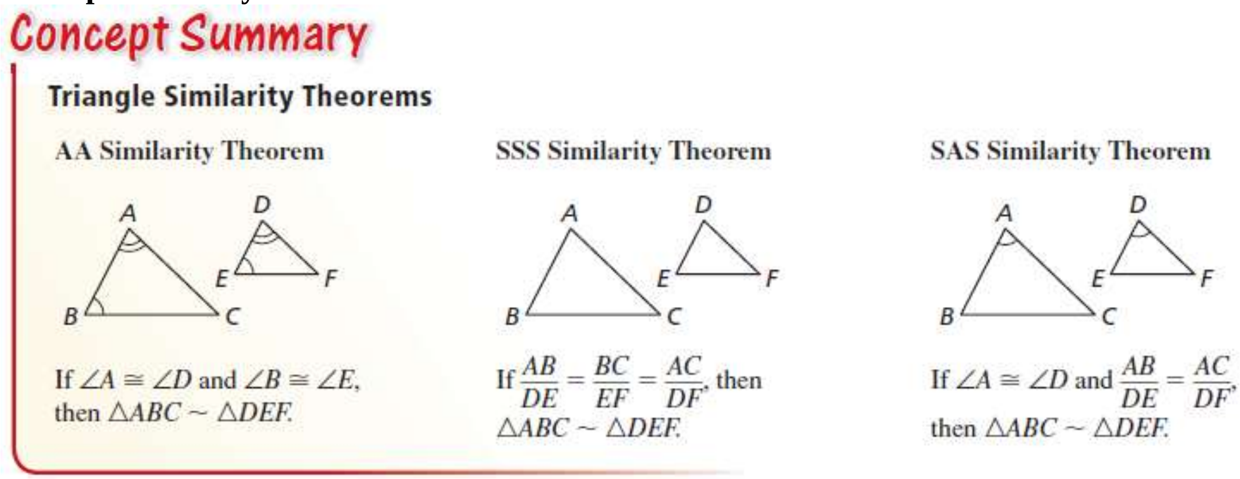
Example 4:

Is ΔDEF similar to ΔUVW?



Chapter 8: Similarity

Concept Summary:



Khan Academy Videos:

1. Solving similar triangles
2. Solving similar triangles: same side plays different roles

Homework: Triangle Similarity WS 1

Reading: student notes section 8-4

Section 8-4: Proportionality Theorems

SOL: G.7

Objective:

Use the Triangle Proportionality Theorem and its converse
Use other proportionality theorems

Vocabulary:

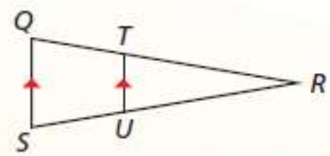
- *Midsegment: a segment whose endpoints are the midpoints of two sides of the triangle; it is parallel to one side of the triangle and its length is half of the length of that side.*

Key Concept:

Theorems

Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

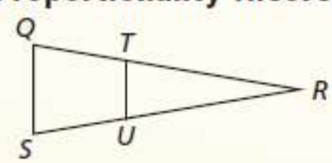


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof Ex. 27, p. 415

Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof Ex. 28, p. 415

Contrapositive of the Triangle Proportionality Theorem

If $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

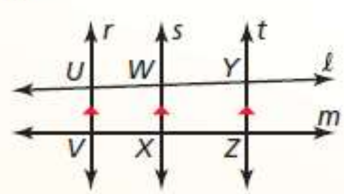
Inverse of the Triangle Proportionality Theorem

If $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$.

Theorem

Theorem 8.8 Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



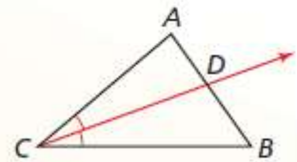
$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof Ex. 32, p. 415

Theorem

Theorem 8.9 Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



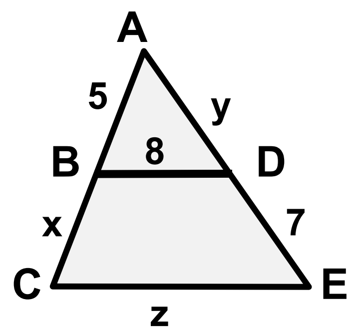
$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof Ex. 35, p. 416

Examples:

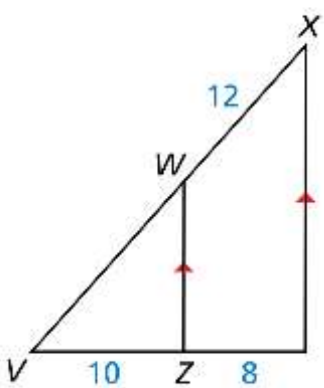
Example 0:

In the diagram *BD* is a mid-segment, Find *x*, *y*, *z*.



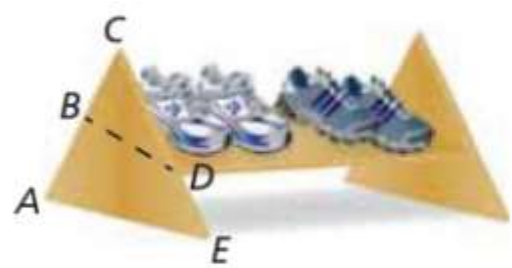
Example 1:

In the diagram $\overline{WZ} \parallel \overline{XY}$, $WX = 12$, $VZ = 10$, and $ZY = 8$. What is the length of $\overline{VW}$?

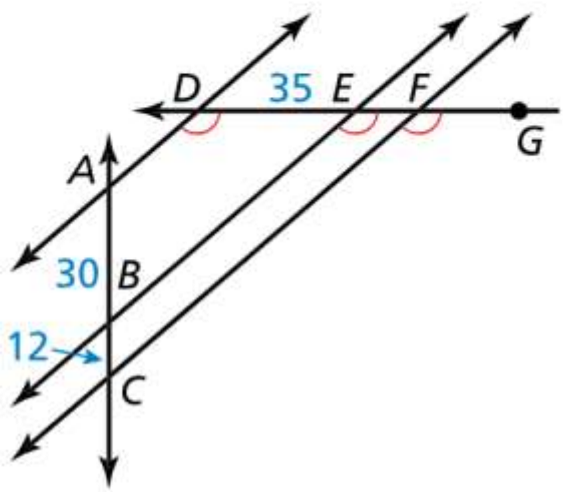


Example 2:

$BA = 35cm$, $CB = 25cm$, $CD = 20cm$, and $DE = 28cm$. Explain why the shelf is parallel to the floor.



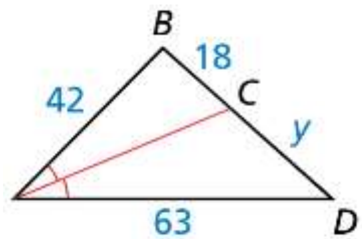
Example 3:



In the diagram, $\angle ADE$, $\angle BED$, and $\angle CFG$ are all congruent. $AB = 30$, $BC = 12$, and $DE = 35$. Find DF.

Example 4:

In the diagram, $\angle BAC \cong \angle CAD$. Use the given lengths to find the length of $\overline{CD}$.



Concept Summary:

- A segment that intersects two sides of a triangle and is parallel to the third side divides the two intersected sides in proportion
- If two lines divide two segments in proportion, then the lines are parallel
- If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal
- Corresponding parts (medians, altitudes, perimeters) are in the same ratio as corresponding sides in similar triangles

Khan Academy Videos:

1. Introduction to angle bisector theorem
2. Using the angle bisector theorem
3. Using similar and congruent triangles

Homework: Triangle Similarity WS 2

Reading: student notes chapter review section

Chapter 8: Similarity

Section 8-R: Chapter Review

SOL: G.7

Objective: Chapter review

Vocabulary: none new

Key Concept:

Ratios:
To get a proportion, we must set up a ratio that has the corresponding parts (parts from the same triangle have to be in either the top or the bottom). Solve using cross-multiplication.

$\frac{a}{c} = \frac{b}{d}$ ad = cb a and b must come from same triangle (c and d from other) !

Triangle Similarity Theorems:

All similar triangles must have their corresponding angles congruent !!
All sides must have the same *scaling factor* with their corresponding side
AA – (includes ASA and AAS) – if two angles are congruent in a triangle then the third angle must be congruent
SAS – sides must have the same scaling factor (be in the same ratio); included angle
SSS – all sides must have the same scaling factor

Proofs:

Use similar steps to congruent triangle proofs.
Need to show angles congruent (parallel lines, vertical angles, etc) and sides having the same ratio (scaling factor)

Similar triangles (or figures) problem solving:

1) Draw a picture of triangles, if you are not given one or if the picture given is too complex
2) Find corresponding parts (angles must be congruent and order still rules!)
3) Set up a proportion; make sure the tops (and bottoms) come from the same triangles!
4) Solve using cross multiplication
5) Check answer to make sure it makes sense

Test Taking Tips:

Check your answer and make sure that it makes sense in the picture
If the figure is smaller, then the corresponding part must be smaller than the given piece of the larger

Homework: Quiz Review Worksheet

Reading: student notes chapter review section

Chapter 8: Similarity

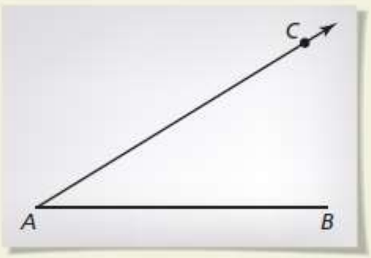
Constructions:

CONSTRUCTION Constructing a Point along a Directed Line Segment

Construct the point L on $\overline{AB}$ so that the ratio of AL to LB is 3 to 1.

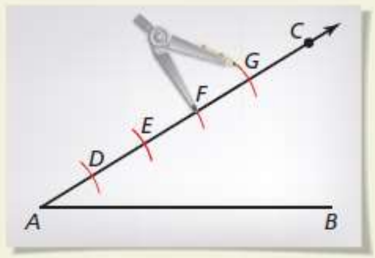
SOLUTION

Step 1



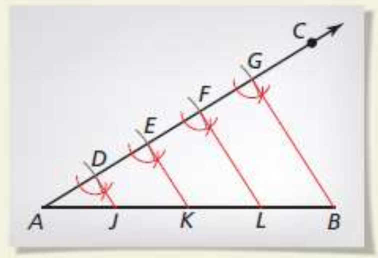
Draw a segment and a ray Draw $\overline{AB}$ of any length. Choose any point C not on $\overleftrightarrow{AB}$. Draw $\overrightarrow{AC}$.

Step 2



Draw arcs Place the point of a compass at A and make an arc of any radius intersecting $\overrightarrow{AC}$. Label the point of intersection D. Using the same compass setting, make three more arcs on $\overrightarrow{AC}$, as shown. Label the points of intersection E, F, and G and note that $AD = DE = EF = FG$.

Step 3



Draw a segment Draw $\overline{GB}$. Copy $\angle AGB$ and construct congruent angles at D, E, and F with sides that intersect $\overline{AB}$ at J, K, and L. Sides $\overline{DJ}$, $\overline{EK}$, and $\overline{FL}$ are all parallel, and they divide $\overline{AB}$ equally. So, $AJ = JK = KL = LB$. Point L divides directed line segment AB in the ratio 3 to 1.

Proportions: Cross-Multiply

Proportions: (seeing the bear trap)

Any time a top or bottom of proportion has a + or – sign in it; put parentheses around it

Wrong way:

$$\frac{3}{x} = \frac{2}{x-2}$$

$3x - 2 = 2x$ (error)

$x \neq 2$

Right way:

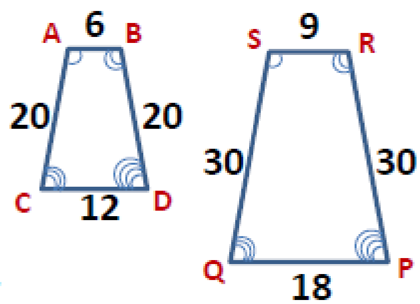
$$\frac{3}{x} = \frac{2}{(x-2)}$$

$3(x - 2) = 2x$

$3x - 6 = 2x$

$x = 6$

Similar Triangles (and Polygons)



Similar (~) Figures

- Same Shape, but not same size
- Corresponding *__angles are congruent__*
- Corresponding *__sides are proportional__*
- Scale Factor is proportion in simplest form
- Order rules again (to find corresponding things)!!

  Quadrilateral ABCD ~ Quadrilateral SRQP

  1 2 3 4   1 2 3 4
- Assign one figure to top of ratio, the other to the bottom;
- Solve using proportions

$$9 = \frac{3}{2} \times 6$$

$$18 = \frac{3}{2} \times 12$$

$$30 = \frac{3}{2} \times 20$$

27

Similar Triangles (and Polygons)

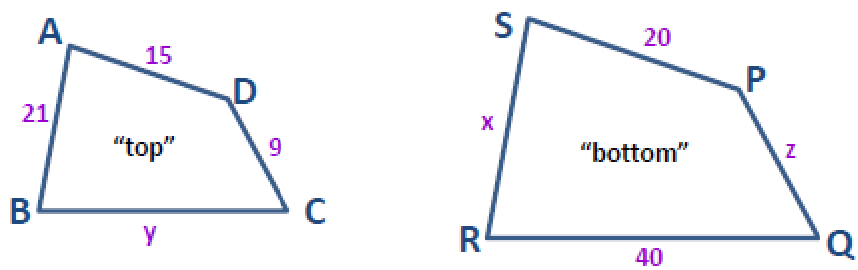
Similar (~) Figures

- Order rules again (to find corresponding things)!!

Quadrilateral ABCD ~ Quadrilateral SRQP
1 2 3 4 — 1 2 3 4

- Assign one figure to top of ratio, the other to the bottom

- Solve using proportions



$$\frac{\overset{1\,2}{AB}}{\underset{1\,2}{SR}} = \frac{\overset{2\,3}{BC}}{\underset{2\,3}{RQ}} = \frac{\overset{3\,4}{CD}}{\underset{3\,4}{QP}} = \frac{\overset{4\,1}{DA}}{\underset{4\,1}{PS}}$$

"top"
"bottom"

$$\frac{21}{x} = \frac{y}{40} = \frac{9}{z} = \frac{15}{20} \qquad \text{scaling factor: } \frac{15}{20} = \frac{3}{4}$$

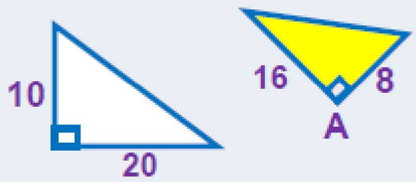
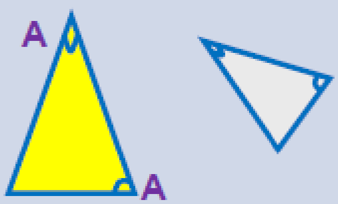
$$\frac{21}{x} = \frac{15}{20} \qquad \frac{y}{40} = \frac{15}{20} \qquad \frac{9}{z} = \frac{15}{20}$$

$15x = 20(21)$	$20y = 15(40)$	$15z = 9(20)$
$15x = 420$	$20y = 600$	$15z = 180$
$x = 28$	$y = 30$	$z = 12$

Similar Triangles

Proving Triangles Similar

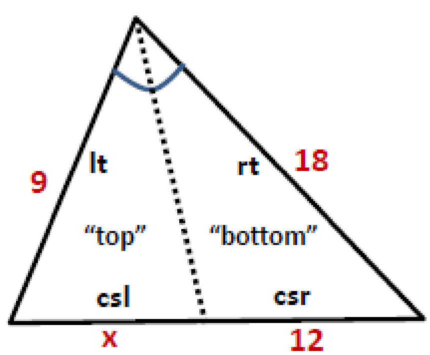
- Angles are congruent
- Sides are proportional (not congruent)

Post/Thrm	Picture	Δ Congruence / Logic
SSS		SSS All sides multiplied by same number (1/3)
SAS		SAS Sides either side of congruent angle multiplied by same number (5/4)
AA		ASA / AAS If two angles congruent, since all three add to 180, then all 3 angles congruent

"Multiplied by same number" is the scaling factor

Similar Triangles (Special Cases)

Angle Bisector Theorem



$$\frac{lt}{csl} = \frac{rt}{csr}$$

Sides
Partial

Alternative proportion

Sides Partial

$$\frac{lt}{rt} = \frac{csl}{csr}$$

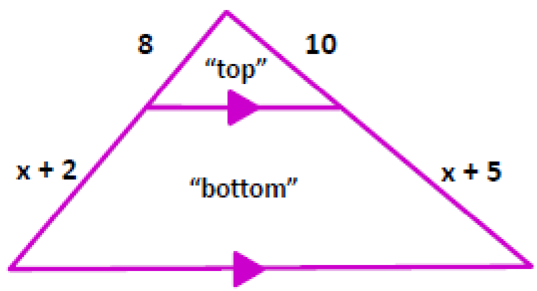
Example

$$\frac{9}{18} = \frac{x}{12}$$

18(x) = 9 (12)
18x = 108
x = 6

Csl/csr – cut side left or right

Transversals in Parallel Lines



Left Right

$$\frac{tl}{bl} = \frac{tr}{br}$$

Note: parallel bases cannot use this special case (they must use little vs big triangle proportions)

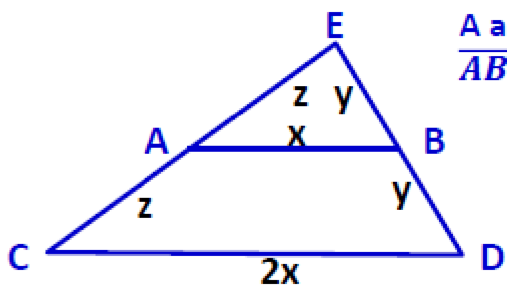
Example

$$\frac{8}{x+2} = \frac{10}{x+5}$$

8(x + 5) = 10 (x + 2)
8x + 40 = 10x + 20
20 = 2x
10 = x

Similar Triangles (Special Cases)

Mid-segment Theorem



A and B are midpoints
$\overline{AB}$ is mid-segment

Example:
Given CD = 20, EA = 8, BD = 7
Find AB, EB, and AC
10, 7, 8

Note: top triangle is half of big triangle