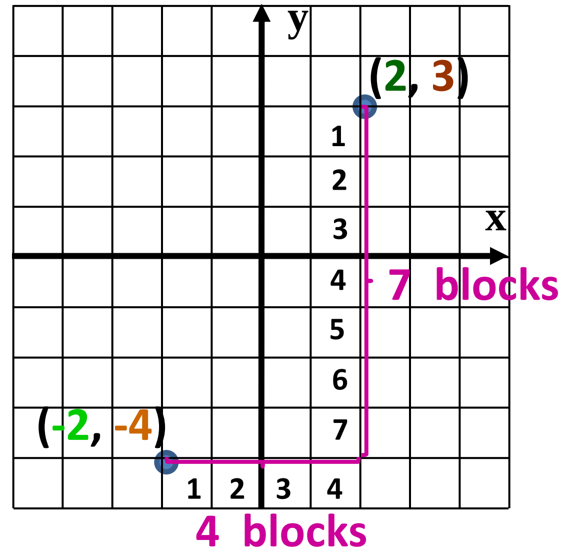
Distance between two points with coordinates

Example: (2, 3) and (-2, -4)



Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Solution using formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{((-2) - 2)^2 + ((-4) - 3)^2}$$

$$d = \sqrt{(-4)^2 + (-7)^2} \quad = \sqrt{16 + 49} \quad = \sqrt{65} \quad = 8.09$$

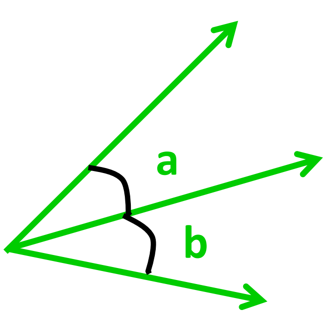
Solution using Pythagorean Theorem: $a^2 + b^2 = c^2$

$4^2 + 7^2 = c^2$ (left or right blocks)^2 + (up or down blocks)^2

$16 + 49 = c^2$

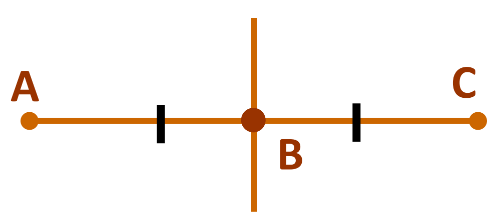
$65 = c^2$ $\quad \sqrt{65} = \sqrt{c^2}$ $\quad \sqrt{65} = 8.09 = c$

Bisector cuts into two congruent (equal) parts



$m\angle a = m\angle b$

$\angle a \cong \angle b$



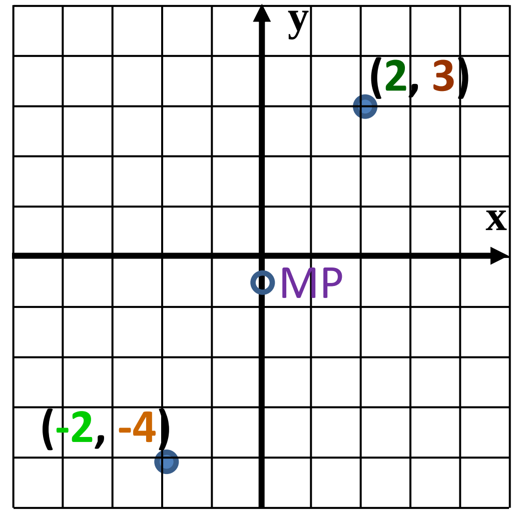
AB = BC

$AB \cong BC$

B is a midpoint!!

Midpoint a point <u>halfway</u> between two coordinate points

Example: Given two endpoints
(2, 3) and (-2, -4)



Formula: $$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

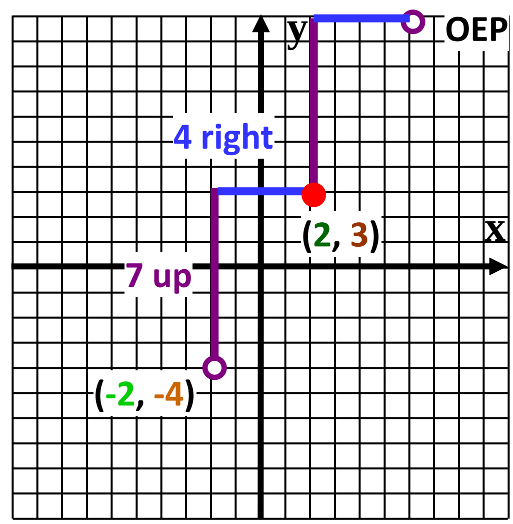
Solution using formula:

$$\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right)$$

$$\left(\frac{0}{2}, \frac{-1}{2}\right) = \left(0, \frac{-1}{2}\right)$$

Example: Given An endpoint and a midpoint (Travel Problem)

MidPoint (2, 3)
and (-2, -4) End point



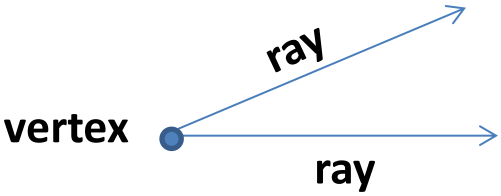
Graphing Solution using triangles:
Midpoint is 7 up and 4 right from the endpoint. Move 7 up and 4 right from midpoint to get to the other endpoint.

Solution using numbers:

MP(2, 3)	MP (2, 3)	MP - midpoint
- EP (-2, -4)	+ Trvl (4, 7)	Trvl – travel distance
Travel (+4, +7)	OEP (6, 10)	OEP – other endpoint

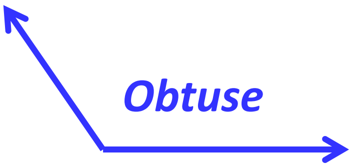
From EP to MP is travel; from MP to OEP is travel

Angles between two rays hinged at a vertex



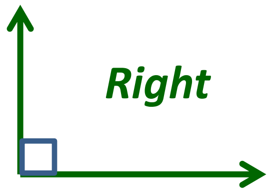
Straight

180° (line)



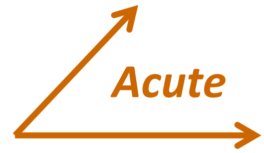
Obtuse

Greater than (>) 90°



Right

Equal (=) 90°

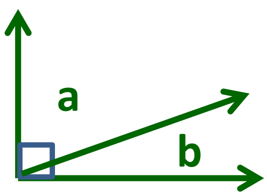


Acute

Less than (<) 90°

Special Angle Pairs

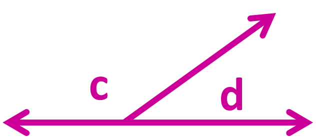
Complementary a pair of angles that sum to 90°



$m\angle a + m\angle b = 90°$

$m\angle a = 90° - m\angle b$ $m\angle b = 90° - m\angle a$

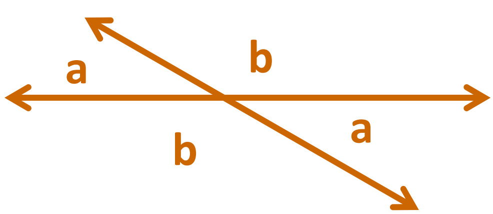
Supplementary a pair of angles that sum to 180°



Linear Pair: $m\angle c + m\angle d = 180°$

$m\angle c = 180° - m\angle d$ $m\angle d = 180° - m\angle c$

Vertical Angles have the same measure (congruent)

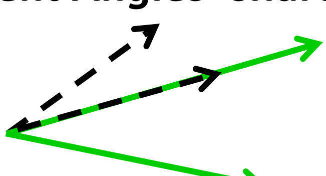


$m\angle a = m\angle a$

$m\angle b = m\angle b$

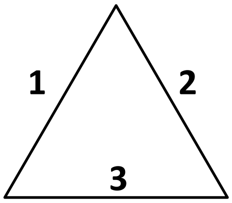
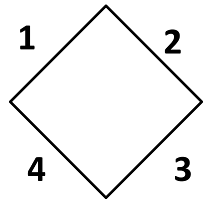
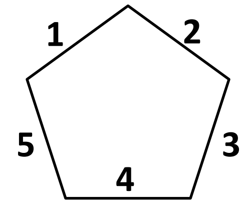
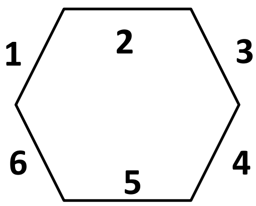
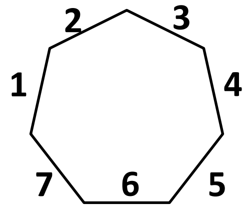
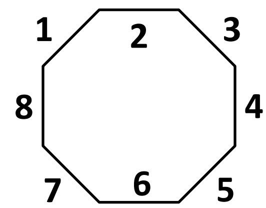
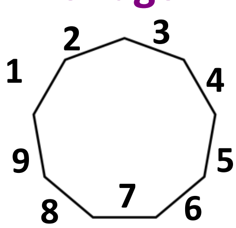
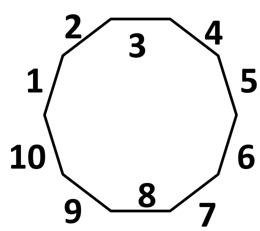
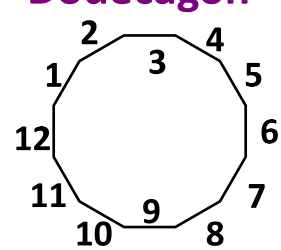
$m\angle a + m\angle b = 180$ (linear pair)

Adjacent Angles share a common side (ray)



Linear Pairs – supplementary adjacent angles

Polygons made up of line segments for sides

Triangle  **3 sides**	**Quadrilateral**  **4 sides**	**Pentagon**  **5 sides**
Hexagon  **6 sides**	**Heptagon**  **7 sides**	**Octagon**  **8 sides**
Nonagon  **9 sides**	**Decagon**  **10 sides**	**Dodecagon**  **12 sides**

All others known as n-gons: 11 – Eleven-gon or 20 – Twenty-gon or 50 – Fifty-gon

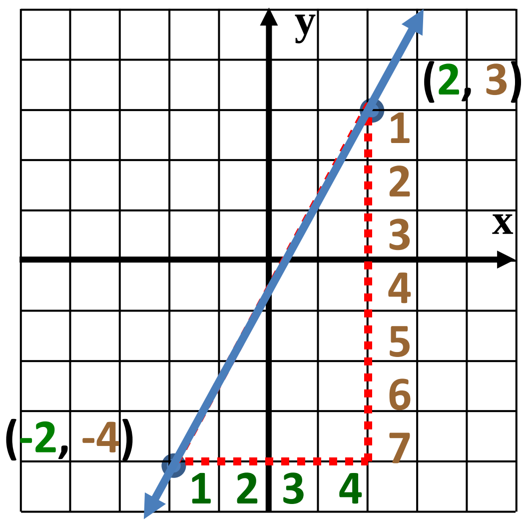
Regular polygons all sides (and angles) are equal

Irregular polygons the sides (or angles) are not all equal

Perimeter is sum (add up) of all sides

Slope – steepness and direction of a line

Formula: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{up\ or\ down}{right\ or\ left}$



From right triangle:

$$m = \frac{blocks\ up\ or\ down}{blocks\ right\ or\ left}$$

$$m = \frac{7}{4}$$

From slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{2-(-2)} = \frac{3+4}{2+2} = \frac{7}{4}$$

Using calculator:

L1	L2
2	3
-2	-4

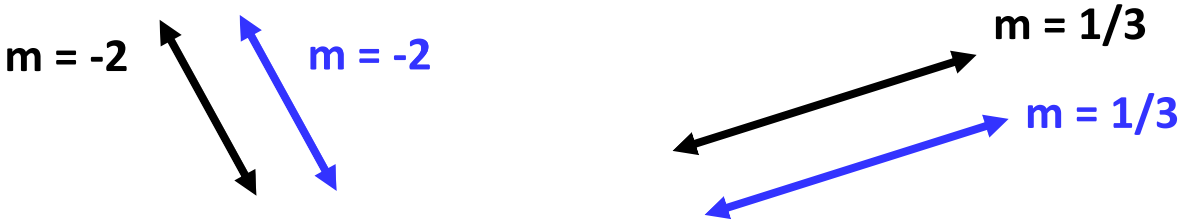
STAT, EDIT
Enter x-data into L1 and y-data into L2
STAT, CALC
Option 4: LinReg **y=ax+b**
Enter 5 times **a=1.75** **a = Slope**
b=-.5 **b = Y-intercept**

If decimal, put decimal in calculator and hit Math Key, enter, enter to convert back to fraction **1.75 -> Frac** **7/4**

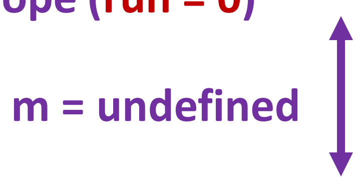
Slope – steepness and direction of a line

Formula: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x} = \dfrac{up\ or\ down}{right\ or\ left}$

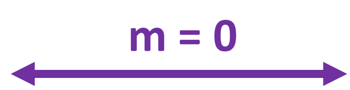
Parallel – same (equal) slopes



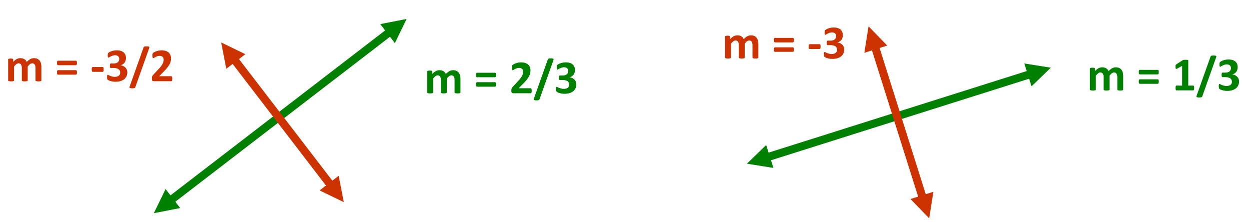
vertical lines (|)have an undefined slope (run = 0)



horizontal lines (—) have no slope (= 0) (rise = 0)



Perpendicular (90°) – negative reciprocals (flip and negate)



Slope – steepness and direction of a line

Graphing Lines

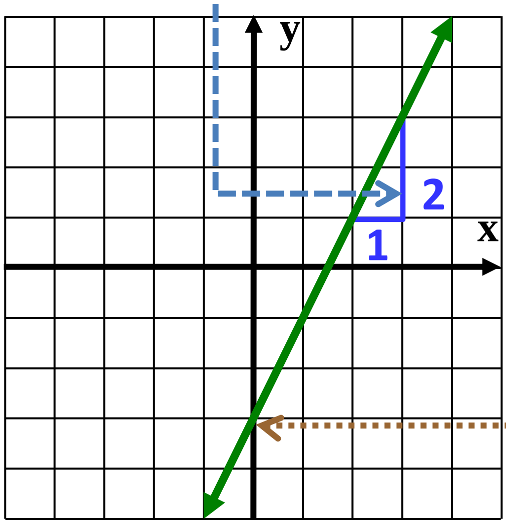
Y-intercept (when x = 0)

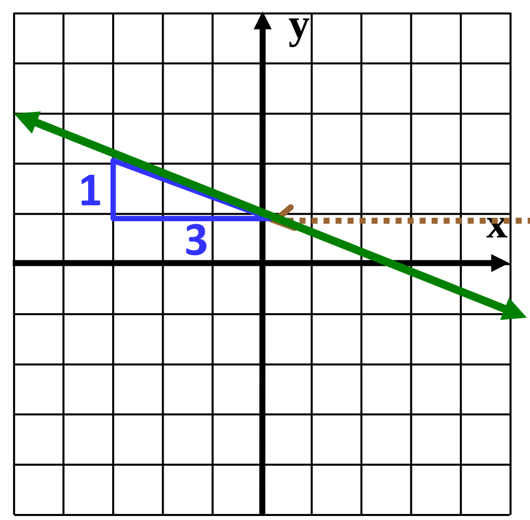
Slope-Intercept Form: $y = mx + b$

Slope

$y = 2x - 3$

$y = (-1/3)x + 1$



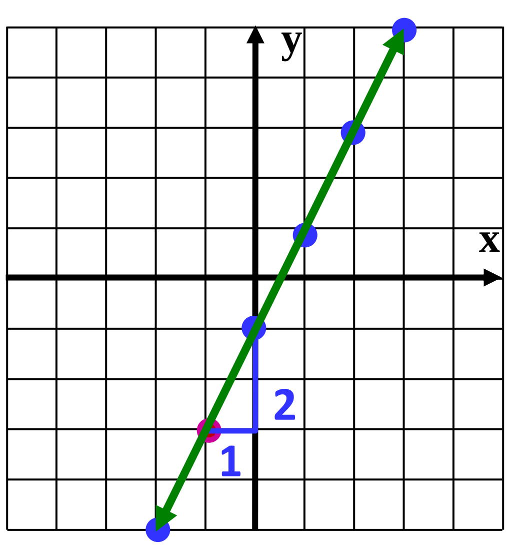


Slope

Point-Slope Form: $y - y_1 = m(x - x_1)$

Point

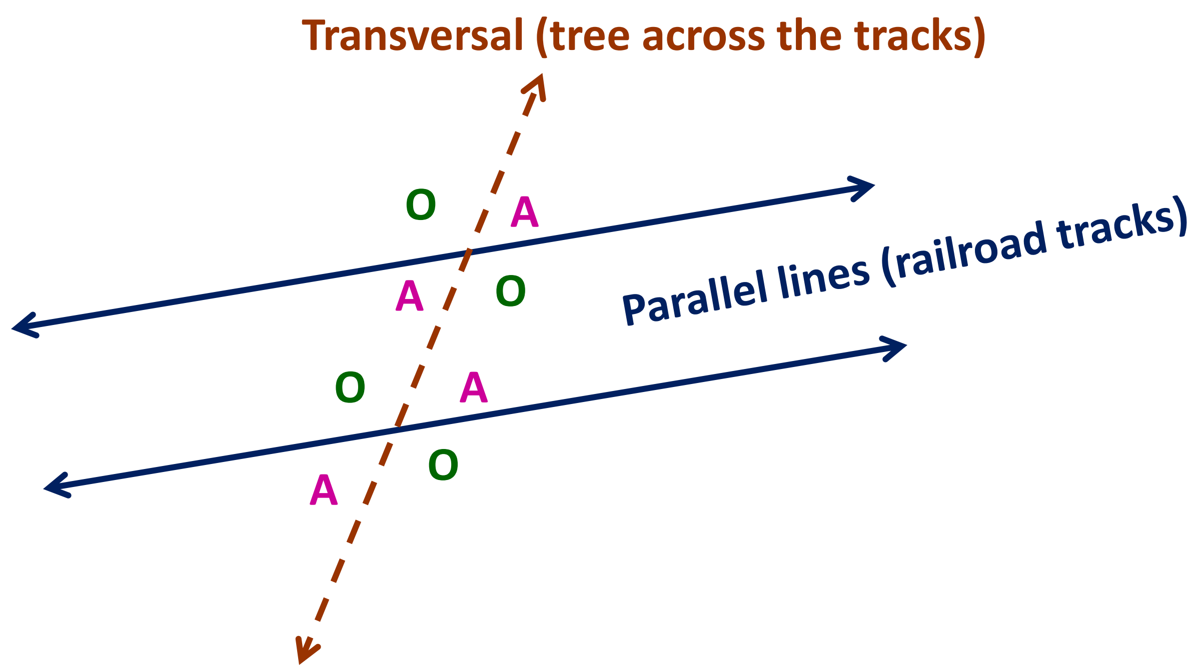
(x_1, y_1)



$Y + 3 = 2(x + 1)$

Point (-1, -3)
Slope: m = 2

Parallel Lines



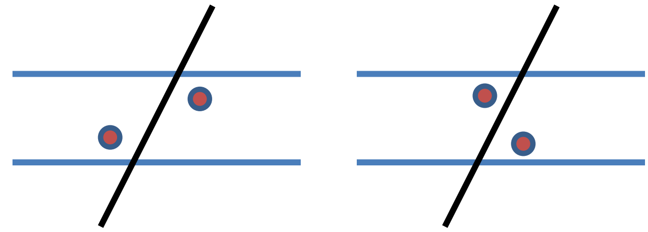
8 angles: 4 congruent acute (little angles)
and 4 congruent obtuse (big angles)
Or 8 right angles

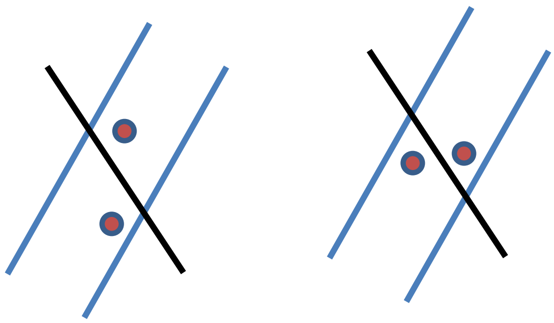
Pairs of A's and O's across the "x" are vertical angles (=)
Big = Big or Little = Little
One of each that form a "y" are a linear pair (o + a = 180)
Big + Little = 180
Vertical angles and Linear pair angles only involve ***two*** lines!!!

Special Angle Pairs with Parallel Lines (3 lines)

Alternate Interior Angles (A- opposite side) Congruent (≅)

Inside railroad tracks
Opposite sides of tree

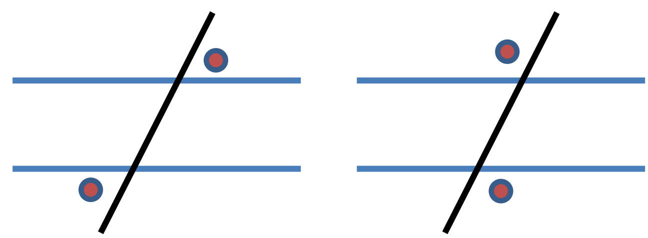


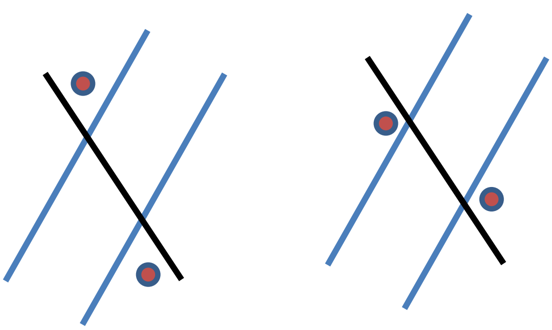


Angle = Angle

Alternate Exterior Angles (A- opposite side) Congruent (≅)

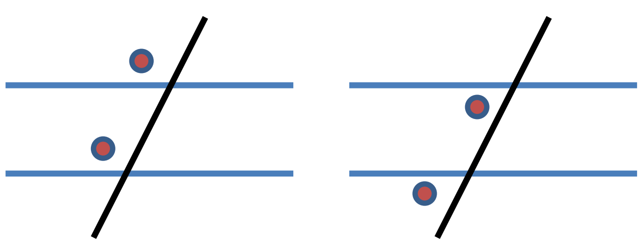
Outside railroad tracks
Opposite sides of tree





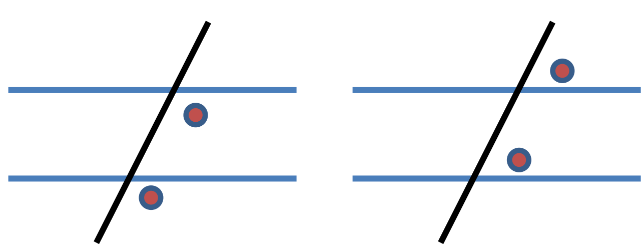
Angle = Angle

Corresponding Angles (C - same side) Congruent (≅)



Angle = Angle

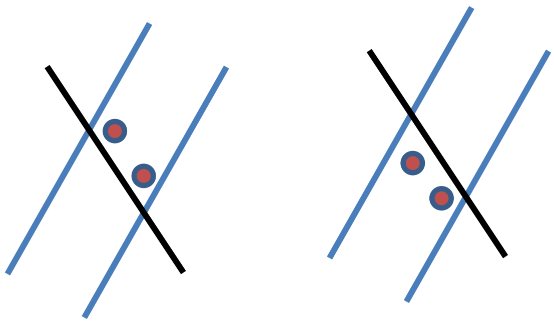
Same location
Upper right
Lower right
Upper left
Lower left

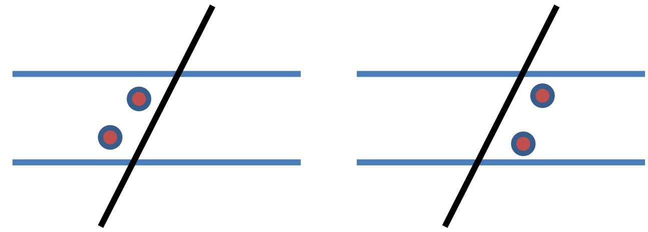


Special Angle Pairs with Parallel Lines (3 lines)

Consecutive Interior Angles (C – same side) Supplementary

Inside railroad tracks
Same side of tree

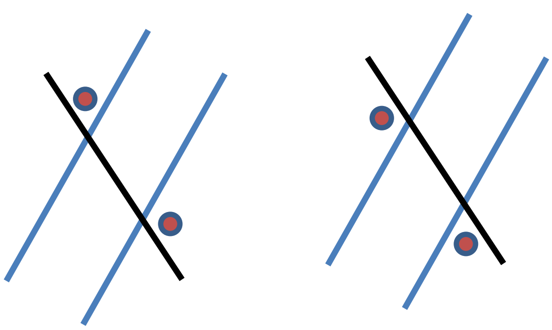


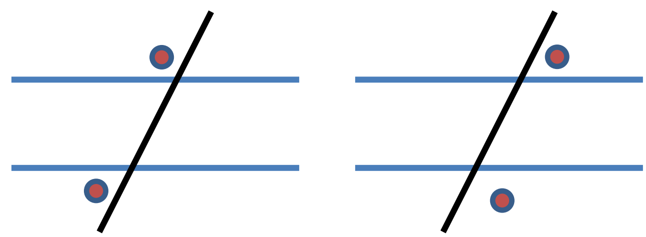


Angle + Angle = 180

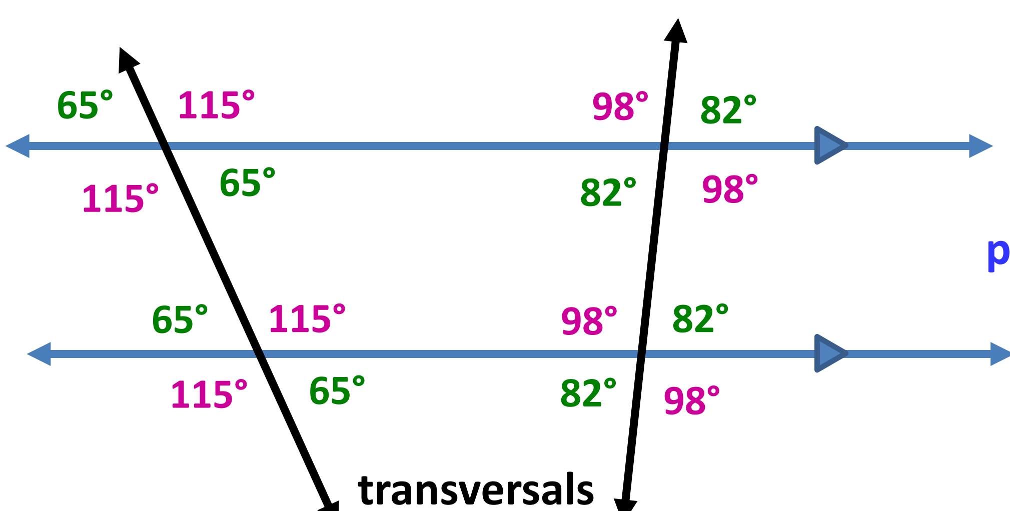
Consecutive Exterior Angles (C – same side) Supplementary

Outside railroad tracks
Same side of tree

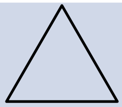
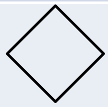
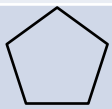
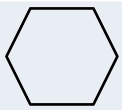
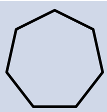
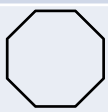
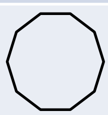
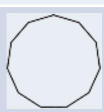
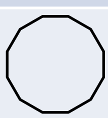




Angle + Angle = 180

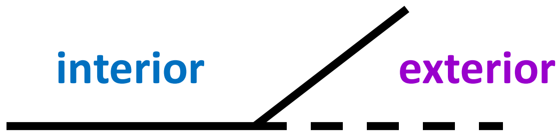


Angles with Polygons

Name	Pic	Nr Sides	Int Sum	Ext Sum	One Int	One Ext
Triangle		3	180	360	60	120
Quadrilateral		4	360	360	90	90
Pentagon		5	540	360	108	72
Hexagon		6	720	360	120	60
Heptagon		7	900	360	128.57	51.43
Octagon		8	1080	360	135	35
Nonagon		9	1260	360	140	40
Decagon		10	1440	360	144	36
Eleven-gon		11	1620	360	147.27	32.72
Dodecagon		12	1800	360	150	30
N-gon		n	$(n-2)\times180$	360	180-Ext	360/n

Exterior angles always <u>sum</u> to 360 (once around a circle).

Interior and Exterior always make a linear pair (adds to 180°)

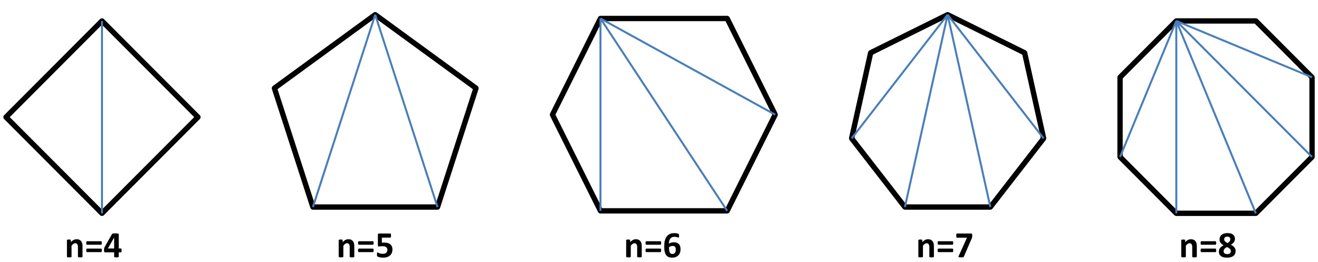


Interior angle + Exterior angle = 180
Exterior angle = 180 – interior angle

To find number of sides: 360 divided by exterior angle

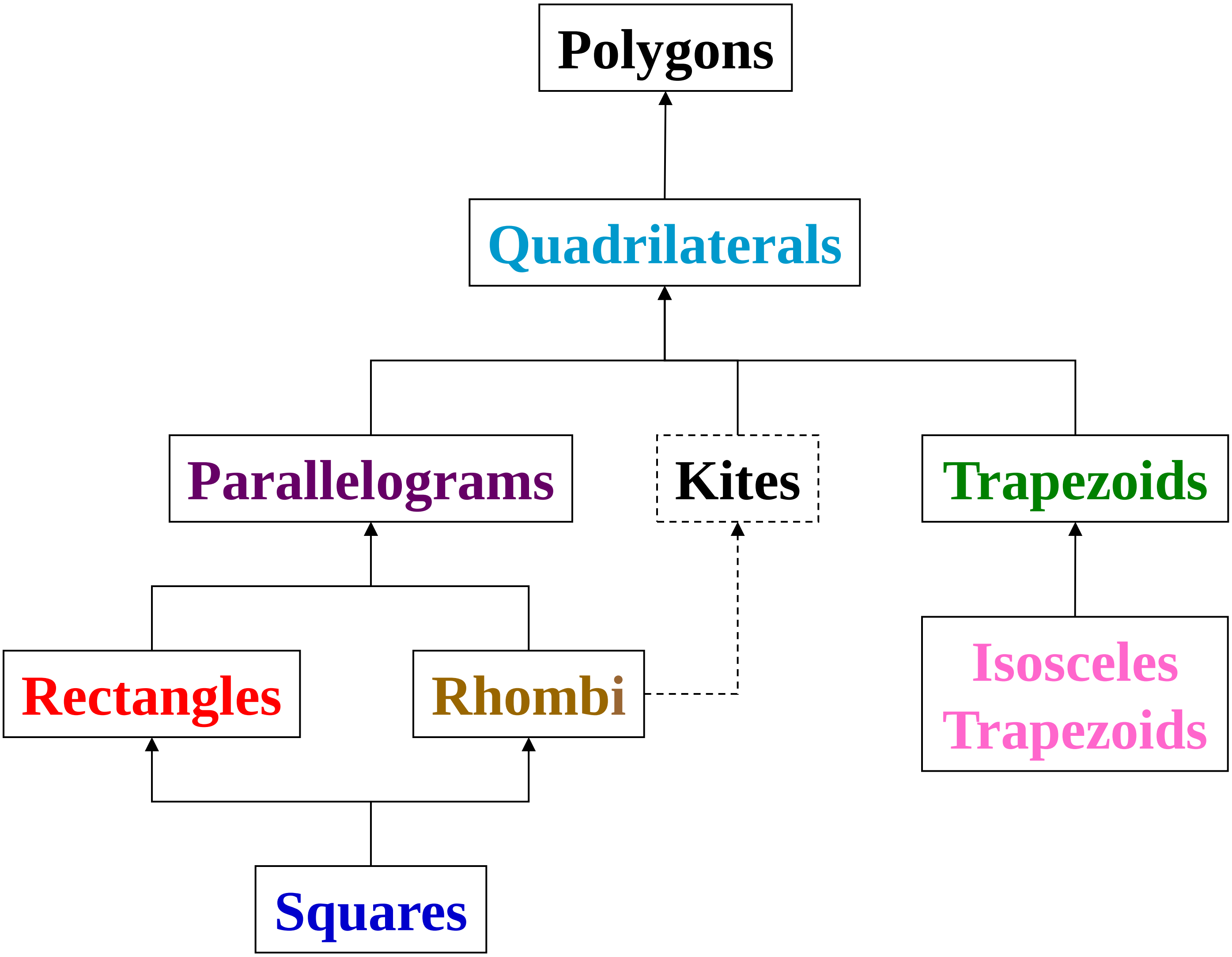
n = 360 / Ext ∠

Sometimes use Int + Ext = 180 to find Ext angle



Sides	3	4	5	6	7	8
Triangles	1	2	3	4	5	6
Sum of Interior Angles = (n – 2) x 180						

Polygon Hierarchy



Polygons are closed figures
with line segments as sides
Exterior Angles add to 360

Quadrilaterals are 4-sided figures
Interior Angles add to 360

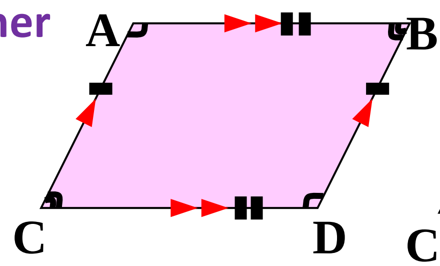
Parallelogram:

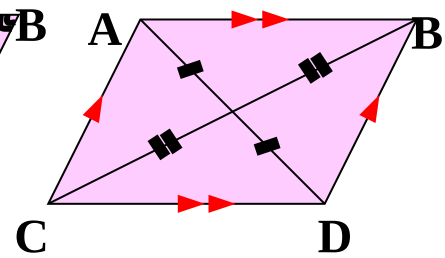
Sides: Opposite sides parallel and congruent

Angles: Opposite angles congruent

Consecutive angles supplementary

Diagonals: Bisect each other





Rectangle:

Sides: Opposite sides parallel and congruent

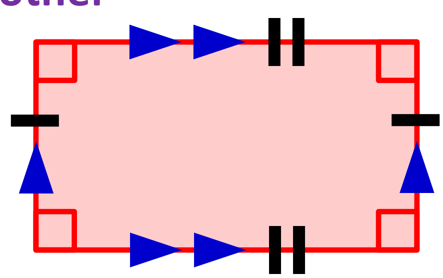
Angles: Opposite angles congruent

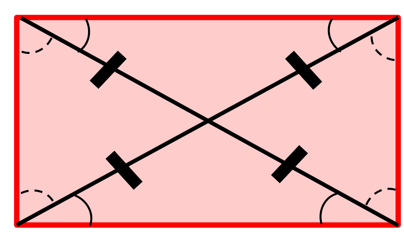
Consecutive angles supplementary

Corner angles = 90°

Diagonals: Bisect each other

Congruent





Rhombus

Sides: Opposite sides parallel and congruent

All four sides equal

Angles: Opposite angles congruent

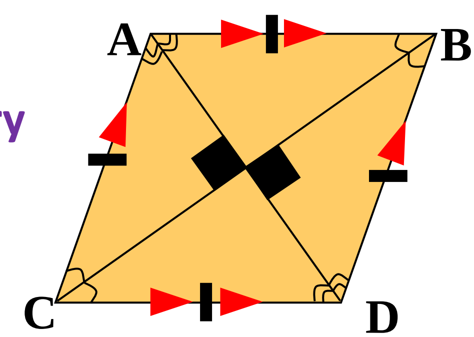
Consecutive angles supplementary

Diagonals: Bisect each other

Bisect opposite angles

Perpendicular to each other

Divides into 4 congruent triangles

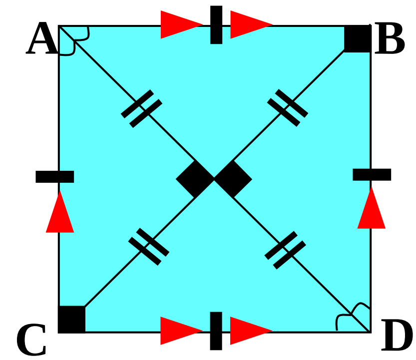


Square:

Sides: Opposite sides parallel and congruent
All four sides equal

Angles: Opposite angles congruent
Consecutive angles supplementary
Corner angles = 90°

Diagonals: Bisect each other
Congruent
Bisect opposite angles
Perpendicular to each other
Divides into 4 congruent triangles



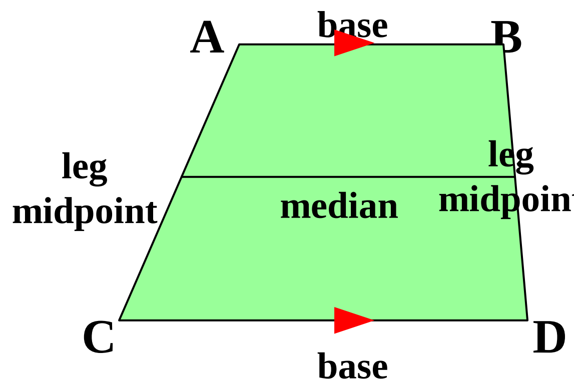
Trapezoid

Sides: Bases are parallel
Legs are not parallel

Angles: Leg angles supplementary

Diagonals: nothing special

Median: parallel to the bases
connects leg midpoint to other leg midpoint
formula: $Median = \frac{Base1+Base2}{2}$



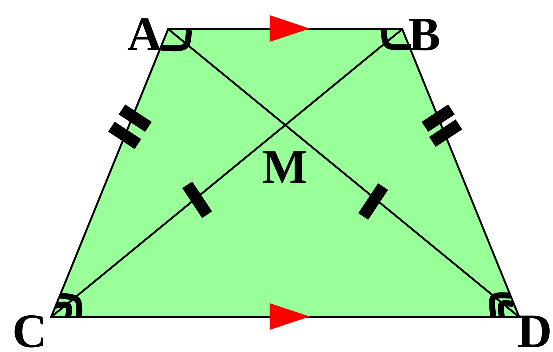
Isosceles Trapezoid

Sides: Bases are parallel
Legs are not parallel
but are congruent

Angles: Leg angles supplementary
Base angle pairs are congruent

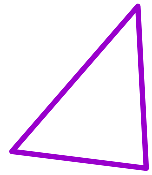
Diagonals: congruent

Median: parallel to the bases
connects leg midpoint to other leg midpoint
formula: $Median = \frac{Base1+Base2}{2}$



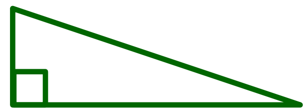
Classify Triangles by Angles: (by largest angle only)

Acute: All 3 angles acute

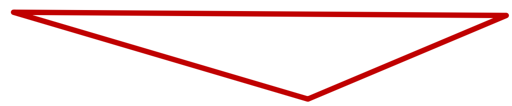


Equiangular: All angles are equal (60°)

Right: Right angle in triangle

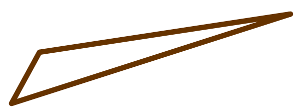


Obtuse: One angle obtuse in triangle

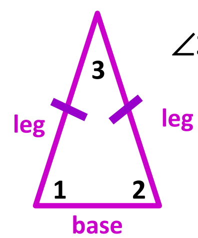


Classify Triangles by Sides: (how many sides equal)

Scalene: no sides equal



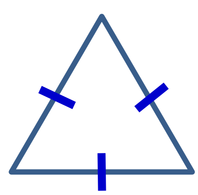
Isosceles: 2 sides equal



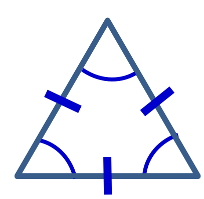
$\angle 3$ is vertex angle

Base angles: $\angle 1 \cong \angle 2$

Equilateral: 3 sides equal

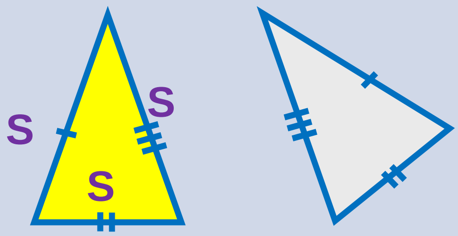
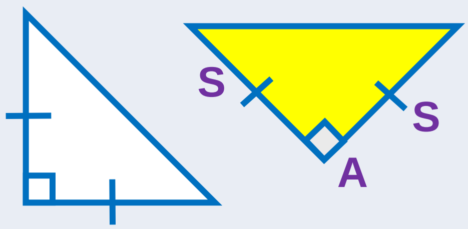
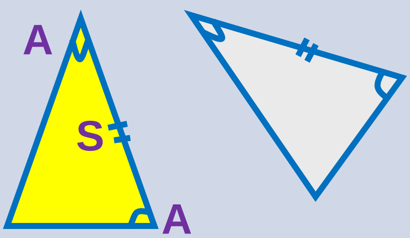
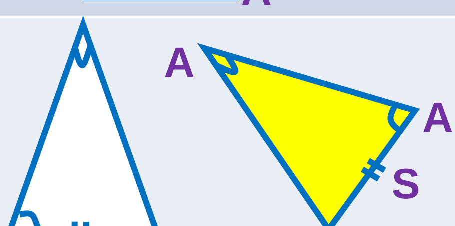
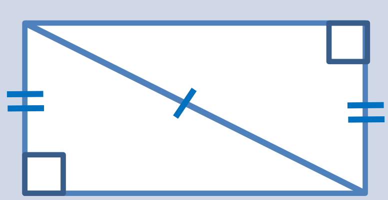


Equilateral means Equiangular and vice versa



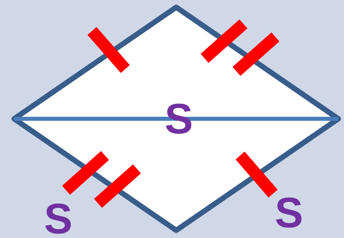
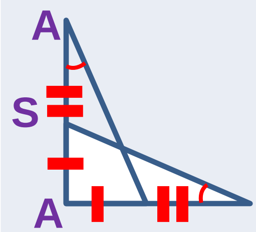
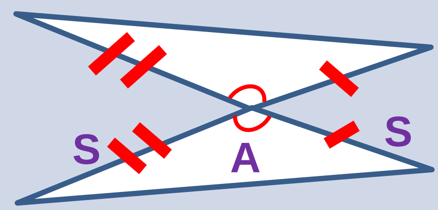
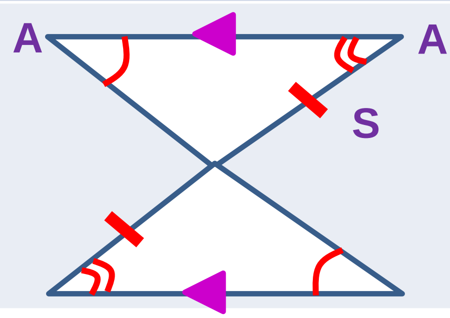
Triangle Congruence

If proving triangles congruent, always mark *__only one triangle__* up with S (for sides) and A (for angles) based on the items you have congruent

Post/Thrm	Picture	Hidden Features
SSS		**Shared Side**
SAS		**Shared Side** **Vertical Angles** **// Lines → Alt Int ∠'s**
ASA		**Shared Side** **Vertical Angles** **// Lines → Alt Int ∠'s**
AAS		**Shared Side** **Vertical Angles** **// Lines → Alt Int ∠'s**
HL		**Shared Side**

HL – Hypotenuse Leg is a short cut for SSS because of the Pythagorean Theorem

Triangle Congruence Hidden Features

Feature	Picture	Triangles ≅ by
Shared Side (Reflexive Prop)		**SSS**
Shared Angle (Reflexive Prop)		**ASA**
Vertical Angles ("Bow Tie")		**SAS**
Parallel Sides (◀ lines) **Alternate Interior Angles**		**AAS**

NEVER *AAA or ASS or SSA*

No Cars, donkeys, profanity or Social Security Administration

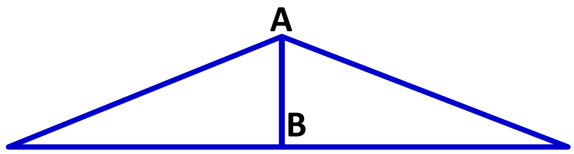
ΔABC ≅ ΔLMN ***Order Rules!!! (first to first, second to second, etc)***

Angles match by <u>one</u> letter ***Sides match by <u>two</u> letter groups***

"CPCTC" – corresponding parts of congruent triangles are congruent

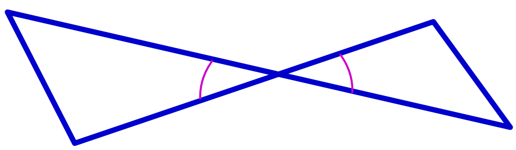
9 Common Properties, Definitions & Theorems for Triangles

1. Reflexive Property AB = AB



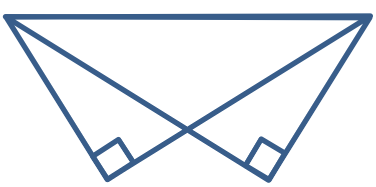
When triangles share a side or an angle

2. Vertical Angles Congruent



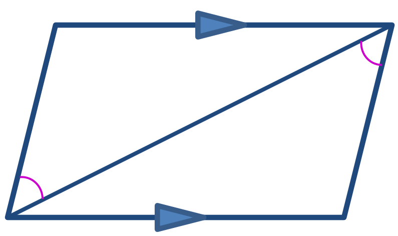
When two lines intersect (“bowtie”)

3. Right Angles Congruent



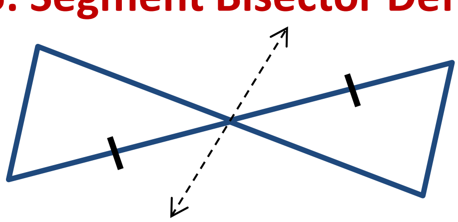
When you are given right triangles and/or a square/rectangle

4. Alt Interior Angles Congruent



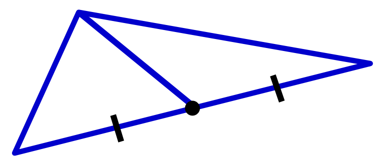
When two sides are parallel in given

5. Segment Bisector Definition



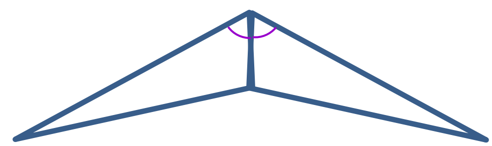
Results in 2 segments being congruent

6. Midpoint Definition



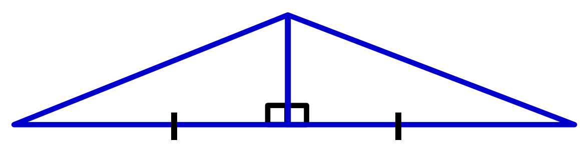
Results in 2 segments being congruent

7. Angle Bisector Definition



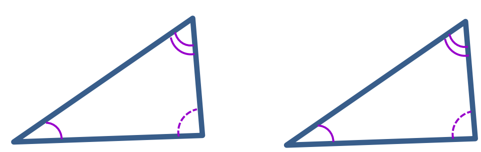
Results in 2 angles being congruent

8. Perpendicular Bisector Defn.



Results in 2 segments being congruent and two right angles

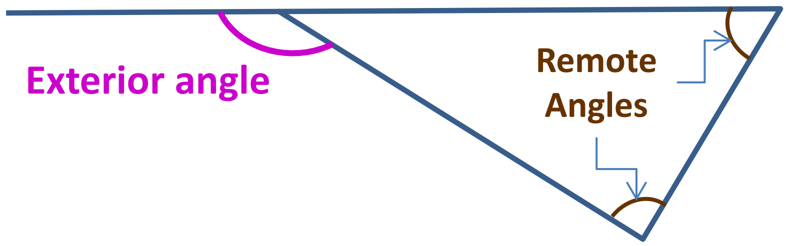
9. Third angle Theorem



If 2 angles of a triangle are congruent to 2 angles in another triangle, then the 3^{rd} angles are congruent (Δ's $\angle$s sum to 180)

Note: DO NOT ASSUME ANYTHING, IF IT IS NOT GIVEN

Exterior Angle in Triangles



Exterior angle = Remote angle + Other Remote angle

Given 3 numbers, do they make a triangle?

1. Put in order from smallest to largest
2. Is Small + Middle > Large?
   A. Yes, then it makes a triangle
   B. No, then it does not make a triangle

Example A: 8, 11, 4
1. Ordered: 4, 8, 11
2. 4 + 8 = 12 > 11 so triangle

Example B: 9, 13, 4
1. Ordered: 4, 9, 13
2. 4 + 9 = 13 = 13 so <u>no</u> triangle

Given 2 numbers, find range of the third side

1. Put in order from smallest to largest
2. (Large – Small) < Third Side < (Large + Small)

Example: Given 3 and 9 find third side range

9 – 3 = 6 < Third Side < 12 = 9 + 3

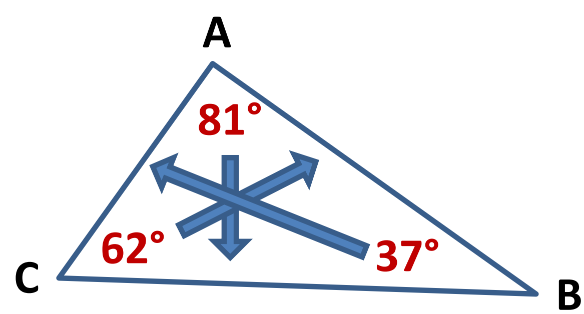
Angles versus Sides in Triangles

1. Largest side is opposite the largest angle
2. Middle side is opposite the middle angle
3. Smallest side is opposite the smallest angle

Steps to assure order
1. Put given information in order requested
   (smallest to largest   or   largest to smallest)
2. Substitute the corresponding letters
   (1 Angles; 2 for Sides)
3. Put in the missing letters
   (from the 3 in triangle)

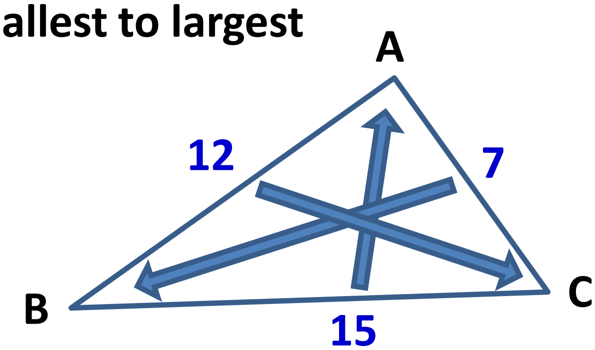
Sides Example: Find Sides largest to smallest

Lg > Md > Sm
1. 81° > 62° > 37°
2. ∠A > ∠C > ∠B
3. BC > AB > AC



Angles Example: Find Angles smallest to largest

Sm < Md < Lg
1. 7 < 12 < 15
2. AC < AB < BC
3. ∠B < ∠C < ∠A



Special Segments in Triangles

Segment	Point of Concurrency	Special Characteristic	Starts	Finishes
Perpendicular Bisector	Circumcenter	Equidistant from vertices	Nowhere Special	Midpoint
Angle Bisector	Incenter	Equidistant from sides	Vertex	Nowhere Special
Median	Centriod	Center of Gravity	Vertex	Midpoint
Altitude	Orthocenter	Nothing Special	Vertex	Nowhere Special

Point of concurrency is where the three segments of the same type come together (3 Medians of a triangle cross at the Centroid)

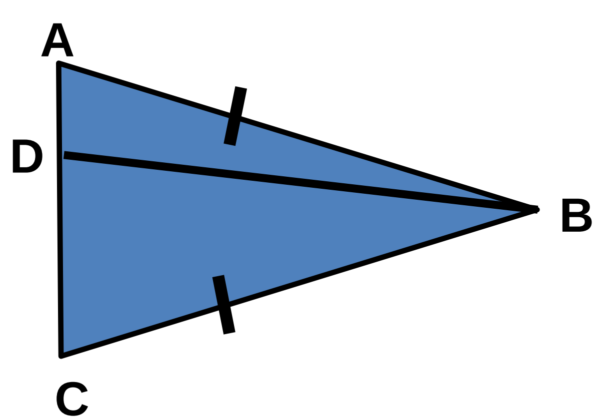
"Bigger angle opposite bigger side"

- *SAS Inequality, or Hinge Theorem*

If ∠ABD < ∠CBD, then AD < DC

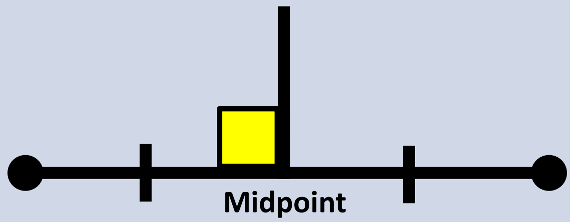
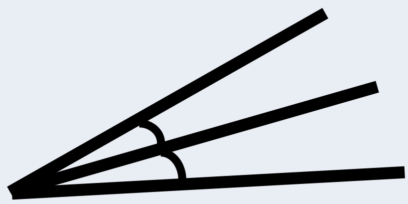
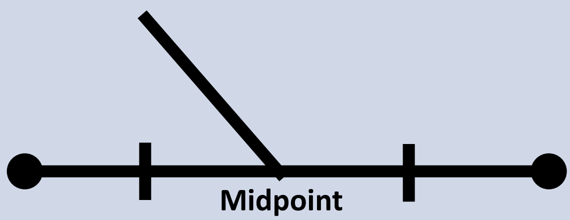
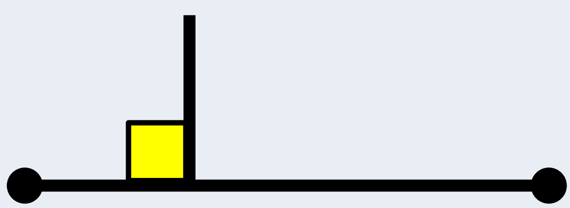
- *SSS Inequality*

If AD < DC, then ∠ABD < ∠CBD



This is our virtual alligator problem

Special Segments in Triangles (Pictures and Possible Problems)

Segment	Picture	Problems
Perpendicular Bisector		Angle = 90° ½ Sides = each other
Angle Bisector		½ Angles = each other Total = 2 (1/2 angle)
Median		½ Sides = each other
Altitude		Angle = 90°

Logic Symbols

$\wedge$ --- and $\vee$ --- or $\therefore$ --- therefore

~ --- not $\rightarrow$ --- if …, then $\leftrightarrow$ --- if and only if

English to Symbols

Let P = We have a test and let Q = It snows tomorrow

R = we will do makeup work and T = finish our test

If it snows tomorrow, then we don't have a test.

$$Q \rightarrow \sim P$$

If we don't have a test, then we will do makeup work or finish our test.

$$\sim P \rightarrow R \vee T$$

If it snows tomorrow, then we don't have a test and we will not do makeup work.

$$Q \rightarrow \sim P \wedge \sim R$$

Logic Symbols

$\wedge$ --- and $\vee$ --- or $\therefore$ --- therefore

~ --- not $\rightarrow$ --- if …, then $\leftrightarrow$ --- if and only if

Logic Laws (help us draw correct conclusions)

Detachment: (like school rules)

$A \rightarrow B$ is true. A is true, therefore B is true

If you have more than 3 tardies, you get MIP. Jon has 3 tardies; so Jon will get MIP.

Syllogism: (like transitive property of equality – removes the middle man)

$A \rightarrow B$ and $B \rightarrow C$, so $A \rightarrow C$

If you have more than 10 absences, you have to take the final. If you have to take the final, then you don't get out early. So, if you have more than 10 absence, you don't get out early.

Special If … , then … statements

Converse: converse --- change order (flips)

If it snows, then we get out early.

If we get out early, then it snows.

Inverse: inverse --- insert nots (negate: ~ --- not)

If it snows, then we get out early.

If it does not snow, then we do not get out early.

Contrapositive: contrapositive --- change order and nots (both)

If it snows, then we get out early.

If we did not get out early, then it did not snow.

More proof stuff

	Equality	Congruence
Reflexive Property	x = x	$\overline{AD} \cong \overline{AD}$
Symmetric Property	7 = x so x = 7	$\overline{AD} \cong \overline{JK}$ so $\overline{JK} \cong \overline{AD}$
Transitive Property	a = b, b =c so a = c	$\overline{AD} \cong \overline{BC}$, $\overline{BC} \cong \overline{XY}$ so $\overline{AD} \cong \overline{XY}$

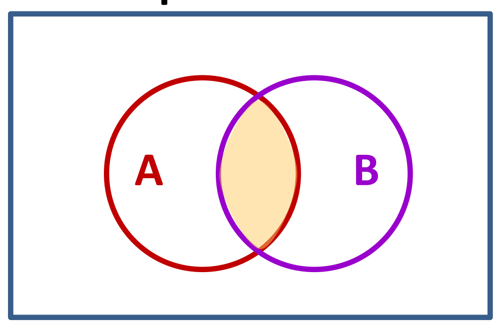
Alphabetical order (RST) increase in number of = or ≅ signs (1,2,3)

Venn Diagrams

"Some of A is B"

Example: A: Scalene triangles
B: Obtuse triangles
Some obtuse triangles are scalene

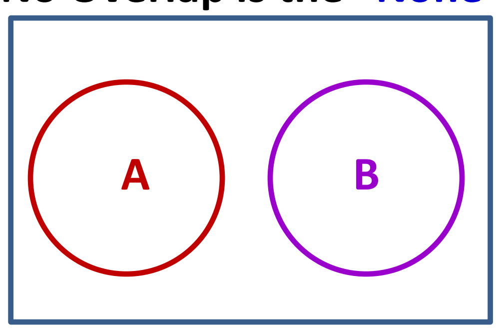
Overlap is the "Some"



"None of A is B"

Example: A: Equilateral triangles
B: Obtuse triangles
No obtuse triangle is equilateral

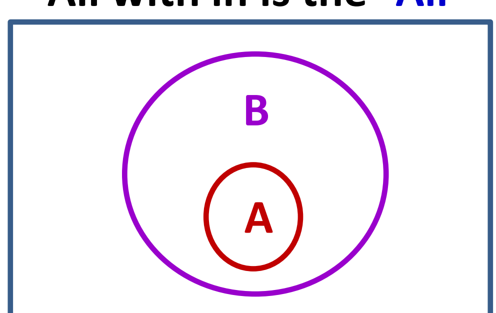
No Overlap is the "None"



"All of A is B"

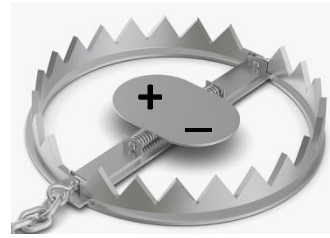
Example: A: Equilateral triangles
B: Isosceles triangles
All equilateral triangles are isosceles

All with in is the "All"



Proportions: Cross-Multiply

Proportions: (seeing the bear trap)
Any time a top or bottom of proportion has a + or – sign in it; put parentheses around it



Wrong way:

$$\frac{3}{x} = \frac{2}{x-2}$$

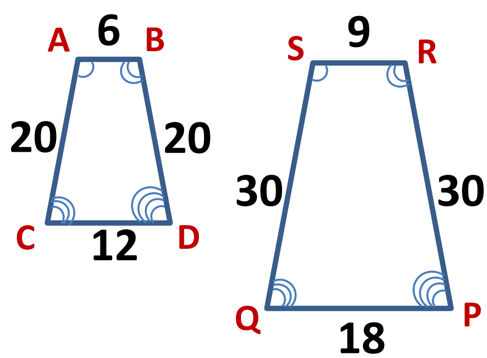
3x – 2 = 2x (error)
x ≠ 2

Right way:

$$\frac{3}{x} = \frac{2}{(x-2)}$$

3(x – 2) = 2x
3x – 6 = 2x
x = 6

Similar Triangles (and Polygons)



Similar (~) Figures

- Same Shape, but not same size
- Corresponding ***angles are congruent***
- Corresponding ***sides are proportional***
- Scale Factor is proportion in simplest form
- Order rules again (to find corresponding things)!!

$$9 = \frac{3}{2} \times 6$$

$$18 = \frac{3}{2} \times 12$$

$$30 = \frac{3}{2} \times 20$$

Quadrilateral ABCD ~ Quadrilateral SRQP
1 2 3 4 1 2 3 4

- Assign one figure to top of ratio, the other to the bottom;
- Solve using proportions

Similar Triangles (and Polygons)

Similar (~) Figures

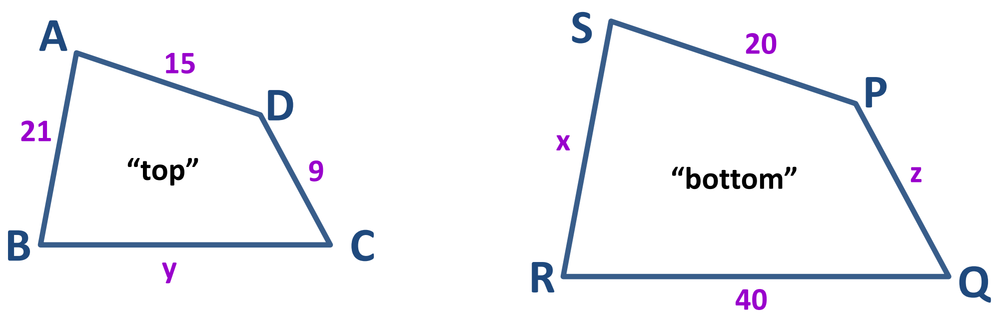
- Order rules again (to find corresponding things)!!

  Quadrilateral ABCD ~ Quadrilateral SRQP

  $\underset{1\,2\,3\,4}{ABCD} \sim \underset{1\,2\,3\,4}{SRQP}$

- Assign one figure to top of ratio, the other to the bottom

- Solve using proportions



$$\frac{\overset{1\,2}{AB}}{\underset{1\,2}{SR}} = \frac{\overset{2\,3}{BC}}{\underset{2\,3}{RQ}} = \frac{\overset{3\,4}{CD}}{\underset{3\,4}{QP}} = \frac{\overset{4\,1}{DA}}{\underset{4\,1}{PS}}$$

"top"

"bottom"

$$\frac{21}{x} = \frac{y}{40} = \frac{9}{z} = \frac{15}{20}$$

scaling factor: $\frac{15}{20} = \frac{3}{4}$

$$\frac{21}{x} = \frac{15}{20} \qquad \frac{y}{40} = \frac{15}{20} \qquad \frac{9}{z} = \frac{15}{20}$$

$15x = 20(21)$

$15x = 420$

$x = 28$

$20y = 15(40)$

$20y = 600$

$y = 30$

$15z = 9(20)$

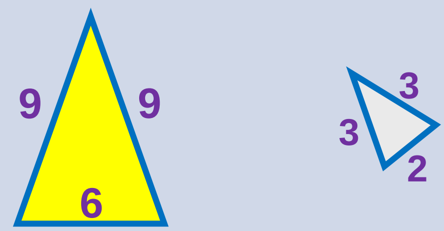
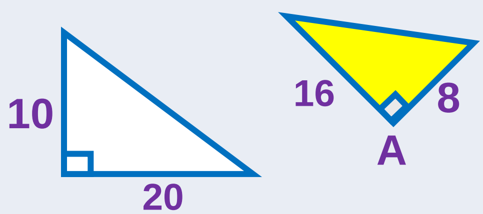
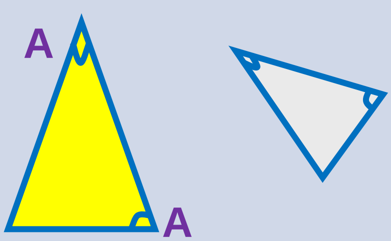
$15z = 180$

$z = 12$

Similar Triangles

Proving Triangles Similar

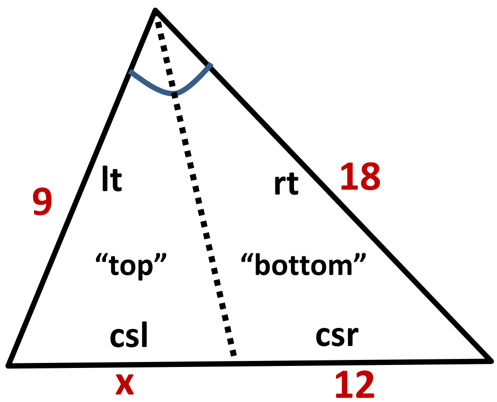
- Angles are congruent
- Sides are proportional (not congruent)

Post/Thrm	Picture	Δ Congruence / Logic
SSS		SSS All sides multiplied by same number (1/3)
SAS		SAS Sides either side of congruent angle multiplied by same number (5/4)
AA		ASA / AAS If two angles congruent, since all three add to 180, then all 3 angles congruent

"Multiplied by same number" is the scaling factor

Similar Triangles (Special Cases)

Angle Bisector Theorem



$$\frac{lt}{csl} = \frac{rt}{csr}$$

Sides
Partial

Alternative proportion

Sides Partial

$$\frac{lt}{rt} = \frac{csl}{csr}$$

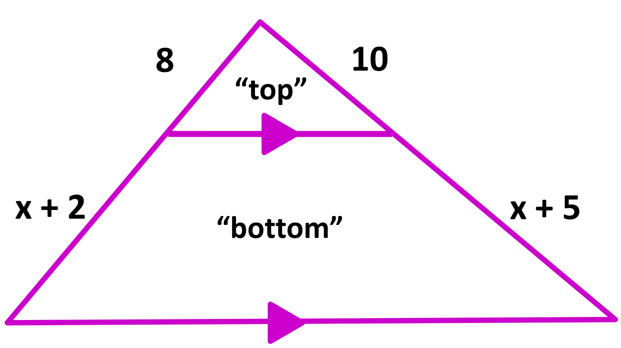
Csl/csr – cut side left or right

Example

$$\frac{9}{18} = \frac{x}{12}$$

18(x) = 9 (12)
18x = 108
x = 6

Transversals in Parallel Lines



Left Right

$$\frac{tl}{bl} = \frac{tr}{br}$$

Example

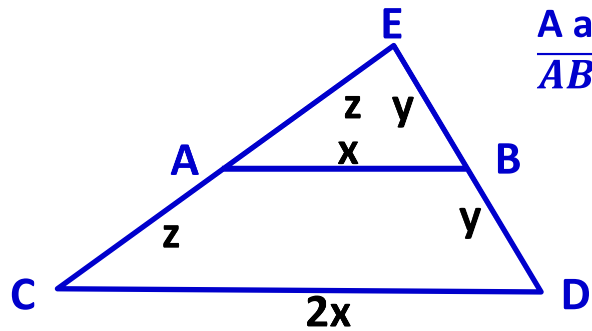
$$\frac{8}{x+2} = \frac{10}{x+5}$$

8(x + 5) = 10 (x + 2)
8x + 40 = 10x + 20
20 = 2x
10 = x

Note: parallel bases cannot use this special case (they must use little vs big triangle proportions)

Similar Triangles (Special Cases)

Mid-segment Theorem



A and B are midpoints

$\overline{AB}$ is mid-segment

Example:

Given CD = 20, EA = 8, BD = 7

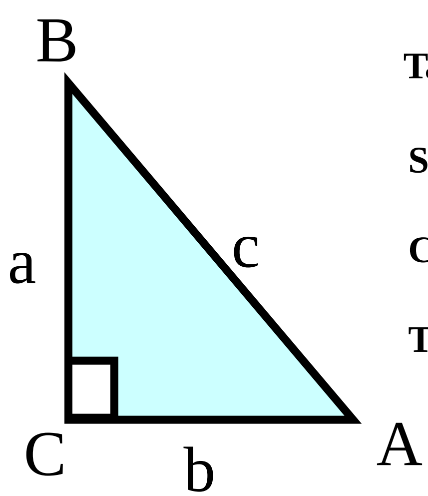
Find AB, EB, and AC

10, 7, 8

Note: top triangle is half of big triangle

Trigonometry in Triangles

Trig Relationship between two acute angles



Sin A is opposite over hypotenuse**:** a/c

Cos A is adjacent over hypotenuse**:** b/c

Tan A is opposite over adjacent**:** a/b

Sin B is opposite over hypotenuse**:** b/c

Cos B is adjacent over hypotenuse**:** a/c

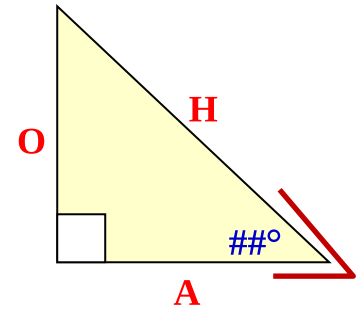
Tan B is opposite over adjacent**:** b/a

So, Sin A = Cos B and Cos A = Sin B

$a^2 + b^2 = c^2$ (from Pythagorean Theorem)

$m\angle A + m\angle B = 90°$ (3∠'s of Δ = 180°)

Trigonometry in Triangles



Steps for solving Trig problems

1. Mark perspective (given) angle

2. Label each side of the triangle as
   - H for hypotenuse (opposite 90° and usually the diagonal side)
   - A for side adjacent to given angle (A & H form the given angle)
   - O for the side opposite the given angle

3. Determine using the information (sides and angles) given in the problem which of the trig functions you need to solve for variable

   $\sin(\text{angle}) = \dfrac{\text{opp}}{\text{hyp}}$ $\quad\cos(\text{angle}) = \dfrac{\text{adj}}{\text{hyp}}$ $\quad\tan(\text{angle}) = \dfrac{\text{opp}}{\text{adj}}$

   SOH $S = \frac{O}{H}$ $\quad$ CAH $C = \frac{A}{H}$ $\quad$ TOA $T = \frac{O}{A}$

4. Set up an equation using the trig function and the variable

$$\text{Trig function (angle°)} = \frac{\text{Some side}}{\text{Some other side}}$$

5. Solve for the variable (based on where the variable is)

   Variable on top; multiply both sides by the bottom

   Example 1 on next page

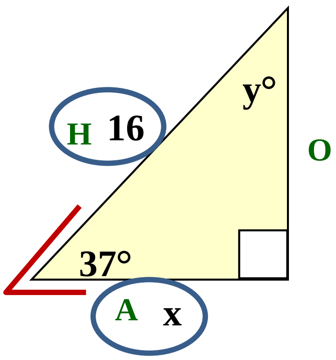
   Variable on bottom: variable and trig function trade places

   Example 2 on next page

   Variable is angle: use inverse trig (2nd key then trig key)

   Example 3 on next page

Example 1: (variable on top)



2) 16 is H , x is A and no value for O

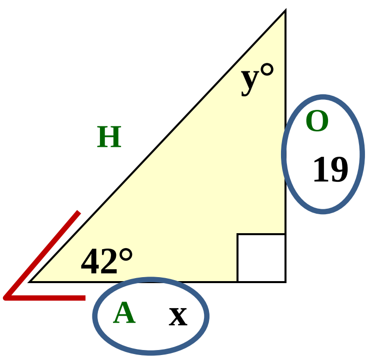
3) Since we have A and H we need to use cos

4) $\cos(37°) = \dfrac{x}{16}$ (x is on top multiply both sides by bottom)

5) 16 cos (37°) = x = 12.78

Use 90 – 37 = 53 to find the other angle, y

Example 2: (variable on bottom)



2) 19 is O , x is A and no value for H

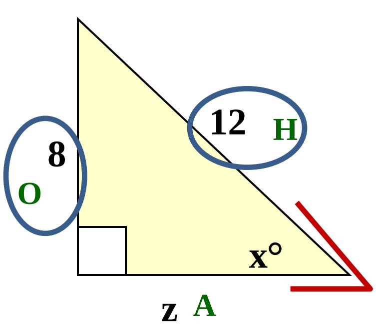
3) Since we have O and A (no H) we need to use tan

4) $\tan(42°) = \dfrac{19}{x}$ (x is on bottom then switch it with the trig function)

5) $x = \dfrac{19}{\tan(42°)} = 21.10$

Use 90 – 37 = 53 to find the other angle, y

Example 3: (variable is the angle)



2) 12 is H , 8 is O and no value for A -- x is the angle !

3) Since we have O and H we need to use sin

4) $\sin(x°) = \dfrac{8}{12}$ (x is angle use inverse sin)

5) $x = \sin^{-1}(8/12) = 48.19°$

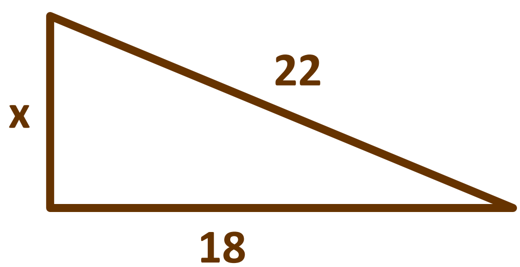
Use Pythagorean Theorem to find one missing side

$12^2 = z^2 + 8^2 \rightarrow 144 = z^2 + 64 \rightarrow 80 = z^2 \rightarrow 8.94 = z$

Pythagorean Theorem

On formula sheet: $a^2 + b^2 = c^2$ **(c is the hypotenuse)**
Variable in problem is not always the hypotenuse!!

Pythagorean Triple: **3 whole numbers that satisfy the formula**
Common examples:
3, 4, 5 5, 12, 13 6, 8, 10 7, 24, 25 8, 15, 17



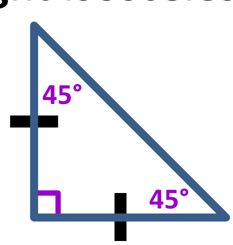
$$x^2 + 18^2 = 22^2$$
$$x^2 + 324 = 484$$
$$x^2 = 160$$
$$x \approx 12.65$$

Special Case Right Triangles

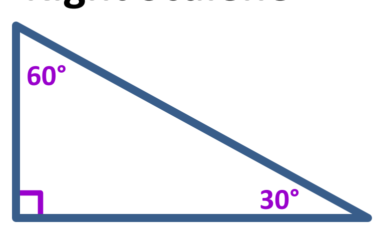
Sometimes can be solved using either Trig or Pythagorean Thrm

Two cases:

45 – 45 – 90
Right Isosceles



30 – 60 – 90
Right Scalene

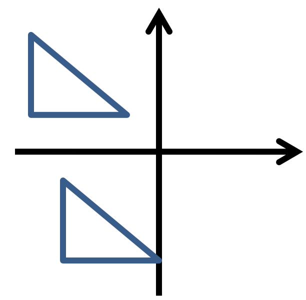


Angle	Side Opposite	If hyp = 20, then
30	½ hypotenuse	½(20) = 10
45	½ hypotenuse × √2	½(20) × √2 = 14.14
60	½ hypotenuse × √3	½(20) × √3 = 17.32

Transformations

Translation – "Slides"

Up – adds to y
Down – subtracts from y
Right – adds to x
Left – subtracts from x



Translation function: (x, y) → (x ± h, y ± k)

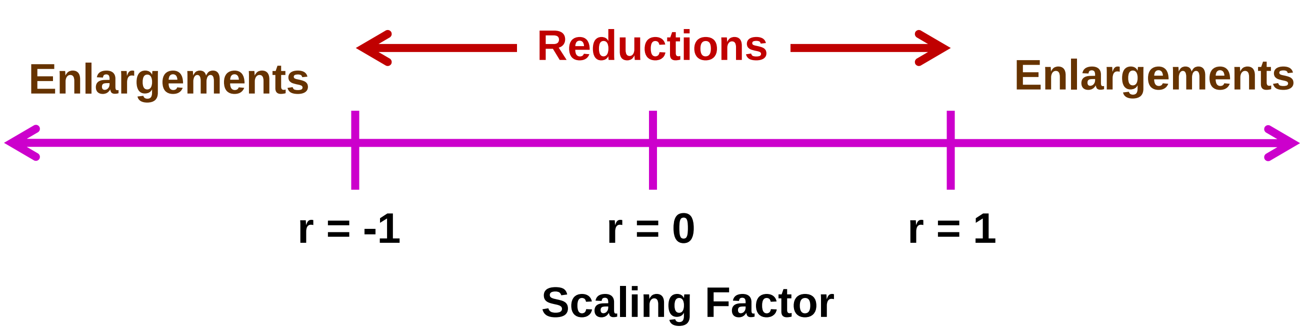
In picture above: Down 7 and right 3: (x, y) → (x + 3, y – 7)

Dilations – "Shrinks or Blow-ups"

Similar figures (from before)

Scaling factor r: if |r| > 1 then enlargement
if |r| < 1 then reduction
if |r| = 1 then congruence transformation

Negative numbers flip figure over dilation center point



Transformations

Rotations – "Turns" (usually around the origin)

Clockwise:

Counterclockwise:

90° increment turns can be done without using trig

180° rotation (in either direction) is same
as reflection across the origin

<table>
<tr><th>Clockwise</th><th>Counterclockwise</th><td rowspan="4">Same places turning in different directions</td></tr>
<tr><td>90°</td><td>270°</td></tr>
<tr><td>180°</td><td>180°</td></tr>
<tr><td>270°</td><td>90°</td></tr>
</table>

Transformations

Mirror Image across a line or point

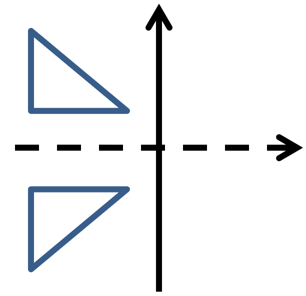
Reflections – “Flips”

Equal distance from reflection point/line

Over x-axis (or horizontal lines)

Equation: $(x, y) \rightarrow (x, -y)$

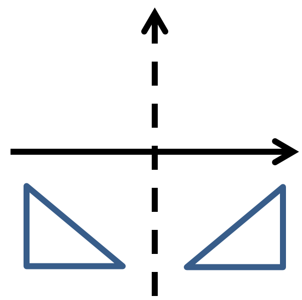
Up/down distance



Over y-axis (or vertical lines)

Equation: $(x, y) \rightarrow (-x, y)$

Left/right distance

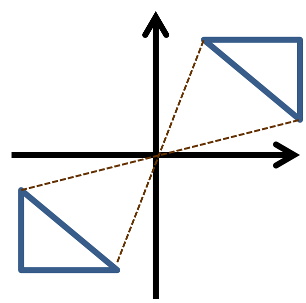


Over origin

Equation: $(x, y) \rightarrow (-x, -y)$

Origin is ***midpoint*** of before and after points

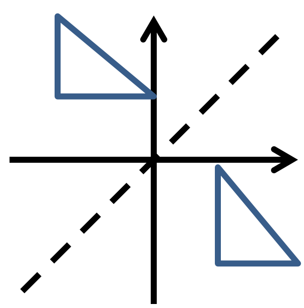
Both distances



Over (diagonal) line y = x

Equation: $(x, y) \rightarrow (y, x)$

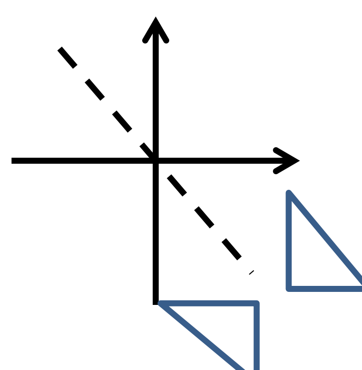
Diagonal distance



Over (diagonal) line y = -x

Equation: $(x, y) \rightarrow (-y, -x)$

Diagonal distance



Symmetry

Line symmetry – 1) folding the figure in half
2) regular figures have lines of symmetry equal to the number of sides

Point symmetry – 1) where two lines of symmetry intersect
2) midpoint of every point and its reflection
3) even-sided regular figures have it

Rotational symmetry – number of times a figure can be turned

Order – in regular figures equal to the number of sides

Magnitude – equal to 360/order (degrees per turn)
(equal to the exterior angle of figure)

Tesselations

A figure, or series of figures, tessellates a plane by covering it without gaps or overlaps (like tile on a floor)

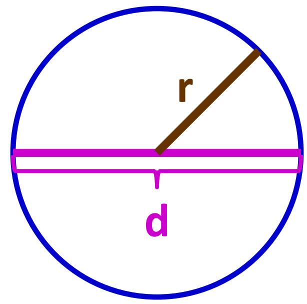
Gaps – interior angles add up to less than 360

Overlaps – interior angles add up to more than 360

Only 3 regular figures tessellate: Triangle, Square, Hexagon

Circles

All points equidistant from the center



$d = 2r$

$C = 2\pi r = d\pi$

$A = \pi r^2$

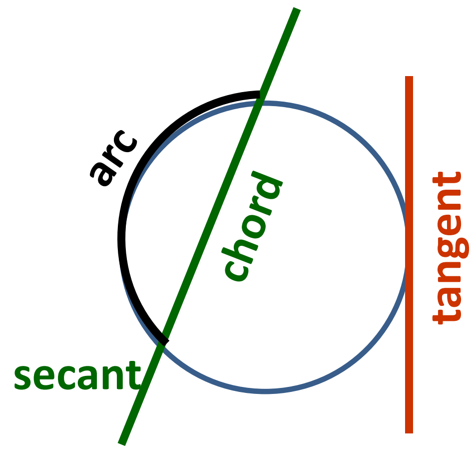
<u>Terms</u>

A = Area

C = Circumference (perimeter)

d = diameter

r = radius

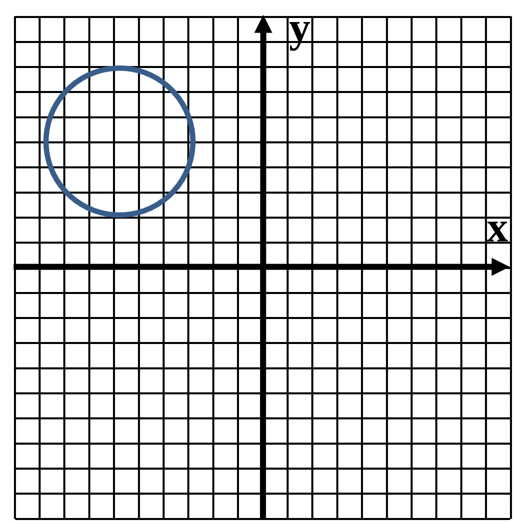


secant touches circle twice
called a chord inside the circle
tangent touches circle once

Arc = part of circumference

Equation: $(x - h)^2 + (y - k)^2 = r^2$

Example to the right



center (-6, 5) radius = 3

$(x - (-6))^2 + (y - 5)^2 = 3^2$

$(x + 6)^2 + (y - 5)^2 = 9$

Circles

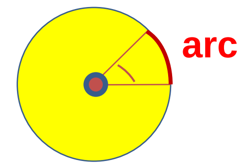
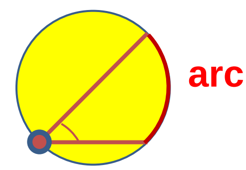
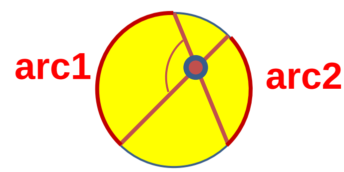
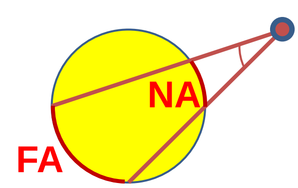
Segments

Location	Formula	
Inside	Part x part = part x part	
Outside	Outside x whole = Outside x whole	
Outside	Outside x whole = Outside x whole	

Tangents

Circles

Angles

Angle	Vertex Location	Sides	Formula (arcs)	Picture
Central	Center	Radii	= arc	
Inscribed	Edge	Chords	= ½ arc	
Interior	Inside (not at center)	Chords	= ½ (arc1+arc2)	
Exterior	Outside	Secants Tangents	= ½ (Far arc – Near arc)	

Surface Area and Volume notes

Use and read carefully the formula sheet

B – area of the base
usually a square base in pyramids
b – base of triangle or trapezoids

Pythagorean Theorem used in

Cones:

Slant height ^{2} = radius ^{2} + height ^{2}

Pyramids:

Slant height ^{2} = (1/2 side) ^{2} + height ^{2}

Constructions

Perpendicular segments and Angle Bisectors – Building a Rhombus

Constructions

Perpendicular segments and Angle Bisectors – Building a Rhombus

Constructions

Perpendicular segments and Angle Bisectors – Building a Rhombus

Constructions

Perpendicular segments and Angle Bisectors – Building a Rhombus