Modified and Animated By Chris Headlee
Dec 2011

CHAPTER 9 SOL PROBLEMS

SSM: Super Second-grader Methods

SOL Problems; not Dynamic Variable Problems

13 **Given the following measures of the sides of triangles, which is a right triangle?**

A 41 cm, 40 cm, 9 cm
B 45 ft, 40 ft, 35 ft
C 52 in., 50 in., 11 in.
D 45 yd, 35 yd, 25 yd

SSM:

- **no common p-triples (no help)**

Pythagorean Thrm:

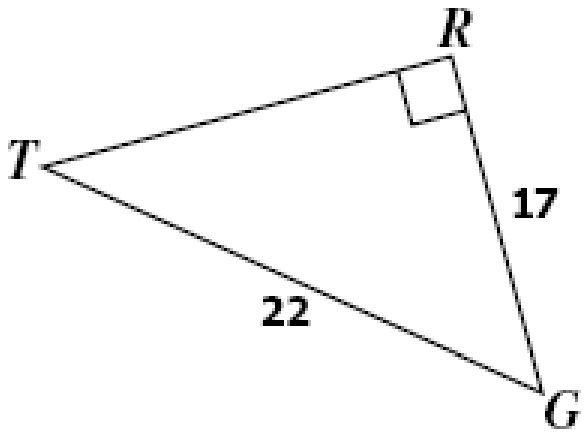
$9^2 + 40^2 = 41^2$

$81 + 1600 = 1681$

$1681 = 1681$

others do not satisfy Pythagorean Theorem

19 $\triangle TRG$ is a right triangle.



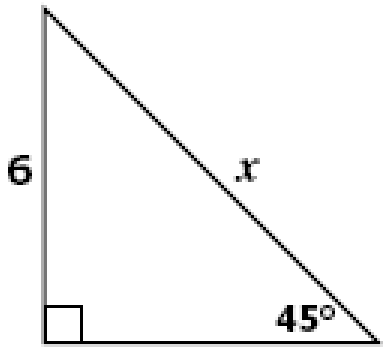
Which is closest to the length of $\overline{RT}$?

A 5
B 11
C 14
D 28

SSM:
- x < 22 ; eliminates D
- answer A doesn't form a triangle

Pythagorean Theorem:

$x^2 + 17^2 = 22^2$
$x^2 + 289 = 484$
$x^2 = 195$
$x = 13.96$



SSM:
- **answer A is wrong; x must be bigger than 6**

In the figure, what is the value of x ?

F 6

G $6\sqrt{2}$

H $6\sqrt{3}$

J 12

Pythagorean Thrm;
45-45-90 triangle (isosceles)

$6^2 + 6^2 = x^2$

$36 + 36 = x^2$

$72 = x^2$

$8.49 = x \quad (6\sqrt{2})$

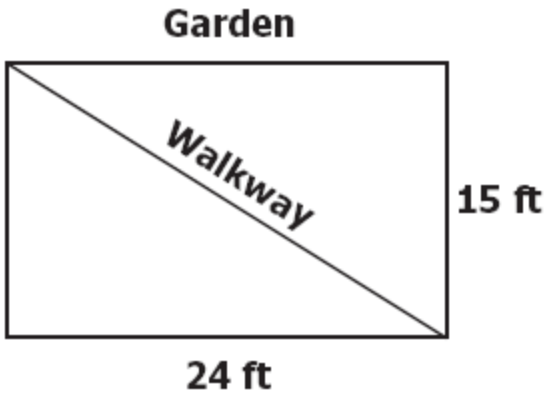
Special Case right triangles:
side opposite 45° angle is ½ hyp √2
so $6 = \frac{1}{2} \times \sqrt{2}$

$12 = x\sqrt{2}$

$6\sqrt{2} = x$

Trig:
6 is O; x is H; use sin
sin 45 = 6 / x
x = 6 / (sin 45) = 8.49

13 Mr. Ammons is constructing a walkway through his rectangular garden. The walkway runs diagonally as shown in the diagram.



Which is closest to the length of the walkway?

A 18.7 ft
B 28.3 ft
C 30.0 ft
D 39.0 ft

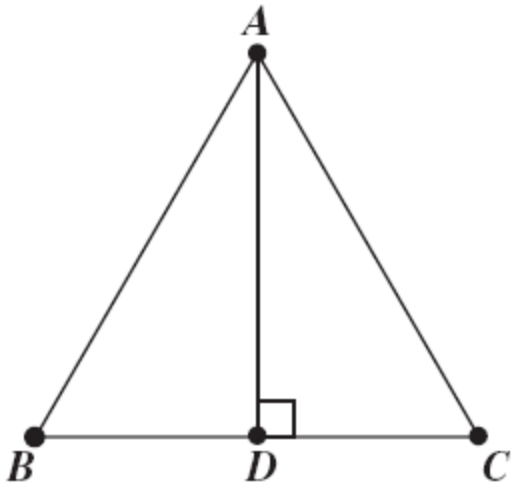
Pythagorean Thrm:

$$15^2 + 24^2 = W^2$$
$$225 + 576 = W^2$$
$$801 = W^2$$
$$28.3 = W$$

SSM:

- Our eyes tell us that the walkway must be bigger than both sides (A is wrong)
- Answer D does not form a triangle (so it is wrong)

17 Triangle ABC is an equilateral triangle with side lengths of 10 inches.



What is the length, in inches, of $\overline{AD}$?

A 5

B $\frac{10\sqrt{3}}{3}$

C $5\sqrt{2}$

D $5\sqrt{3}$

SSM:
- measure AC and compare to AD
- compare with answers

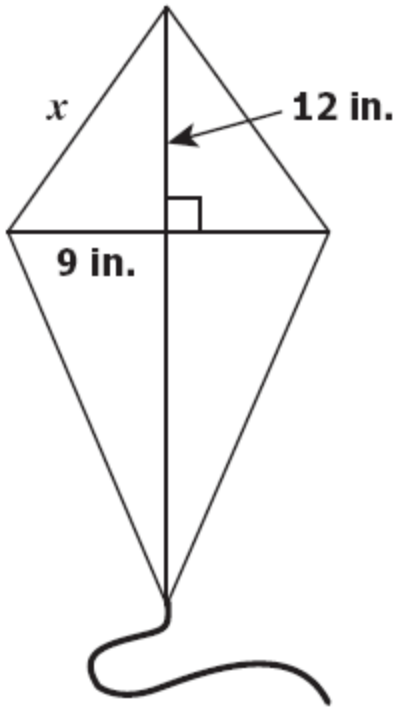
Special case right triangle

side opposite 60 is ½ hyp √3

½ (10) √3

5 √3

19 A drawing of Mark's kite is shown below.



What is the length of the short section of the outer frame indicated by x in the drawing?

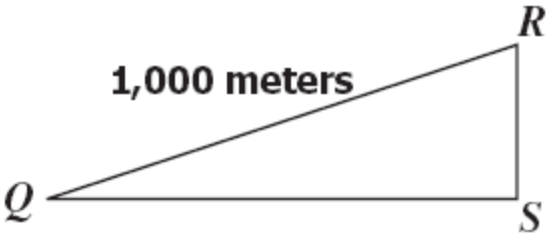
A 16 in.
B 15 in.
C 14 in.
D 13 in.

SSM:
- $x > 12$
- Pythagorean triple $(3 - 4 - 5) \times 3$

Pythagorean Theorem:

$$9^2 + 12^2 = x^2$$
$$81 + 144 = x^2$$
$$225 = x^2$$
$$15 = x$$

23 Given: $\triangle QRS$ where $m\angle Q = 20°$ and $m\angle S = 90°$



What is the length, to the nearest meter, of $\overline{RS}$?

A 342 m
B 364 m
C 500 m
D 940 m

SSM:
- **RS is smallest side and less than ½ hypotenuse**
- **answers A or B**

Trigonometry problem: **SOH CAH TOA**

label the sides of the triangle:

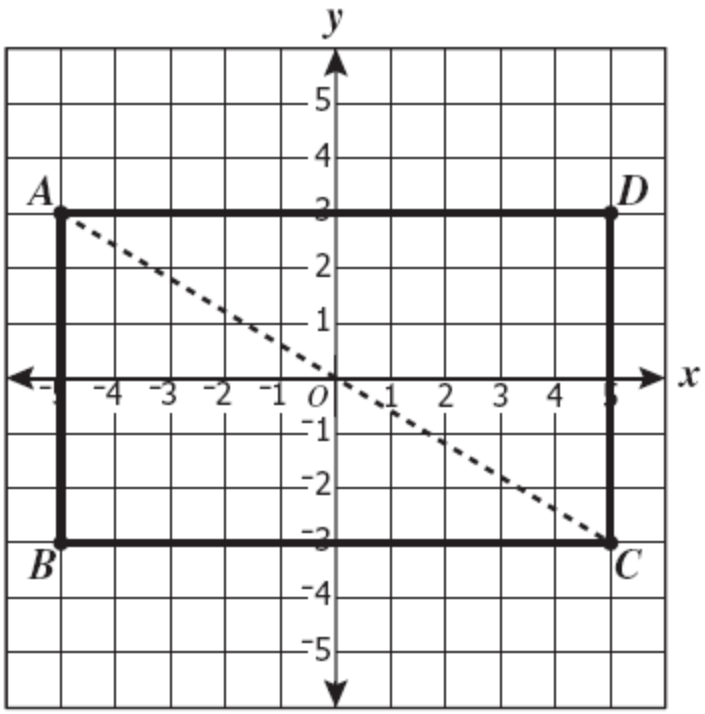
QR (1000) is hyp
RS (x) is opp
QS is adj

must use sin **sin 20° = x / 1000**

1000 (sin 20°) = x

342 = x

41 Rectangle $ABCD$ is placed on a grid as shown.



Which is *closest* to the length of diagonal $\overline{AC}$?

A 8.0
B 10.0
C 11.3
D 11.7

SSM:
- measure AC
- use graph to estimate

Pythagorean Theorem

$6^2 + 10^2 = AC^2$

$36 + 100 = AC^2$

$136 = AC^2$

$11.67 = AC$

or

Distance formula

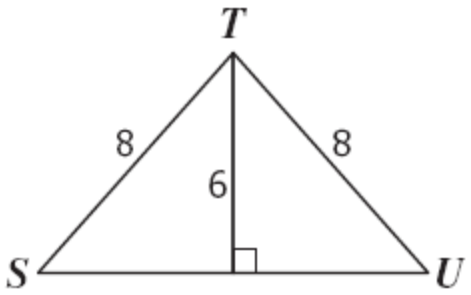
$\sqrt{(-5 - 5)^2 + (3 - (-3))^2}$

$\sqrt{(-10)^2 + (6)^2}$

$\sqrt{(100 + 36)}$

$\sqrt{136} = 11.67$

21



What is the length of $\overline{SU}$?

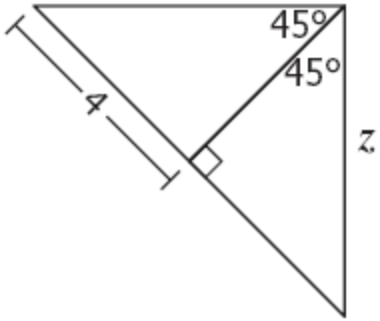
A $2\sqrt{7}$ cm
B 7 cm
C $4\sqrt{7}$ cm
D 20 cm

SSM:
- **SU is bigger than 8 eliminates A and B**

Pythagorean Theorem:

$6^2 + x^2 = 8^2$
$36 + x^2 = 64$
$x^2 = 28$
$x = 2\sqrt{7}$ **need to double it for SU**

22



What is the value of z ?

F $2\sqrt{2}$

G $2\sqrt{3}$

H $4\sqrt{2}$

J $8\sqrt{2}$

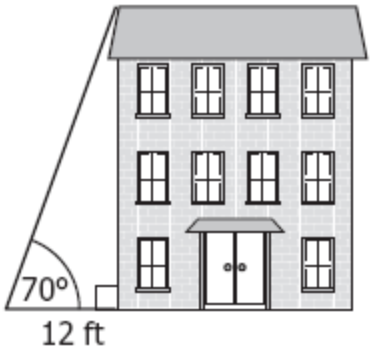
SSM:

- **Measure:**
  z is bigger than 4
  eliminates F and G
- **but less than 8, which**
  eliminates J

Pythagorean Theorem:

$$4^2 + 4^2 = z^2$$
$$16 + 16 = z^2$$
$$32 = z^2$$
$$4\sqrt{2} = z$$

23 From a point 12 feet from the base of a building, the angle of elevation from the ground to the top of the building is 70°.



$\sin 70° \approx 0.940$
$\cos 70° \approx 0.342$
$\tan 70° \approx 2.75$

Which is *closest* to the height of the building?

A 24 ft
B 33 ft
C 35 ft
D 41 ft

SSM:

- Use 12 as the measure and estimate the height of the building
- Less than 3 times, but more than 2; eliminates A and D

Trigonometry problem:

label the sides of the triangle:

12 is adj
h is opp
diagonal is hyp

must use tan

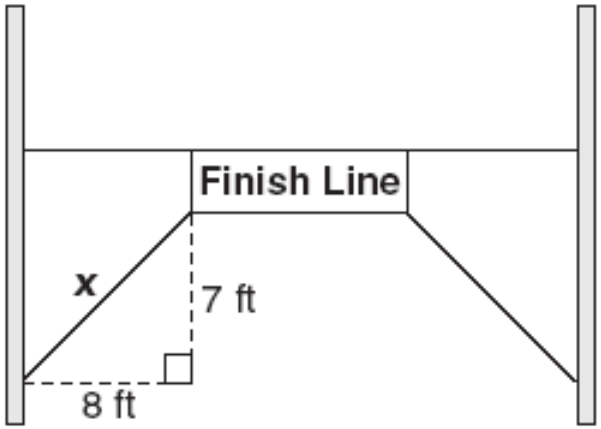
SOH CAH TOA

$\tan 70° = x / 12$

$12 (\tan 70°) = x$

$33 = x$

21 To mark the end of a race, a finish-line banner is stretched across the road as shown in the drawing.



SSM:

- measure 7 with scrap paper
- measure 8 with scrap paper
- If to scale (or close) then measure x with scrap paper
- Estimate answer

Which is closest to the length of the support rope designated by *x* in the drawing?

A 9.5 ft
B 10.6 ft
C 12.0 ft
D 15.0 ft

Pythagorean Theorem:

$7^2 + 8^2 = x^2$

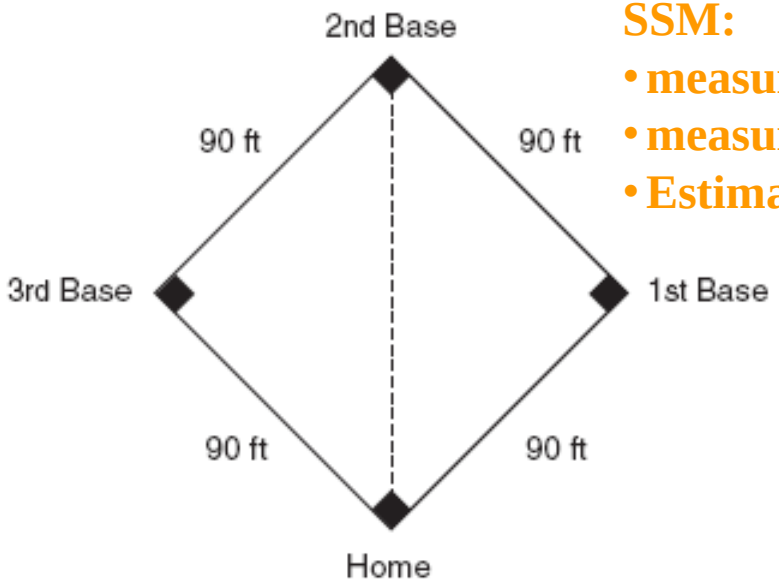
$49 + 64 = x^2$

$113 = x^2$

$10.6 = x$

$(10 < x < 11)$ without a calculator

22 **A baseball diamond is in the shape of a square, 90 feet on a side.**



SSM:
- **measure 90 with scrap paper**
- **measure H-2B with scrap paper**
- **Estimate answer**

What is the direct distance from home plate to second base?

F 90 ft

G $90\sqrt{2}$ ft

H $90\sqrt{3}$ ft

J 180 ft

Pythagorean Theorem:

$90^2 + 90^2 = x^2$

$8100 + 8100 = x^2$

$16200 = x^2$

$90\sqrt{2} = x$

Special Case Right Δs

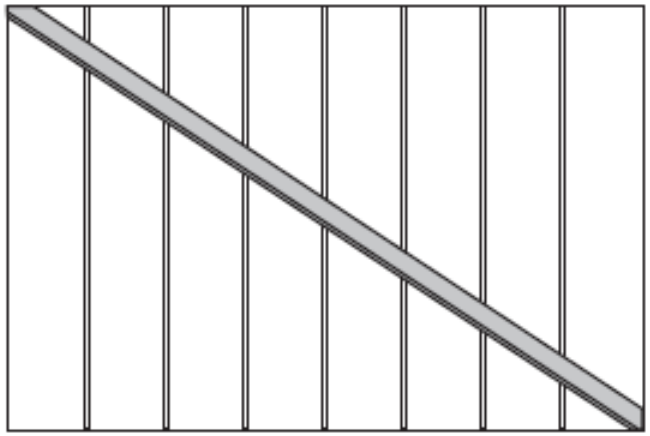
$90 = \frac{1}{2}\ hyp\ \sqrt{2}$

$180 = hyp\ \sqrt{2}$

$180 / \sqrt{2} = hyp$

$90\sqrt{2} = hyp$

23



SSM:

- measure 9 with scrap paper
- measure 12 with scrap paper
- If seems to scale (or close), then measure diagonal
- Estimate answer

What is the length of a diagonal brace that could be used for a wall 9 feet high and 12 feet long?

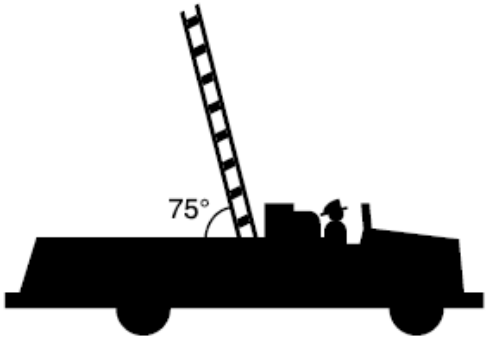
A 12 ft
B 13 ft
C 14 ft
D 15 ft

Pythagorean Theorem:

$9^2 + 12^2 = x^2$
$81 + 144 = x^2$
$225 = x^2$
$15 = x$

Pythagorean Triple!
(3-4-5) × 3

21



A fire truck has a ladder that can extend to 60 feet in length. The ladder can be safely raised to a maximum angle of 75° with the horizontal. Disregarding the height of the fire truck itself, which is closest to the maximum height that the ladder can safely reach?

$\sin 75° \approx 0.966$
$\cos 75° \approx 0.259$
$\tan 75° \approx 3.73$

A 15.53 ft

B 57.96 ft

C 60.00 ft

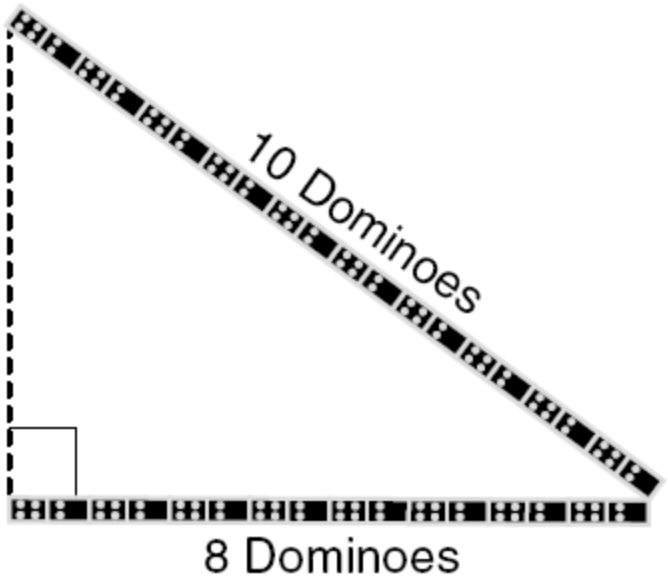
D 62.12 ft

SSM:

- **a 60 foot ladder, raised at a 75° angle will be close to, but not equal (or greater than) 60 feet**
- **only answer B is close**

height = 60 sin 75° = 57.96

22 **Scotty is making a train of dominoes on the floor.**



How many dominoes are needed to complete the triangle?

F 6
G 12
H 18
J 36

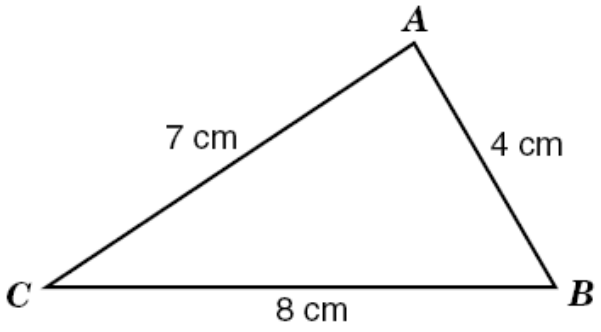
SSM:
- **Side has to be less than 10**
- **only answer F qualifies**

Pythagorean Theorem:

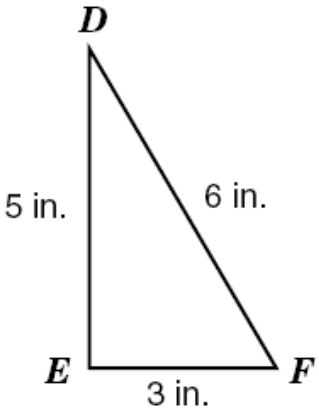
$8^2 + x^2 = 10^2$
$64 + x^2 = 100$
$x^2 = 36$
$x = 6$

23 **Using the measures shown, which triangle must be a right triangle?**

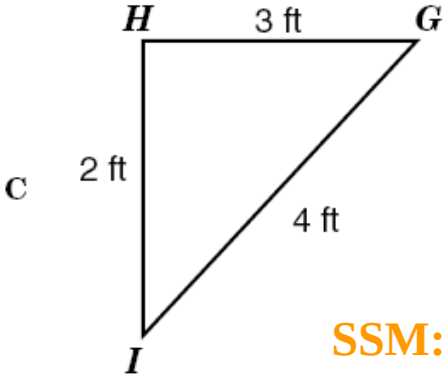
A



B



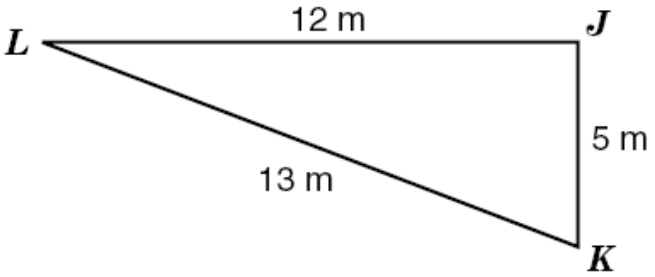
C



SSM:

- **3-4-5 and**
- **5-12-13 Pythagorean Triple**

D

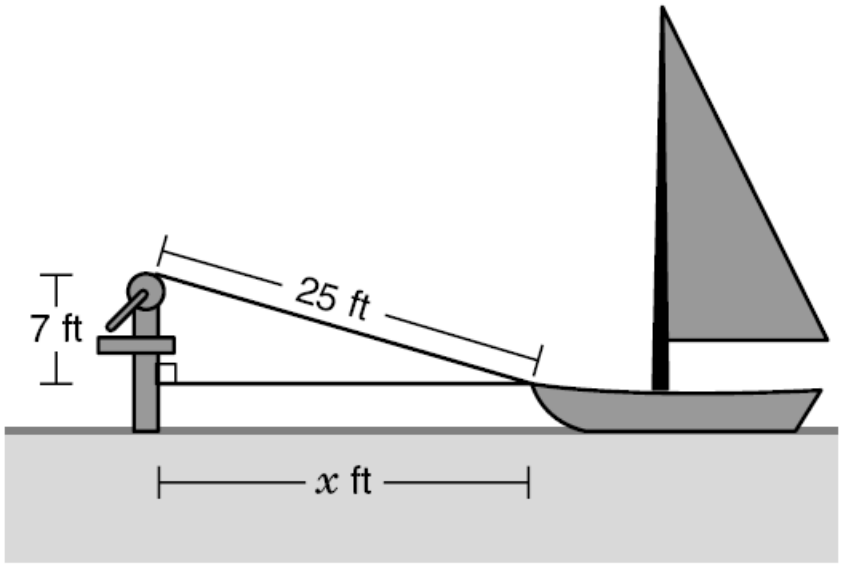


Pythagorean Theorem:

$a^2 + b^2 = c^2$

Check each one

21 **A windlass is used to pull a boat to the dock. The rope is attached to the boat at a point 7 feet below the level of the windlass.**



What is the distance from the boat to the dock when the rope is 25 feet?

A 25 ft
B 24 ft
C 18 ft
D 7 ft

SSM:

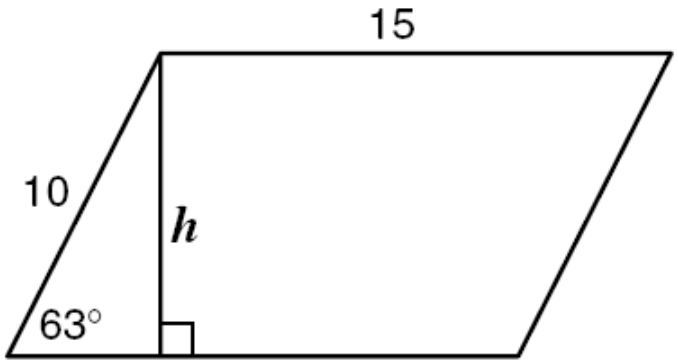
- **measure 7 with scrap paper**
- **estimate x**

- **by sight x > 7 and less than 25 eliminates A and D**

Pythagorean Theorem:

$$7^2 + x^2 = 25^2$$
$$49 + x^2 = 625$$
$$x^2 = 576$$
$$x = 24$$

22 **The parallelogram has the measurements shown.**



Which is closest to the length of the altitude, *h*?

F 19.63
G 8.91
H 8.67
J 6.81

SSM:
- **measure 10 with scrap paper**
- **then measure h with scrap paper**
- **Estimate answer B or C**

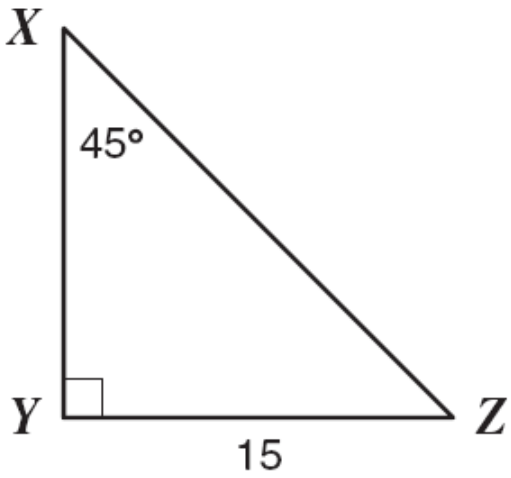
Trigonometry:

h is opp of 63° so use sin

sin 63 = h / 10

10 sin 63 = h = 8.91

23



For the triangle represented by the above drawing, what is the length of $\overline{XZ}$?

A $7.5\sqrt{2}$

B $7.5\sqrt{3}$

C $15\sqrt{2}$

D $15\sqrt{3}$

SSM:

- **measure 15 with scrap paper**
- **estimate XZ based on that**
- **need decimal answers**

Special Case Right Triangle

side opposite 45 is ½ hyp √2

15 = ½ hyp √2

30 = hyp √2

30 / √2 = 15√2 = hyp

21 The top of a ladder is leaning on a building at a point 12 feet above the ground; the bottom of the ladder is 5 feet from the base of the building. What is the length of the ladder?

A 19 ft
B 17 ft
C 13 ft
D 7 ft

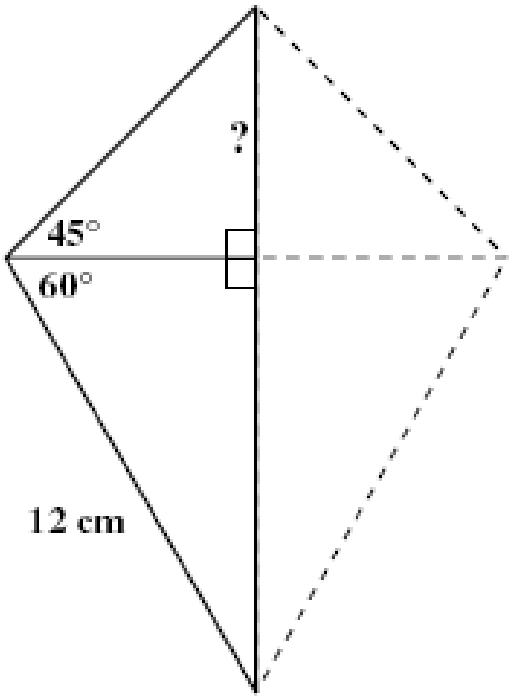
SSM:

- **5-12-13 Pythagorean Triple**

Pythagorean Theorem:

$$5^2 + 12^2 = x^2$$
$$25 + 144 = x^2$$
$$169 = x^2$$
$$13 = x$$

22



A design is formed by joining isosceles right triangles and 60°-30° right triangles as shown in the diagram. If the hypotenuse of the 60°-30° triangle is 12 centimeters, which is *closest* to the length of one leg of the isosceles right triangle?

F 6 cm
G 7.2 cm
H 8.5 cm
J 10.4 cm

SSM:

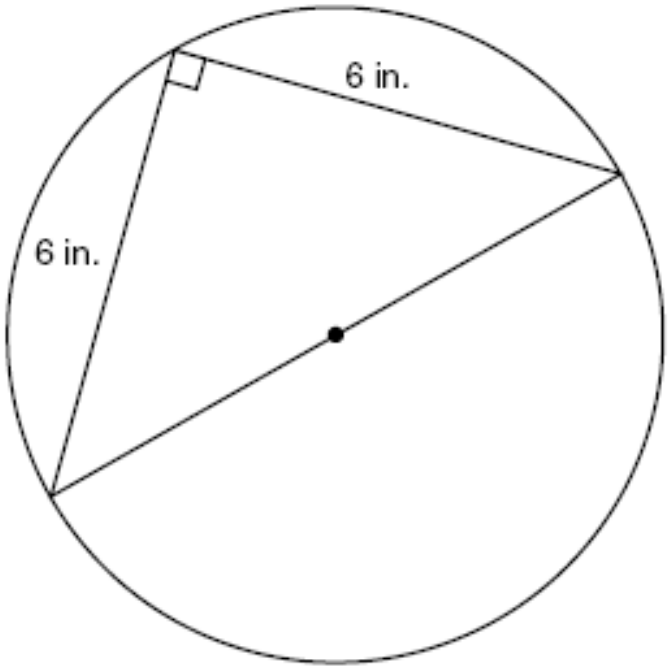
- measure 12 with scrap paper
- measure ? with scrap paper
- Estimate answer

Special Case Right Triangles

side opposite 30° angle = ½ hyp
so side bordering 45 and 60 is 6

? side is in a 45-45-90 right isosceles triangle so it must be equal to 6

23



What is the diameter of the circle shown?

A $3\sqrt{2}$ in.

B $3\sqrt{3}$ in.

C $6\sqrt{2}$ in.

D $6\sqrt{3}$ in.

SSM:

- **Use 6 as a scaling reference**
- **diameter is over 6 but less than 9**

Use Pythagorean theorem:

$6^2 + 6^2 = 36 + 36 = 72 = d^2$

$6\sqrt{2} = d$

45 The distance between the points $(-2, -4)$ and $(3, 8)$ is —

A $\sqrt{17}$
B 13
C 17
D 169

SSM:
- plot points on graph paper
- measure distance with scratch paper
- use graph paper to estimate distance
- Answers A & D wrong

Pythagorean Theorem

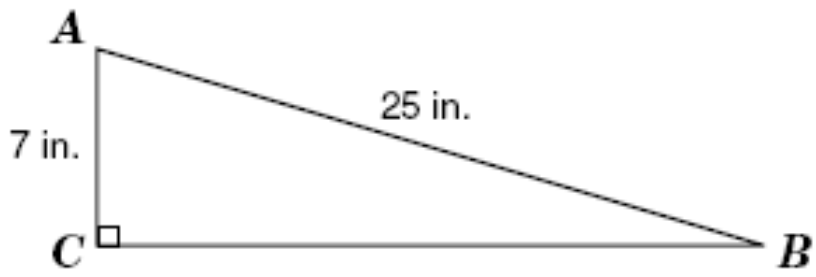
$5^2 + 12^2 = AC^2$
$25 + 144 = AC^2$
$169 = AC^2$
$13 = AC$

or

Distance formula

$\sqrt{(-2 - 3)^2 + (-4 - 8)^2}$
$\sqrt{(-5)^2 + (-12)^2}$
$\sqrt{(25 + 144)}$
$\sqrt{169} = 13$

17 Triangle ABC is a right triangle with the measures shown.



The length of $\overline{BC}$ is —

A 18 in.
B 24 in.
C 32 in.
D 576 in.

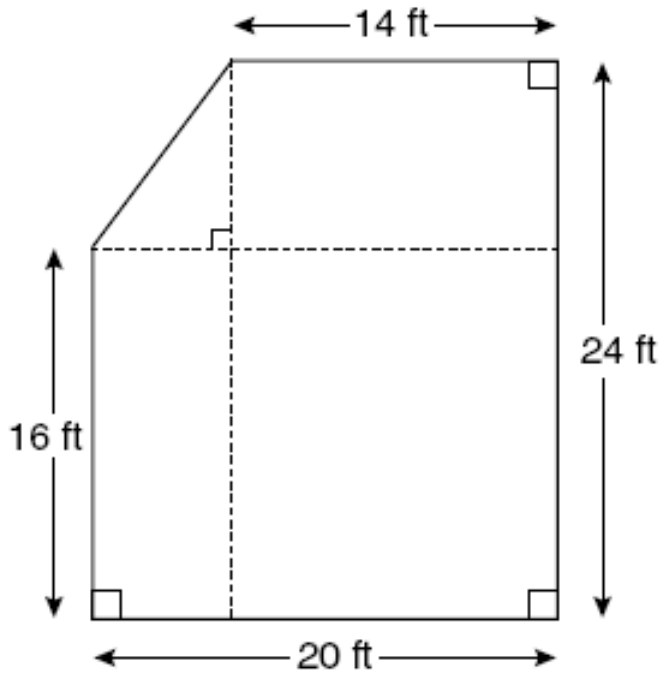
SSM:

- **answers have to be less than 25**
  eliminates C and D

Pythagorean Theorem:

$$7^2 + x^2 = 25^2$$
$$49 + x^2 = 625$$
$$x^2 = 576$$
$$x = 24$$

18 A customer provided this diagram of a patio to a fencing company.



What is the length of the unlabeled side?

F 10 ft
G 11 ft
H 12 ft
J 13 ft

SSM:

- **Measure:**
  ***14* side and compare with unlabeled side**

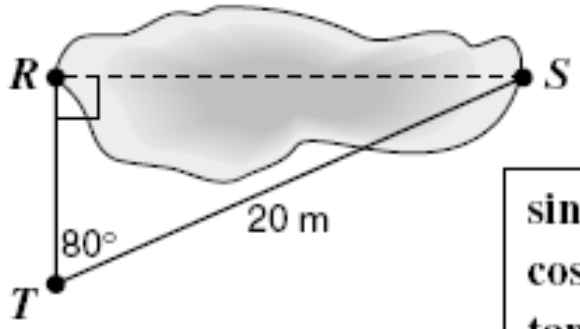
Figure out side lengths:

20 – 14 = 6
24 – 16 = 8

Pythagorean Theorem:

$6^2 + 8^2 = z^2$
$36 + 64 = z^2$
$100 = z^2$
$10 = z$

22 To determine the distance across a pond, Harry made the measurements shown in the diagram.



$\sin 80° \approx 0.985$
$\cos 80° \approx 0.174$
$\tan 80° \approx 5.67$

Which is *closest* to the distance from R to S?

F 3.48 m
G 19.7 m
H 20.3 m
J 113.4 m

SSM:
- has to be less than 20 eliminates H and J
- big angle → big side

Trigonometry problem: SOH CAH TOA

label the sides of the triangle:
20 is hyp
p is opp

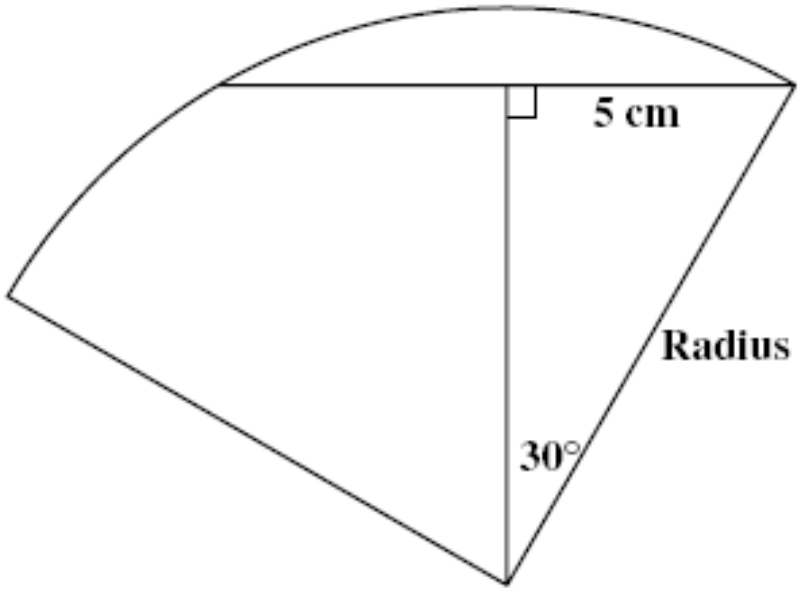
must use sin

$\sin 80° = p / 20$

$20 (\sin 70°) = p$

$19.70 = p$

20



SSM:

- **Use 5 as a scale to measure the Radius about twice as big**

The drawing shows the measurements in a section of a circular design. How long is the radius of the circle?

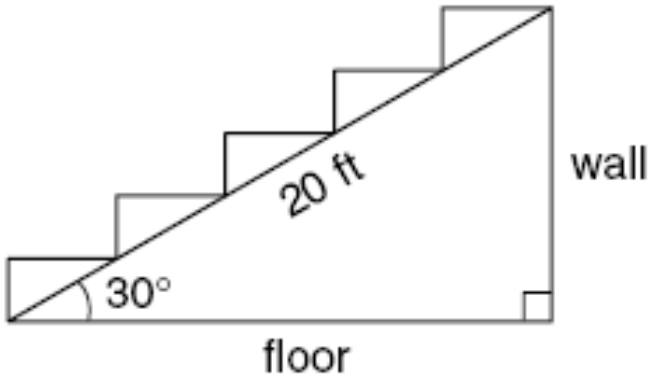
F 10 cm
G 8.7 cm
H 7 cm
J 4.3 cm

Special case Right Triangle:

side opposite 30° angle → ½ hypotenuse

5 = ½ R 10 = R

21 A carpenter is building a flight of stairs as pictured in the drawing.



What is the horizontal distance from the foot of the stairs to the wall?

A 14.1 ft
B 17.3 ft
C 20.0 ft
D 28.3 ft

SSM:

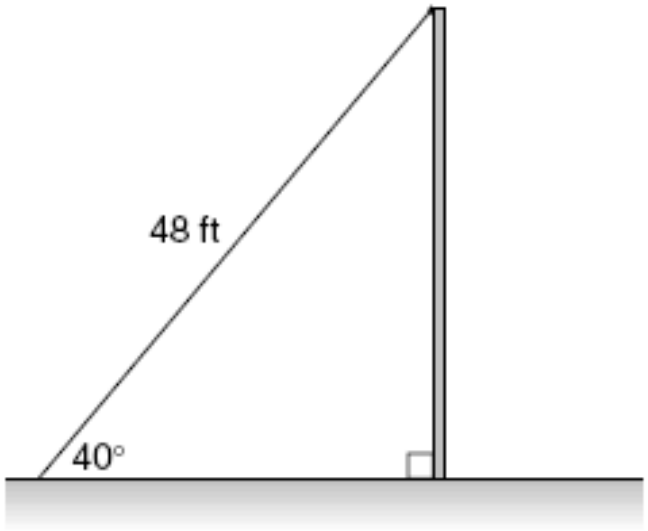
- **Use 20 as a scale to measure the floor**
  slightly less than 20
  eliminates C and D

Special case Right Triangle:

side opposite 60° angle → ½ hypotenuse $\sqrt{3}$

floor = ½ (20)$\sqrt{3}$ = 10 $\sqrt{3}$ = 17.32

23



A cable 48 feet long stretches from the top of a pole to the ground. If the cable forms a 40° angle with the ground, which is closest to the height of the pole?

$\sin 40° \approx 0.642$
$\cos 40° \approx 0.766$
$\tan 40° \approx 0.839$

A 26.4 ft
B 30.9 ft
C 36.8 ft
D 40.3 ft

SSM:

- **Use 48 as the measure and estimate the height of the pole**

Trigonometry problem: **SOH CAH TOA**

label the sides of the triangle:

48 is hyp

h is opp

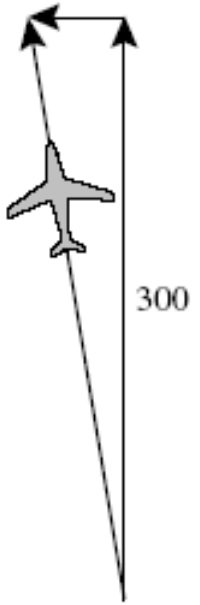
must use sin

$\sin 40° = h / 48$

$48 (\sin 40°) = h$

$30.85 = h$

45 An airplane is headed due north at 300 nautical miles per hour (knots) as shown in the drawing. The wind is directly from the east at 50 knots.



SSM:

- **has to be bigger than 300 eliminates C and D**

Pythagorean Theorem:

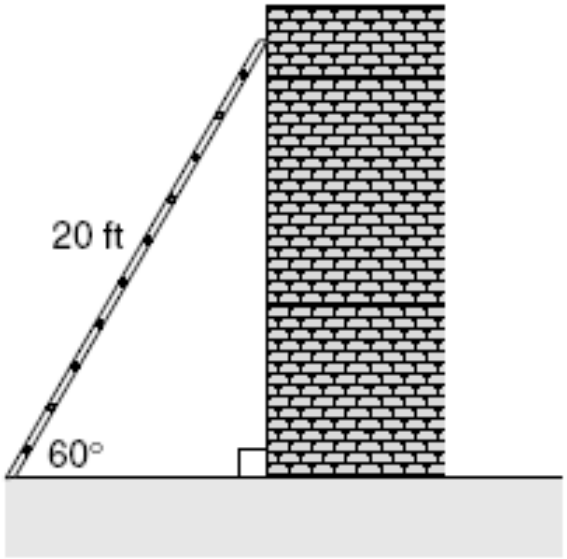
$$50^2 + 300^2 = RS^2$$
$$2500 + 90000 = RS^2$$
$$92500 = RS^2$$
$$304.14 = RS$$

Which is closest to the resultant speed of the airplane?

A 314 knots
B 304 knots
C 296 knots
D 286 knots

No longer on Geometry SOLs

20



SSM:

- **Measure the distance from the ladder to the wall**
- **compare it with 20 its about ½**

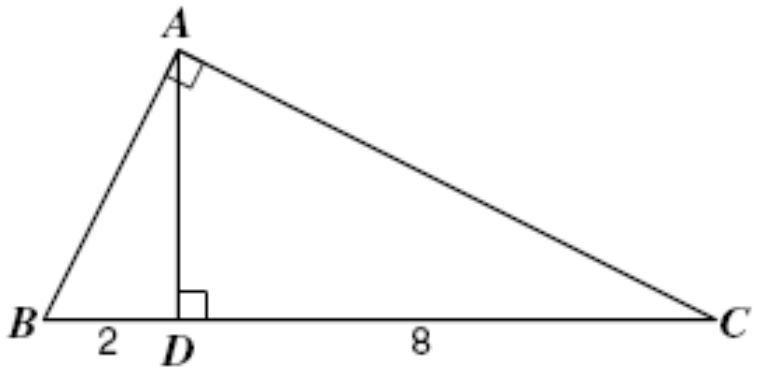
A 20-foot ladder leaning against a building makes an angle of 60° with the ground. How far from the base of the building is the foot of the ladder?

F 5 ft
G 8.2 ft
H 10 ft
J 17.3 ft

Special case right triangles:

the floor piece is opposite a 30° angle
side opposite 30° → ½ hyp
½ (20) = 10

21



SSM:

- **Since the hypotenuse is the largest side of a triangle AC > 8 but AC < 10**
- **only answer C fits**

In the figure, ΔABC is a right triangle. AD is perpendicular to BC, and the measure of $BD = 2$ meters and $DC = 8$ meters. What is the measure of $\overline{AC}$?

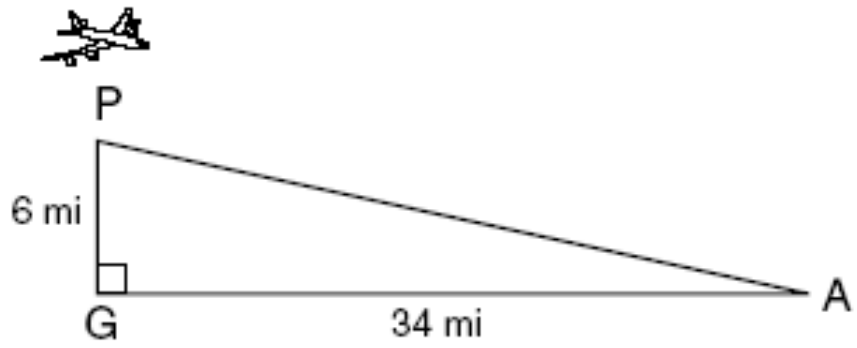
A 2.8 m
B 4.5 m
C 8.9 m
D 10.0 m

First: find AD → AD is an altitude from right angle to hypotenuse
AD = geometric mean of divided hypotenuse
$AD = \sqrt{2 \times 8} = \sqrt{16} = 4$

Second: Use Pythagorean Theorem:

$$4^2 + 8^2 = AC^2$$
$$16 + 64 = AC^2$$
$$80 = AC^2$$
$$8.94 = AC$$

22



An airplane is 34 ground miles from the end of the runway *(GA)* and 6 miles high *(PG)* when it begins its approach to the airport. To the nearest mile, what is the distance *(PA)* from the airplane to the end of the runway?

F 41 mi
G 39 mi
H 37 mi
J 35 mi

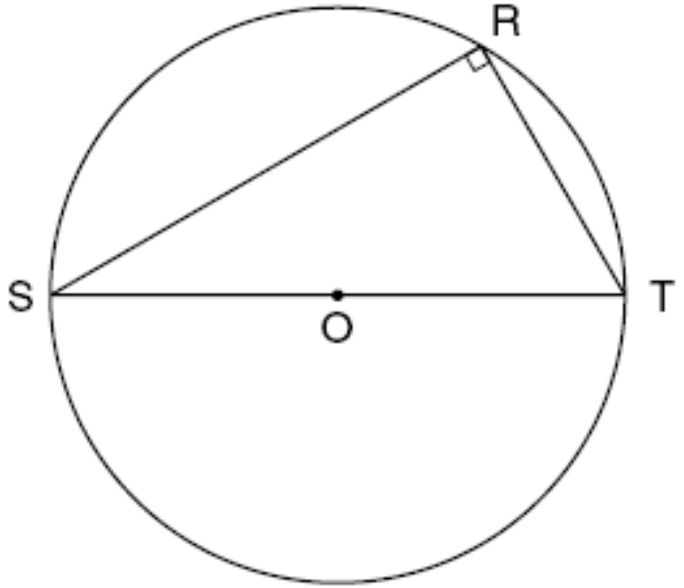
SSM:

- **answer F does not make a triangle, so its wrong**

Use Pythagorean Theorem:

$$34^2 + 6^2 = PA^2$$
$$1156 + 36 = PA^2$$
$$1192 = PA^2$$
$$34.53 = PA$$

23



In circle O, $\angle RST$ formed by chord $\overline{RS}$ and diameter $\overline{ST}$ has a measure of 30°. If the diameter is 12 centimeters, what is the length of chord $\overline{SR}$?

A $12\sqrt{3}$ cm

B $12\sqrt{2}$ cm

C $6\sqrt{3}$ cm

D $6\sqrt{2}$ cm

SSM:

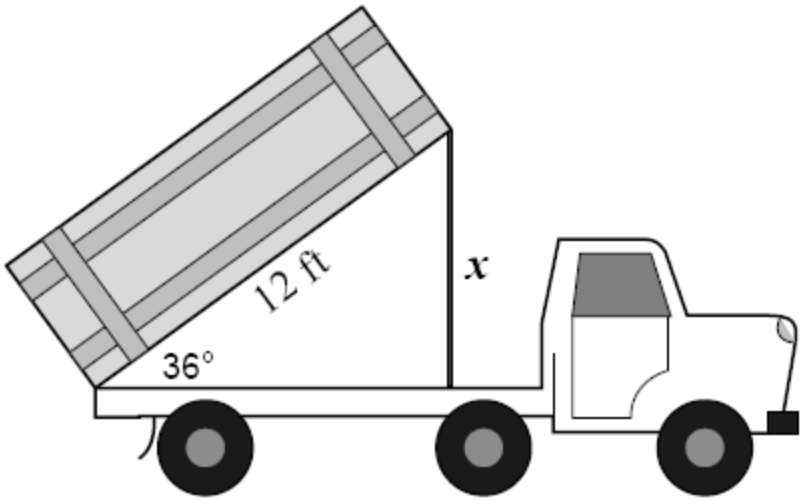
- **Use ST as a reference**
- **SR < ST or SR < 12 eliminates A and B**

Special case right triangle:

side opposite the 60° angle is ½ hyp √3

diameter is hypotenuse so SR = ½ (12)√3 = 6√3

20



The 12-foot bed of a dump truck loaded with heavy stone must rise to an angle of 36° before the stone will spill out. Approximately how high must the front of the bed rise (x) to unload?

$\sin 36° \approx 0.588$ $\cos 36° \approx 0.810$
$\tan 36° \approx 0.727$

F 6 ft
G 7 ft
H 9 ft
J 10 ft

SSM:

- **Use 12 as the measure and estimate the height of the dump bed**

Trigonometry problem: **SOH CAH TOA**

label the sides of the triangle:

12 is hyp

x is opp

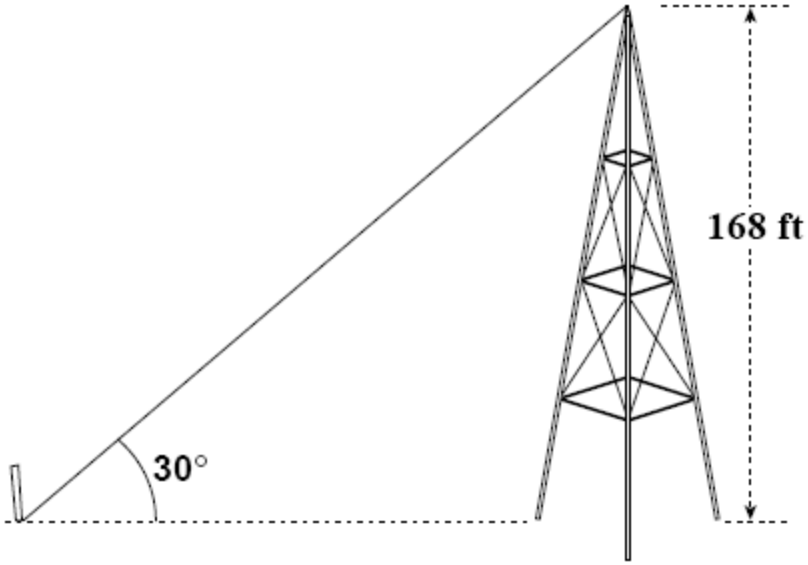
must use sin

$\sin 36° = x / 12$

$12 (\sin 36°) = x$

$7 = x$

21



The angle from a point on the ground to the top of a 168-foot tower is 30°. About how long is a wire that reaches from the top of the tower to the point on the ground?

$\sin 30° \approx 0.5 \quad \cos 30° \approx 0.8666$
$\tan 30° \approx 0.577$

A 146 ft
B 194 ft
C 291 ft
D 336 ft

SSM:
- not to scale (angle too big)
- hyp > 168 (eliminates A)

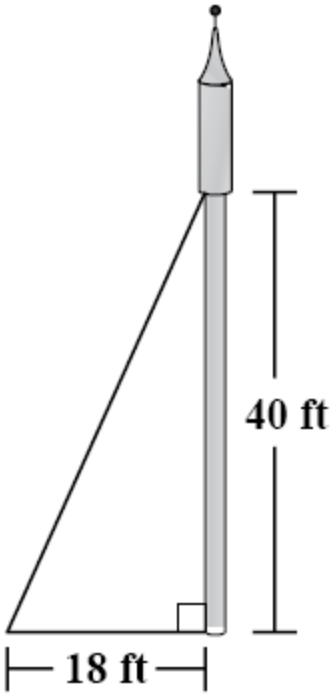
Trigonometry problem: SOH CAH TOA

label the sides of the triangle:
- x is hyp
- 168 is opp
- ground is adj

must use sin

$\sin 30° = 168 / x$
$0.5 = 168 / x$
$336 = x$

22 From a point 18 feet from the base of a tower, a wire is stretched to an attachment 40 feet up the tower.



To the *nearest* foot, how long is the wire?

F 58 ft
G 44 ft
H 36 ft
J 29 ft

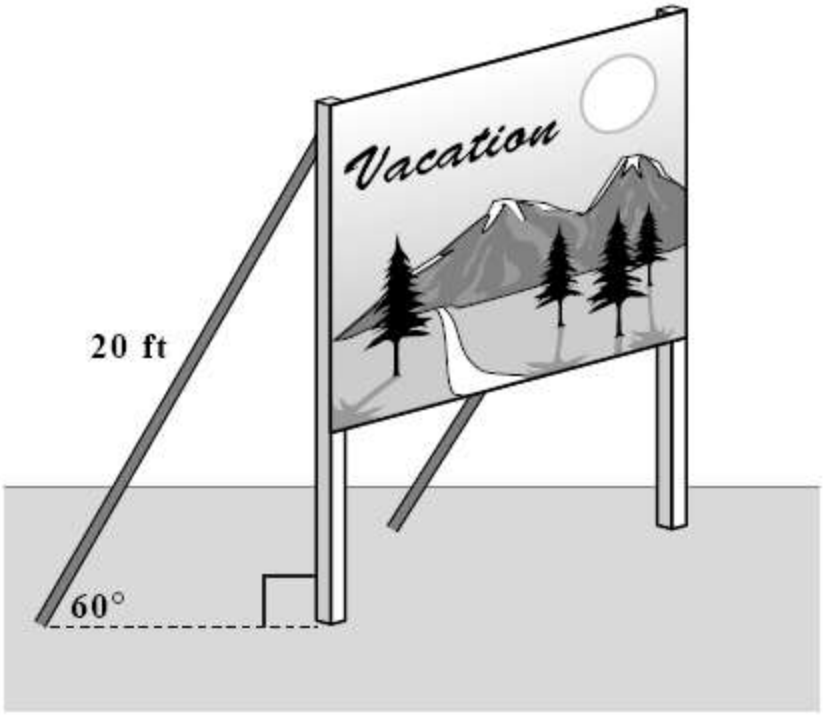
SSM:

- measure 40 with scrap paper
- measure 18 with scrap paper
- If to scale (or close) then measure hyp with scrap paper
- Estimate answer
- Must be > 40

Pythagorean Theorem:

$$40^2 + 18^2 = x^2$$
$$1600 + 324 = x^2$$
$$1924 = x^2$$
$$43.86 = x$$

23



A billboard is supported by 20-foot lengths of tubing at an angle of 60°. How far from the base of the billboard is the bottom end of the brace?

A 5 ft
B 8.7 ft
C 10 ft
D 17.3 ft

SSM:

- Measure how far away the brace is from the billboard
- Use that against the 20 ft brace to estimate the length
- It goes about twice

Special Case Right Triangle:

side opposite 30° → ½ hyp

½ (20) = 10