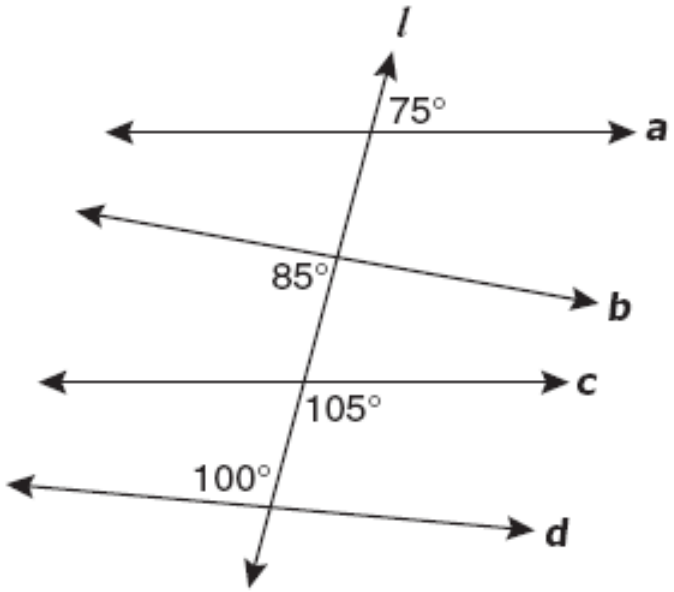
Modified and Animated By Chris Headlee
June 2010

END OF COURSE GEOMETRY

Form M0117, CORE 1

SSM: Super Second-grader Methods

1 Transversal *l* cuts lines *a*, *b*, *c*, and *d*.



SSM:

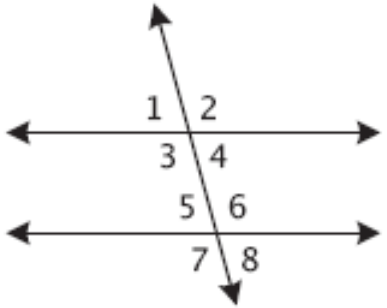
- **pair of angles must involve both lines**
- **only one answer, A, works**

Which two lines are parallel?

A *a* and *c*
B *a* and *d*
C *b* and *c*
D *b* and *d*

Angles 2 and 6 are corresponding
Angles 2 and 3 are vertical angles (always =)
Angles 7 and 8 are a linear pair (always supplementary)
Angle 3 is obtuse so answer D is wrong

2



In the figure above, ∠2 and ∠6 are a pair of —

F consecutive interior angles
G alternate interior angles
H vertical angles
J corresponding angles

SSM:
- **pair of angles involve both lines (eliminates H)**
- **same side of transversal (eliminates G)**
- **both obtuse (eliminates F)**
- **only one answer, J, works**

Angles 2 and 6 are corresponding
same side of the transversal and same side (above) the other line

3 One exterior angle of a regular polygon measures 72°. What is the measure of one interior angle?

A 18°

B 108°

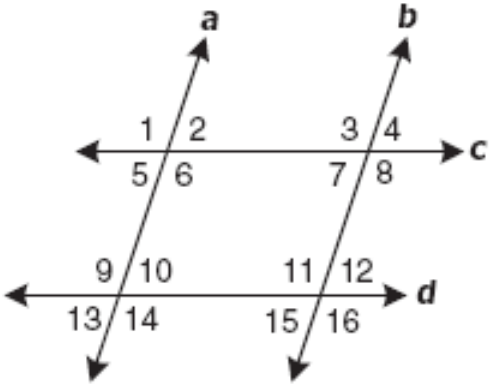
C 360°

D 540°

SSM:
- **interior angle < 180**
- **180 (72 + 108) is a "magic" #**

Interior + Exterior = 180 (linear pair)
180 – 72 = 108

4 In this drawing, $a \parallel b$ and $c \parallel d$.



SSM:
- **angle 1 is obtuse**
- **angle 12 is acute**

Which angle is *not* necessarily congruent to ∠1?

F ∠3

G ∠9

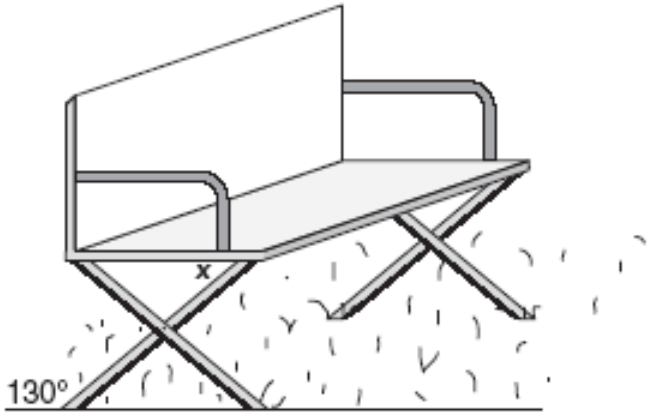
H ∠12

J ∠16

Angles 1, 3, 9, and 16 are obtuse
Angle 12 is acute

Angles 3 and 9 are corresponding
Angle 16 is an alternate exterior angle to Angle 3

5 The support legs on a bench are attached in such a way that the angle made by one leg with the ground is 130°.



SSM:
- **x is acute**
- **180 is magic number**
- **180 – 130 = 50**

What must the measure of the angle marked *x* be in order for the seat of the bench to be parallel to the ground?

A 50°

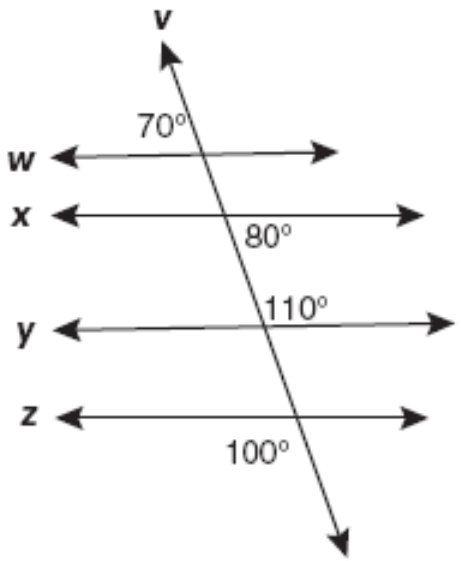
B 65°

C 90°

D 130°

No special relationship between x and 130.
Angle x is alternate interior with the angle that forms a linear pair with 130
So x + 130 = 180, then x = 50

6 Line v is a transversal.



SSM:
- extend lines as far as possible
- w and x intersect
- y and z intersect

Which is a true statement?

F $w \parallel y$ and $x \parallel z$

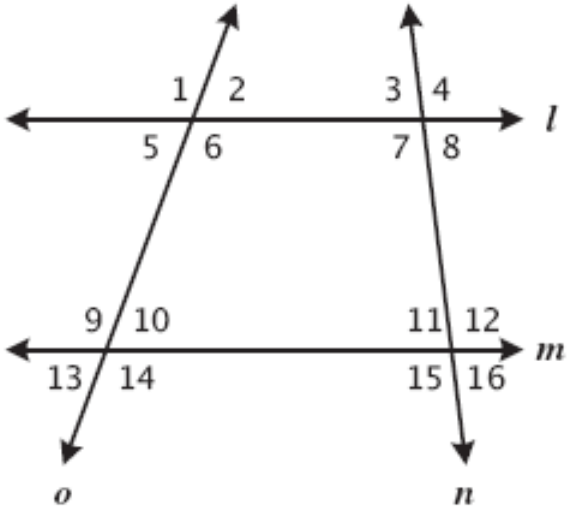
G $w \parallel x$ and $y \parallel z$

H $w \parallel z$ and $x \parallel y$

J $w \parallel x$ and $x \parallel y$

Make linear pair combinations for each line
w: 70, 110; x: 80, 100; y: 110, 70; z: 100, 80
w and y match up and x and z match up

7



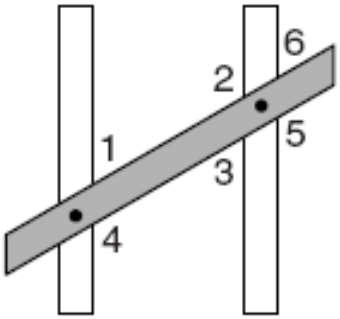
In the drawing above, ∠4 and ∠12 are —

A alternate interior angles
B consecutive interior angles
C corresponding angles
D a linear pair

SSM:
- **4 and 12 are same side of n**
- **4 is exterior and 12 is interior**
- **corresponding**

Angles 4 and 12 are corresponding
They are on the same side of n so A is incorrect
Angle 4 is exterior so B is incorrect
They aren't on the same line so D is incorrect

8 A carpenter nailed a board across two beams, forming the angles shown.



Which equal measures would ensure the beams are parallel?

F $m\angle 1 = m\angle 2$

G $m\angle 1 = m\angle 3$

H $m\angle 2 = m\angle 5$

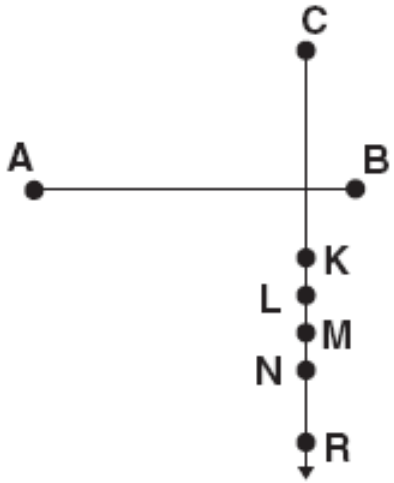
J $m\angle 3 = m\angle 4$

SSM:

- **pair of angles must involve both lines**
- **must be same classification**
- **only one answer, G, works**

Angles 1 and 2 are consecutive interior (supplementary)
Angles 2 and 5 are vertical angles (always =)
Angles 3 and 4 are consecutive interior (supplementary)
Angles 1 and 3 are alternate interior (equal)

9



Which segment is congruent to $\overline{AB}$?

A $\overline{CK}$

B $\overline{CL}$

C $\overline{CM}$

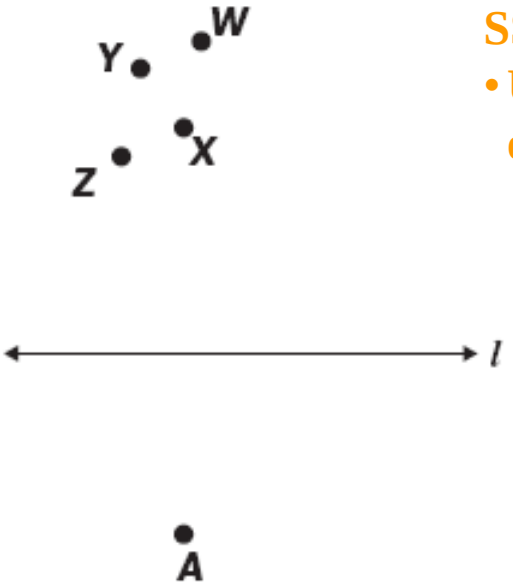
D $\overline{CN}$

SSM:

- **use scratch paper to measure AB**
- **See which point gives same distance**

Use compass to "measure" AB
Place at C and see where it hits

10



SSM:

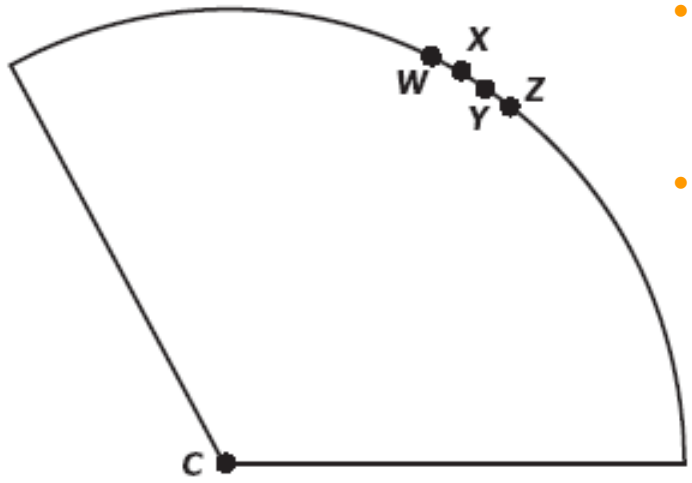
- **Use scratch paper lines and edge to match A with X**

Which point apparently lies on the perpendicular to *l* from *A*?

F *X*
G *Y*
H *Z*
J *W*

Use compass to find two points on line l equal distant from A
Draw two arcs (with same radius) from those points
Connect their intersection and point A

11 One piece of pie is left for two boys to share.



Where should the pie be cut to ensure each gets an equal piece?

A $\overline{CZ}$

B $\overline{CY}$

C $\overline{CX}$

D $\overline{CW}$

SSM:

- Use scratch to measure distance from end of arcs to each point
- The point that is equidistant is the answer

Use compass to find two points on rays equal distant from C
Draw two arcs (with same radius) from those points
Connect their intersection and point C

12 If $p \rightarrow q$, and $q \rightarrow r$, then —

F $r \rightarrow p$

G $p \rightarrow r$

H $\sim r \rightarrow p$

J $r \rightarrow \sim p$

SSM:

- Think transitive property of equality: p = q and q = r so p = r

Law of Syllogism: p must imply r

13 If the conditional statement

"If you have a laptop, then you have a computer"

is represented by $p \rightarrow q$, what is the symbolic representation of

"If you have a computer, then you do not have a laptop"?

A $q \rightarrow \sim p$

B $\sim q \rightarrow p$

C $p \rightarrow \sim q$

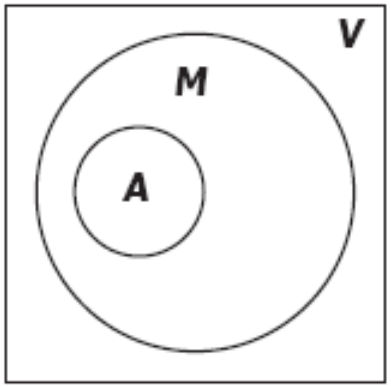
D $\sim q \rightarrow \sim p$

SSM:

- q is second statement
- only one answer has q first

First part represents q and second part represents ~p

14 **In the Venn diagram below, *V* represents the set of all vehicles, *M* represents the set of all motorized vehicles, and *A* represents the set of all automobiles.**



SSM:
- not much help

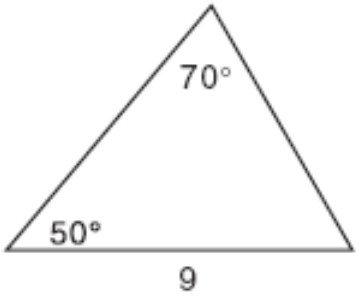
Based on the diagram, which is a valid conclusion?

F All automobiles are motorized vehicles.
G All motorized vehicles are automobiles.
H Some automobiles are not motorized vehicles.
J No automobiles are motorized vehicles.

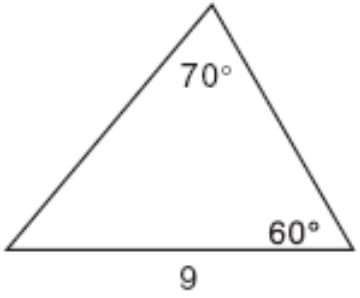
Since all of set A is inside of set M, then all A are M

15 Which triangle below is *not* congruent to the other three triangles?

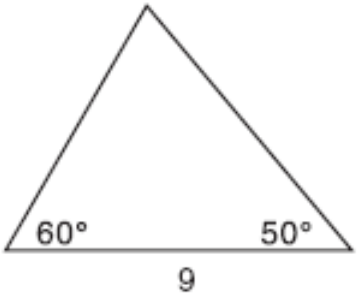
A



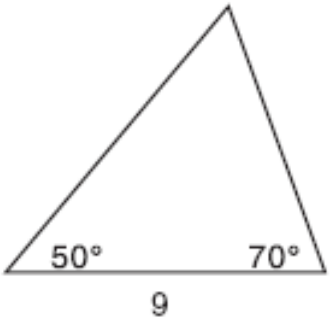
C



B



D

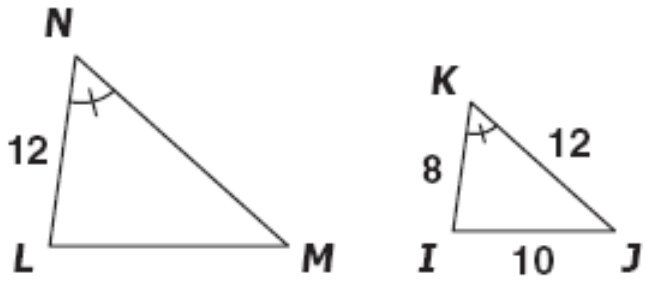


9 has to be opposite the 70° angle (from answer A and C)
Answer B we can figure the missing angle to be 70°
So answer D has 9 opposite a 60° angle

SSM:

- **Use eyes (or scratch paper) to see which triangle is different**

16



Which additional piece of information would prove that $\triangle IJK \sim \triangle LMN$?

F $NM = 18$

G $LM = 18$

H $NM = 15$

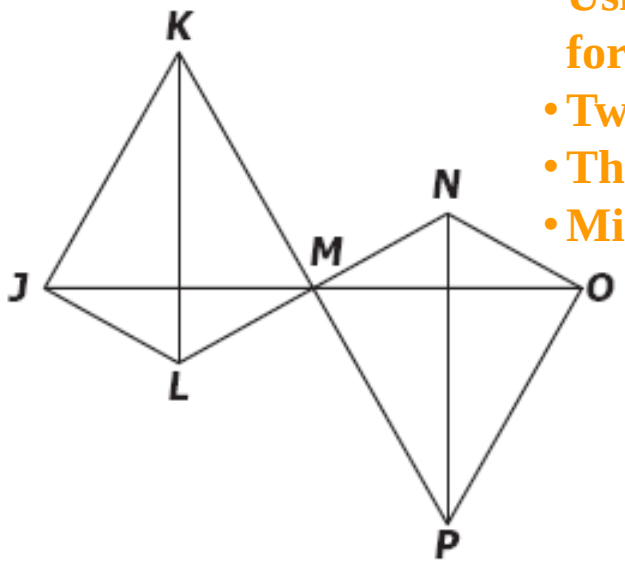
J $LM = 10$

SSM:
- not much help
- Answer J is same value as IJ so it must be wrong

12 is to 8 the same as 18 is to 12 or 15 is to 10

Need to match up correct sides (LM → IJ and NM → KJ)

17 Given: M is the midpoint of $\overline{LN}$ and $\overline{KP}$.



The given information is sufficient to prove $\triangle KML \cong \triangle PMN$ by which postulate/theorem?

A Angle-Side-Angle
B Side-Side-Side
C Side-Angle-Side
D Angle-Angle-Side

SSM:
- Using given info mark S and A for congruent sides or angles
- Two sides → SAS or SSS
- Third S → shared side (NO!)
- Middle A → vertical angle (YES!)

Midpoints divide segments into congruent halves:
so LM = MN and KM = MP
Vertical angles ∠KML ≅ ∠NMP
so SAS

18 **Which of the following could *not* be the lengths of the sides of a triangle?**

F 6 ft, 3 ft, 9 ft
G 3 cm, 4 cm, 5 cm
H 4 in., 6 in., 8 in.
J 5 km, 2 km, 4 km

SSM:
- **not → 3 triangles and one not**

two smallest sides added together > biggest side

A fails because its two smallest sides = biggest side

19 In $\triangle DEF$, m$\overline{DE}$ = 8 inches, m$\overline{EF}$ = 6 inches, and m$\overline{DF}$ = 10 inches. Which lists the angles in order from *smallest* to *largest*?

A $\angle D, \angle E, \angle F$

B $\angle F, \angle D, \angle E$

C $\angle E, \angle F, \angle D$

D $\angle D, \angle F, \angle E$

SSM:
- virtual alligator
- Largest stick → largest angle
- DF is biggest so ∠E is biggest
- EF is smallest so ∠D is smallest

Arrange sides from smallest to largest based on numbers
6, 8, 10
Substitute the side's names (line segments)
EF, DE, DF
Put in missing letter from triangle
D, F, E
Angles are now from smallest to largest

20 In $\triangle ABC$, if $m\angle C < m\angle B < m\angle A$, then —

F $AB < AC < BC$

G $AC < AB < BC$

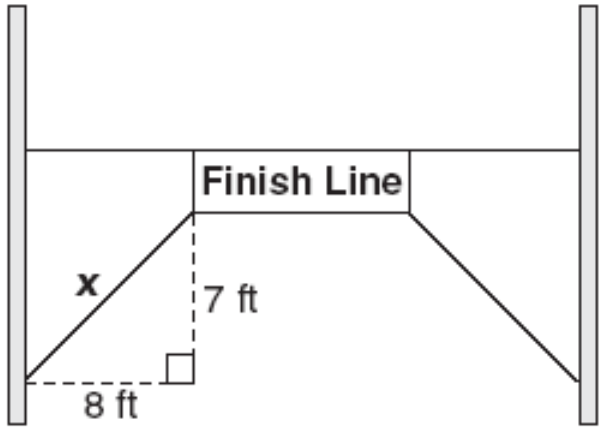
H $AB < BC < CA$

J $BC < AB < CA$

SSM:
- **virtual alligator**
- **Largest angle → largest stick**
- **∠A is biggest so BC is biggest**
- **∠C is smallest so AB is smallest**

Arrange angles from smallest to largest based on given
C, B, A
Put in missing letters from triangle
AB, AC, BC
Sides are now from smallest to largest

21 To mark the end of a race, a finish-line banner is stretched across the road as shown in the drawing.



SSM:

- measure 7 with scrap paper
- measure 8 with scrap paper
- If to scale (or close) then measure x with scrap paper
- Estimate answer

Which is closest to the length of the support rope designated by *x* in the drawing?

A 9.5 ft
B 10.6 ft
C 12.0 ft
D 15.0 ft

Pythagorean Theorem:

$7^2 + 8^2 = x^2$

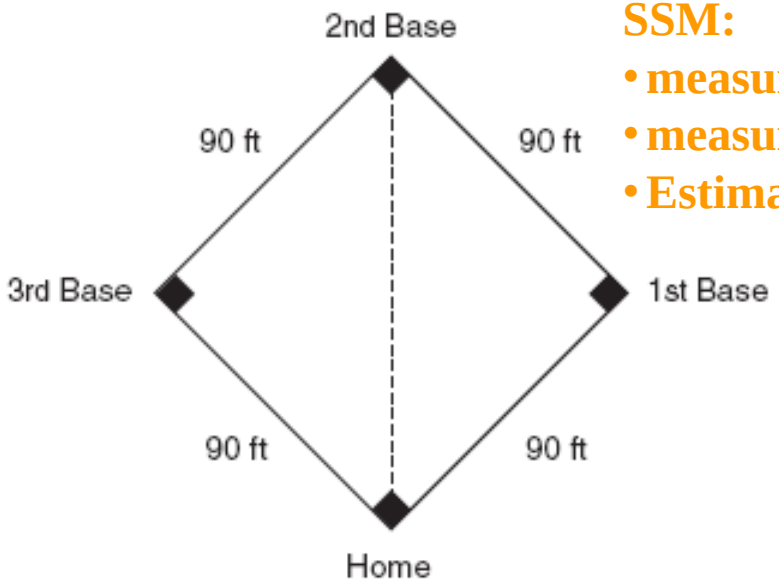
$49 + 64 = x^2$

$113 = x^2$

$10.6 = x$

$(10 < x < 11)$ without a calculator

22 **A baseball diamond is in the shape of a square, 90 feet on a side.**



SSM:
- **measure 90 with scrap paper**
- **measure H-2B with scrap paper**
- **Estimate answer**

What is the direct distance from home plate to second base?

F 90 ft

G $90\sqrt{2}$ ft

H $90\sqrt{3}$ ft

J 180 ft

Pythagorean Theorem:

$90^2 + 90^2 = x^2$

$8100 + 8100 = x^2$

$16200 = x^2$

$90\sqrt{2} = x$

Special Case Right Δs

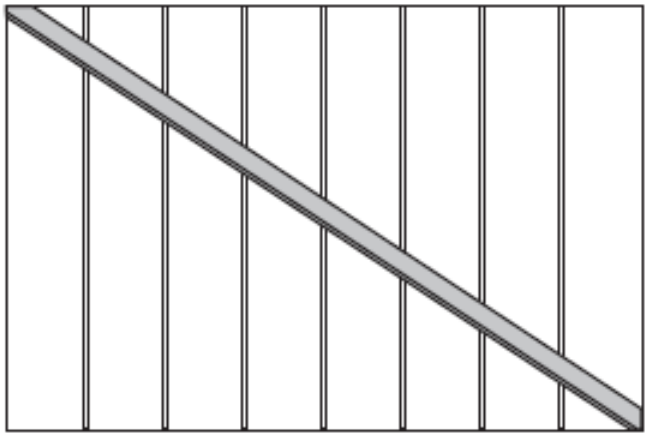
$90 = \frac{1}{2}\ hyp\ \sqrt{2}$

$180 = hyp\ \sqrt{2}$

$180 / \sqrt{2} = hyp$

$90\sqrt{2} = hyp$

23



SSM:

- measure 9 with scrap paper
- measure 12 with scrap paper
- If seems to scale (or close), then measure diagonal
- Estimate answer

What is the length of a diagonal brace that could be used for a wall 9 feet high and 12 feet long?

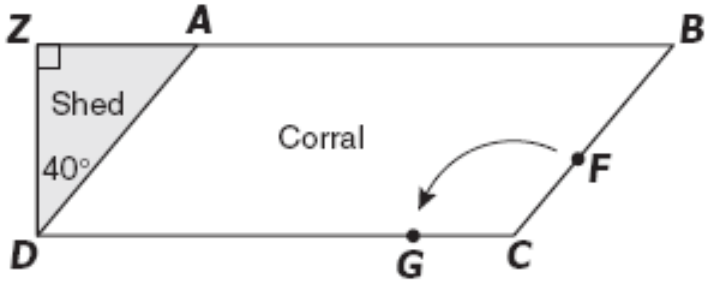
A 12 ft
B 13 ft
C 14 ft
D 15 ft

Pythagorean Theorem:

$9^2 + 12^2 = x^2$
$81 + 144 = x^2$
$225 = x^2$
$15 = x$

Pythagorean Triple!
(3-4-5) × 3

24 Gene's horse corral, labeled *ABCD* in the drawing, is shaped as a parallelogram and is adjacent to the shed, labeled *ZAD*.



SSM:
- angle is obtuse

If a gate, labeled *CF*, opens all the way to the corral fence, position labeled *CG*, through how many degrees does the gate swing?

F 40°
G 50°
H 130°
J 140°

Parallelograms: consecutive angles are supplementary

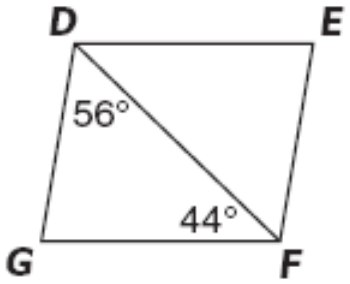
angle D forms a right angle with 40° angle, so angle D = 50
180 – 50 = 130

or

Parallelograms: opposite angles congruent

angle A is exterior angle to shed so angle A = 40 + 90 = 130
angle C must be 130

25 A diagonal of parallelogram *DEFG* forms angles with measures as shown.



SSM:
- **angle is *large* acute**
- **only C applies**

What is the measure of $\angle DEF$?

A 44°
B 56°
C 80°
D 100°

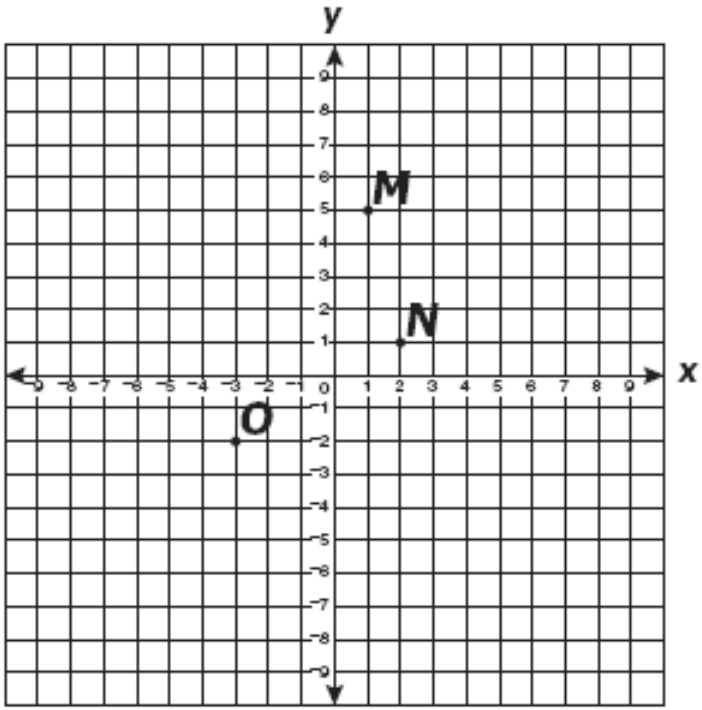
Parallelograms: diagonals form alternate interior angles

so angle D is 56 + 44 = 100

Parallelograms: consecutive angles supplementary

angle E = 180 – 100 = 80

26



SSM:

- **plot each of the answer points (-5, 2), (-4, 2), (-3, 2) and (0,2)**
- **Use eyes to see which is correct**

Quadrilateral *MNOP* is a parallelogram. The coordinates of three of its vertices are $M(1,5)$, $N(2,1)$, and $O(-3,-2)$. If $(x,2)$ are the coordinates of *P*, what is the value of *x* ?

F −5

G −4

H −3

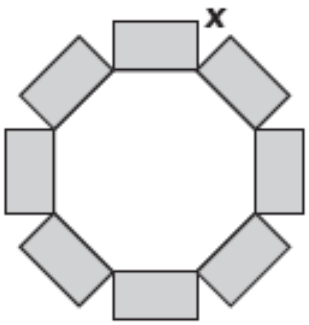
J 0

Parallelograms: opposite side parallel

so slope from MN must be same for PO

over (left) 1 and up 4 gets to (-4, 2)

27 Rectangular flowerbeds are built on each side of a fishpond in the shape of a regular octagon.



SSM:
- angle is acute
- answer D is wrong

What is the measure of the angle, *x*, between two consecutive flowerbeds?

A 30°

B 45°

C 60°

D 90°

Octagon: 360 / 8 = 45 (exterior angle measure)

Once around a point is 360°
Interior angle + exterior angle = 180
two rectangle corners (90+90) = 180

so x must be the same as the exterior angle!

x = 45

28

A portion of a regular polygon is shown. The polygon has —

F 15 sides
G 16 sides
H 18 sides
J 20 sides

SSM:
- no real help

number of sides, n is obtain by:

n = 360 / ext angle
= 360 / 24
= 15

29 **Each interior angle of a regular polygon has a measure of 162°. The polygon has a total of —**

A 17 sides
B 18 sides
C 19 sides
D 20 sides

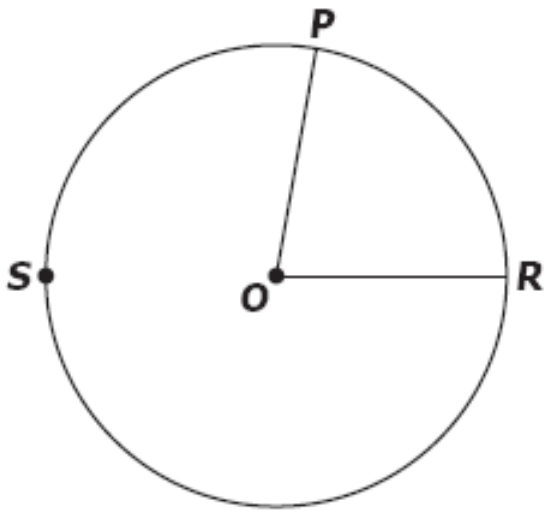
SSM:
- **no real help**

Exterior angle + interior angle = 180
ext + 162 = 180
ext = 18

number of sides, n is obtain by:

n = 360 / ext angle
= 360 / 18
= 20

30 In circle O, the degree measure of $\overset{\frown}{PSR}$ is 280°.



What is the degree measure of $\angle POR$?

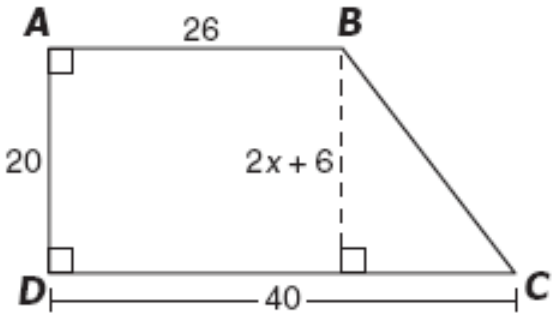
F 160°
G 85°
H 80°
J 40°

SSM:
- angle is *large* acute
- Answers F and J wrong

once around the circle is 360
so arcs PSR + PR = 360
280 + PR = 360
PR = 80

∠POR is a central angle and equal to its arc
∠POR = 80

31



SSM:

- measure side with x in it
- compare with other sides
- must be equal to 20
- plug in answers to see which equals 20

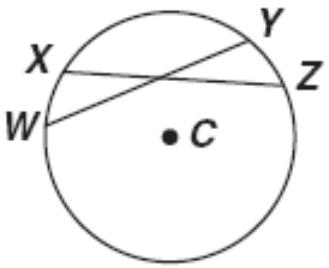
What is the value of *x* in trapezoid *ABCD*?

A 17
B 13
C 10
D 7

side with 2x + 6 forms a rectangle
opposite side of a rectangle are equal

2x + 6 = 20
2x = 14
x = 7

32 In circle C, m $\overset{\frown}{WX} = 25°$, m $\overset{\frown}{XY} = 40°$, m $\overset{\frown}{YZ} = 25°$, and $WY = 24$ centimeters.



SSM:
- measure WY and compare to XZ

What is the length of $\overline{XZ}$?

F 12 cm
G 24 cm
H 25 cm
J 65 cm

chords that cut a circle into equal arcs are equal in length

WY cuts circle into 25 + 40 = 65° arc
XZ cuts circle into 25 + 40 = 65° arc

33 **A pizza has a diameter of 16 inches. Which is closest to the area of one slice if the pizza is divided into 6 equal pieces?**

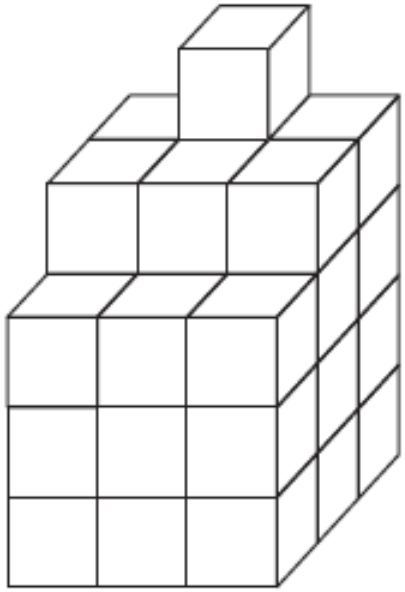
A 134.1 sq in.
B 117.1 sq in.
C 67.2 sq in.
D 33.5 sq in.

SSM:
- **no real help**
- **use equation sheet**
- **r = 8**

area of a circle = πr^2
$= \pi 8^2$
$= 64\pi$

divided by 6 gives $10.67\pi = 33.5$

34



SSM:

- **count the blocks**

Assuming the solid is constructed from cubes measuring 1 unit on each edge and that the figure is completely solid, what is the volume of the cubic solid shown above?

F 12 cubic units

G 34 cubic units

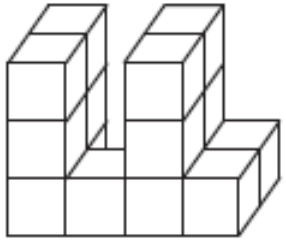
H 59 cubic units

J 68 cubic units

There are 9 blocks in the face and 3 deep = 27 and 7 blocks on top

34 blocks each are 1 cubic units each

35

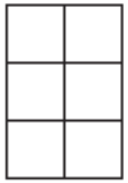


SSM:

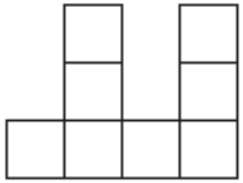
- **look at the tops of each block in picture**

Which could *not* be a two-dimensional view of the block of cubes shown above?

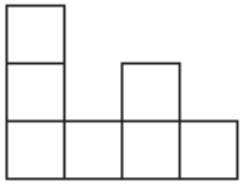
A



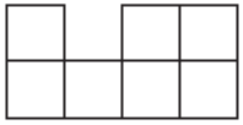
B



C



D



4 rows of blocks; all but the 2nd row has two blocks deep

36 **Which is closest to the volume of a sphere with a radius equal to 8 centimeters?**

F 267.9 cm^3

G 803.8 cm^3

H 1,607.7 cm^3

J 2,143.6 cm^3

SSM:

- **use formula sheet**
- **r = 8**

$$V = (4/3)\pi r^3$$
$$= (4/3)\ \pi 8^3$$
$$= (4/3)\ \pi\ 512$$
$$= 682.67\ \pi$$
$$= 2144.66$$

37 What is the total surface area of a rectangular prism box that measures 5 feet by 1 foot by 1 foot?

A 5 sq ft
B 20 sq ft
C 22 sq ft
D 30 sq ft

SSM:
- use formula sheet
- l = 5; w = 1; h = 1

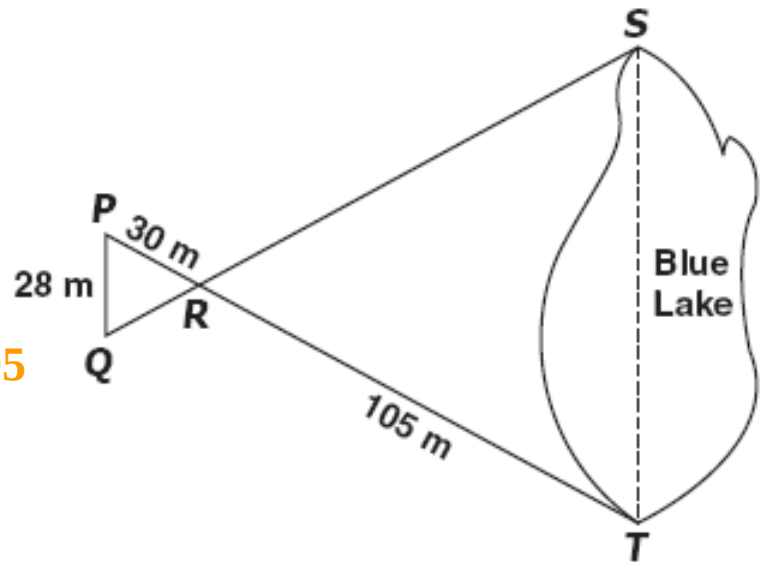
$$SA = 2(lw + lh + wh)$$
$$= 2(5\times1 + 5\times1 + 1\times1)$$
$$= 2(5 + 5 + 1)$$
$$= 2(11)$$
$$= 22$$

38

SSM:

- **measure the 30 side**
- **measure the 105 side**
- **Not to scale!**

- **Answer closest # to 105 since 28 is close to 30**



If $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{ST}$, what is *ST*, the width of the lake?

F 62 meters
G 70 meters
H 84 meters
J 98 meters

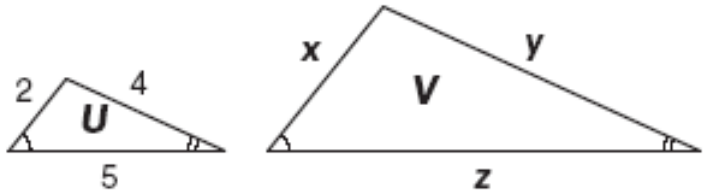
set up similar triangles:

$$\frac{28}{30} = \frac{ST}{105}$$

30 ST = 2940

ST = 98

39 The ratio of the perimeter of $\triangle U$ to the perimeter of $\triangle V$ is 1 : 2.



If the triangles are similar, what is the value of $x + y$?

A 3
B 6
C 12
D 18

SSM:

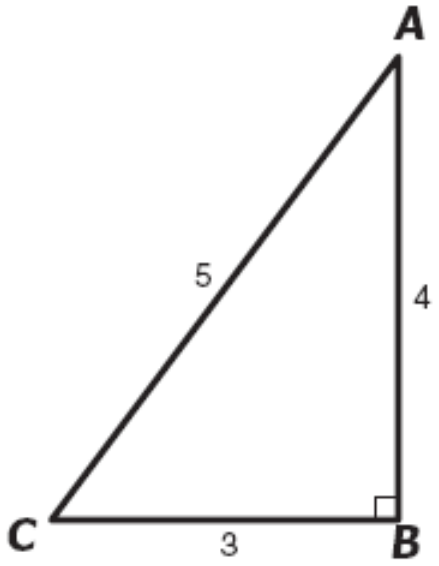
- must be bigger than 6!
- measure the 2 side measure the 4 side seems to scale!
- Use 4 side to measure x and the y

Use scaling factor of similar triangles:

ΔV is twice as big as ΔU

2 + 4 = 6 and 6 × 2 = 12

40 Right triangle *ABC* has the measures shown.



SSM:
- How can the triangle be folded in half?
- It can't!

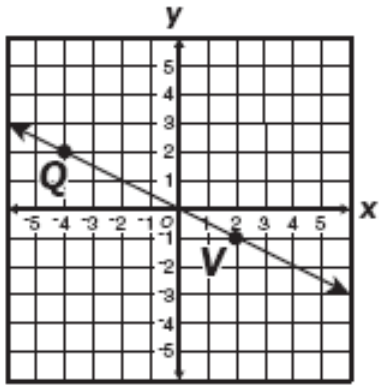
What is the *maximum* number of different lines of symmetry that can be drawn through △*ABC*?

F 0
G 1
H 2
J 3

line of symmetry means to be able to fold in half

scalene triangles can not be folded in half
so no lines of symmetry

41



SSM:
- **falling → negative slope**
- **less than a one to one change**

What is the apparent slope of $\overleftrightarrow{QV}$?

A -2

B $-\frac{1}{2}$

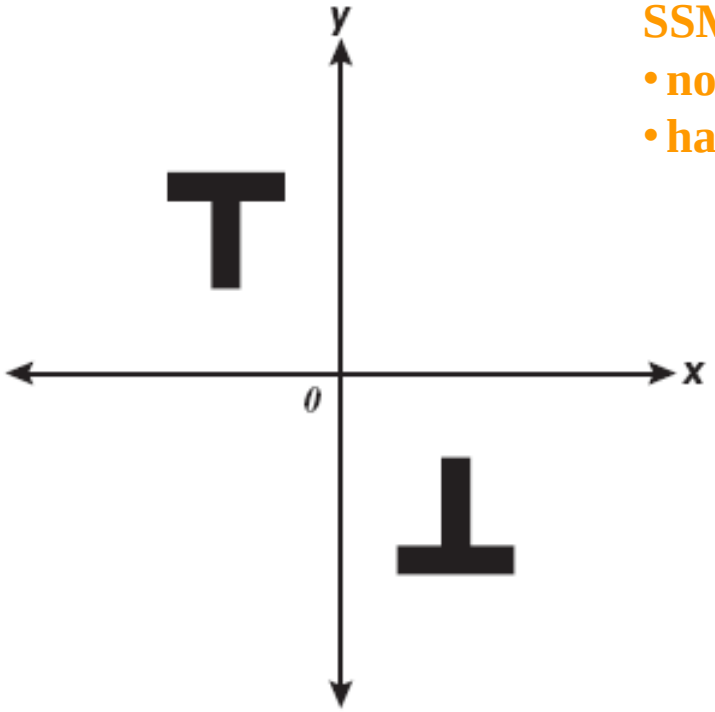
C $\frac{1}{2}$

D 2

slope = $\Delta y / \Delta x$

$$\frac{(2 - -1)}{(-4 - 2)} = \frac{3}{-6} = \frac{1}{-2}$$

42



SSM:
- not a reflection across a line!
- half way around the origin

In relation to one figure, the other figure is apparently a —

F reflection across the line $y = 1$

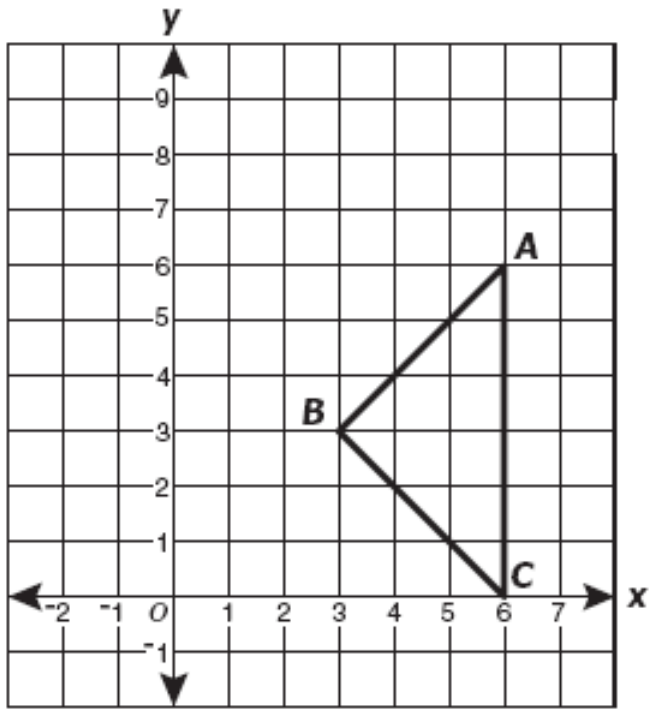
G reflection across the line $y = x$

H 90° rotation about the origin

J 180° rotation about the origin

reflection across the origin is the same as a 180° rotation about the origin

43 Triangle *ABC* is placed on a grid as shown.



The apparent midpoint of $\overline{AB}$ is —

A $(1.5, 1.5)$

B $(3, 3)$

C $(4.5, 4.5)$

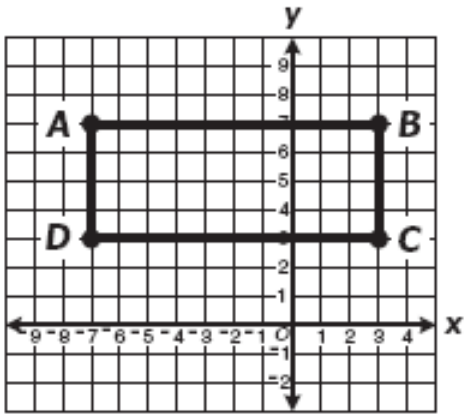
D $(4.5, 1.5)$

SSM:
- **plot each answer on graph paper**
- **only answer C between AB**

midpoint formula: $\left((x_1+x_2)/2 , (y_1+y_2)/2 \right)$

$\left((3+6)/2 , (3+6)/2 \right) = (9/2, 9/2) = (4.5, 4.5)$

44 Rectangle *ABCD* is placed in a coordinate plane as shown.



SSM:
- graph each answer
- which cuts rectangle in half?

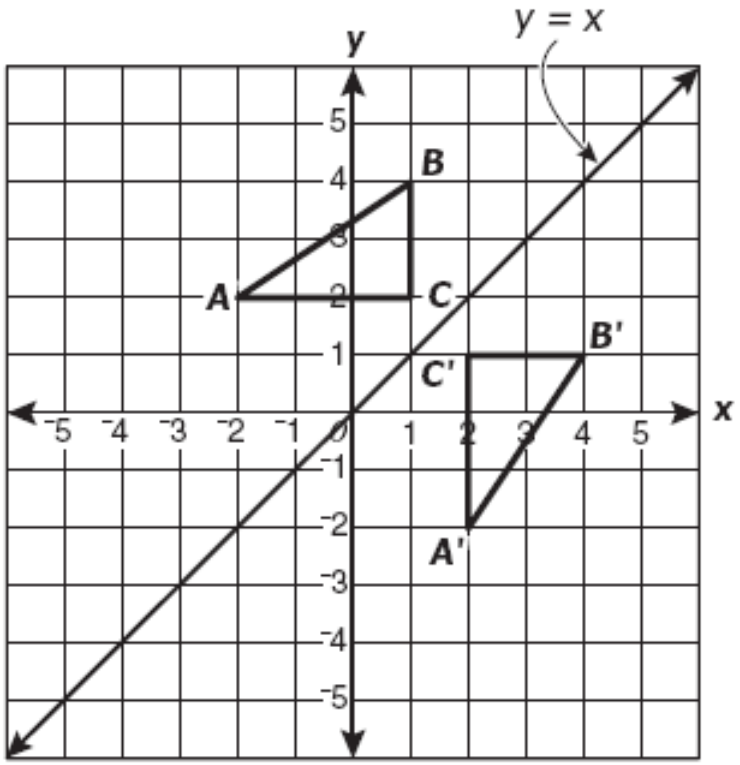
Which equation describes a line of symmetry for rectangle *ABCD*?

F $x = 2$
G $x = 5$
H $y = 5$
J $y = x$

Line y = 5 cuts the rectangle into two halves

so it is a line of symmetry

45



SSM:
- reflections → equal distance
- folded over line y = x

$\triangle A'B'C'$ is apparently the result of —

A reflecting $\triangle ABC$ across the y-axis

B reflecting $\triangle ABC$ across the x-axis

C rotating $\triangle ABC$ about the point (1, 2)

D reflecting $\triangle ABC$ across the line $y = x$

reflection across line y = x

points and their reflections are equal distance from line of reflection