

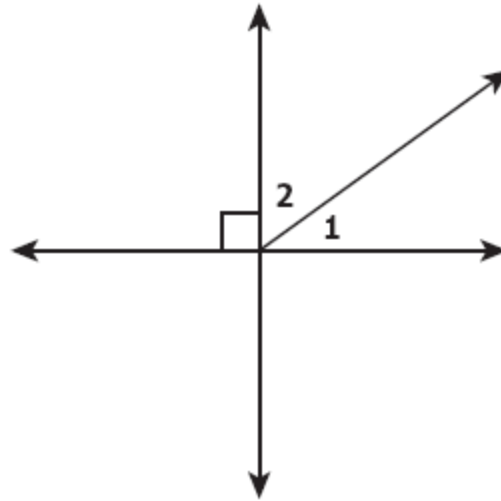
Modified and Animated By Chris Headlee
June 2010

END OF COURSE GEOMETRY

Form M0119, CORE 1

SSM: Super Second-grader Methods

1 In the figure shown, $m\angle 1 = (4x + 12)^\circ$ and $m\angle 2 = (6x + 8)^\circ$.



SSM:

- Corner of scratch paper tells us that $\angle 1 + \angle 2 = 90^\circ$
- Our eyes tell us that $\angle 2 > \angle 1$, but not by much
- Answers A & B are possible, with B fitting better

What is $m\angle 1$?

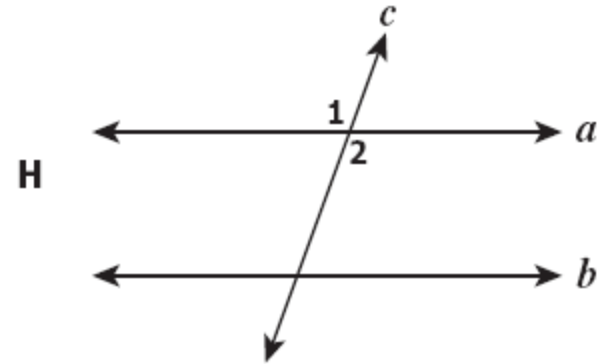
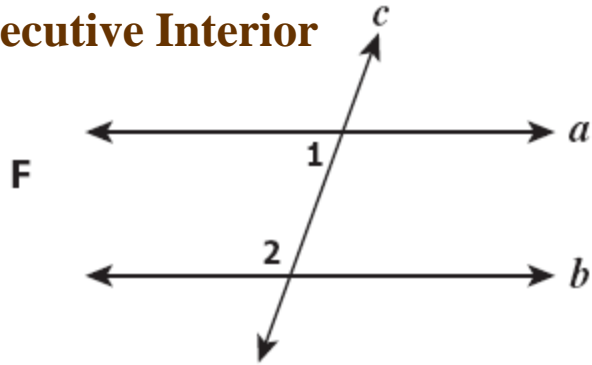
- A 20°
- B 40°**
- C 50°
- D 76°

Angle 1 and Angle 2 are complimentary

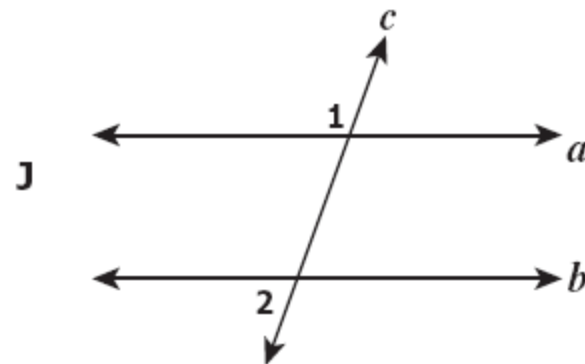
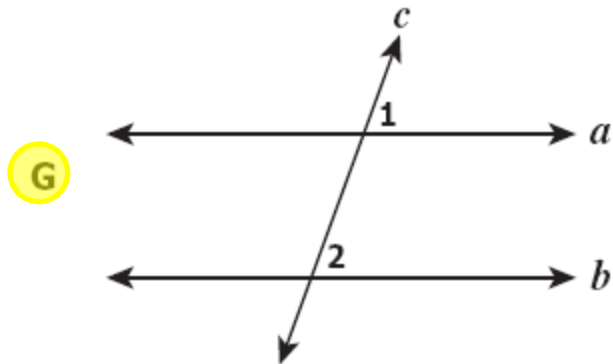
$$\begin{aligned}
 \angle 1 + \angle 2 &= 90^\circ \\
 (4x + 12) + (6x + 8) &= 90 \\
 10x + 20 &= 90 \\
 10x &= 70 \\
 x &= 7 \\
 m\angle 1 &= 4(7) + 12 = 40^\circ
 \end{aligned}$$

2 In each of the following figures, transversal c cuts lines a and b . In which figure are $\angle 1$ and $\angle 2$ corresponding angles?

Consecutive Interior



Vertical



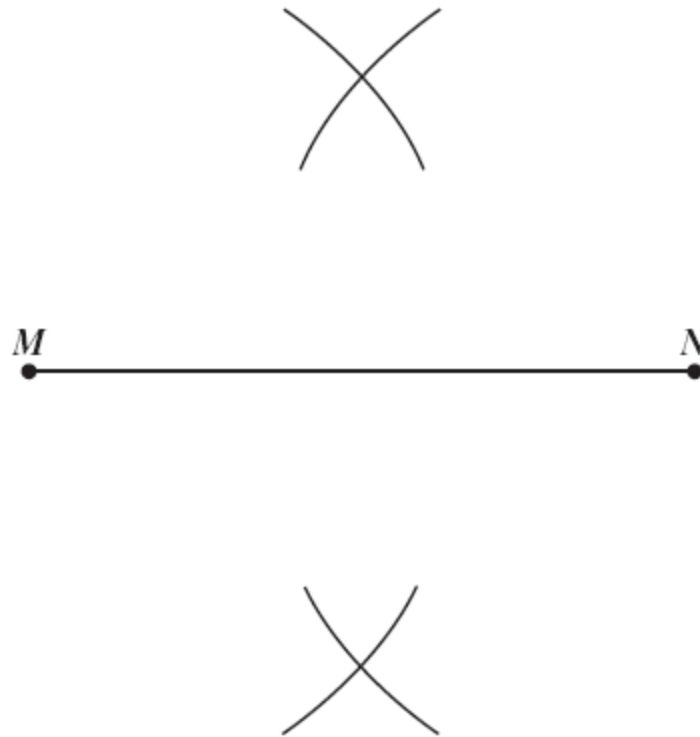
No name

Corresponding Angles:
same side of the transversal
same relationship to other line

SSM:

- Must be same size
- Two different lines involved

3 The arcs for a compass and straightedge construction are shown below.



**Equal Distances from the endpoints M and N:
Isosceles triangle formed
Altitude bisects MN**

Which construction is apparently being made?

- A** Two lines parallel to \overline{MN}
- B** Two congruent angles
- C** A segment congruent to \overline{MN}
- D** The perpendicular bisector of \overline{MN}

SSM:

- **Connect the “x”s**
- **See what’s drawn**

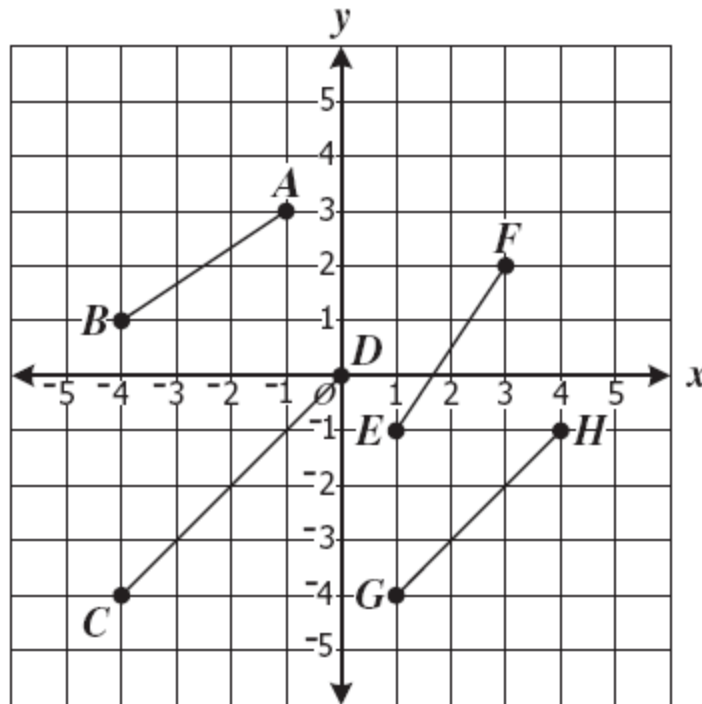
4

$$m_{AB} = 2/3$$

$$m_{CD} = 1$$

$$m_{EF} = 3/2$$

$$m_{AB} = 1$$



SSM:

- Use straight-edge tool to continue lines as far as possible
- Parallel never can cross
EF and AB cross CD
EF crosses GH

Which two segments in the drawing above are most likely parallel?

F \overline{CD} and \overline{GH}

G \overline{CD} and \overline{AB}

H \overline{AB} and \overline{EF}

J \overline{EF} and \overline{GH}





Parallel: same slopes

calculate slopes: $m = \Delta y / \Delta x$

5



Which segment has a measure equal to $\frac{1}{2}(PQ)$?

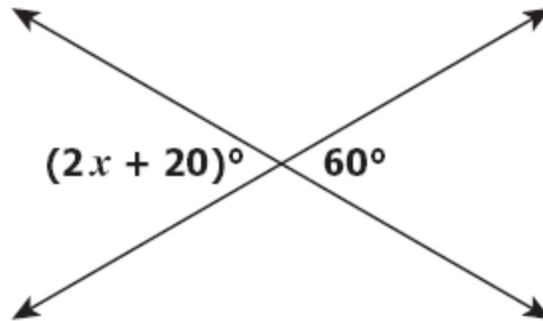
- A 
- B 
- C 
- D** 

SSM:

- Use ruler if available
- Use scratch paper as ruler

double each answer and see which one is the same length as PQ

6 Two lines intersect as shown.



What is the value of x ?

- F** 20
- G** 40
- H** 50
- J** 60

SSM:

- Two acute angles:
assume equal
- Plug answers in for x

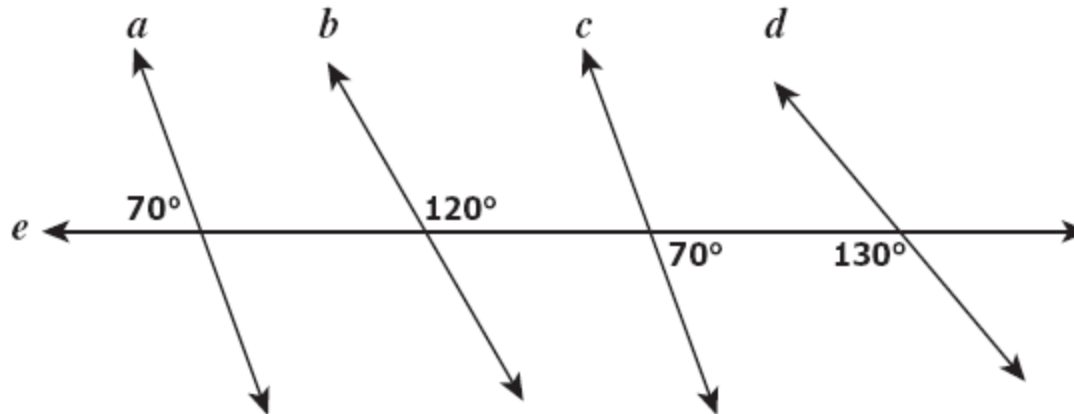
Vertical Angles are equal

$$2x + 20 = 60$$

$$2x = 40$$

$$x = 20$$

7 In this figure, transversal e intersects lines a , b , c , and d .



Which lines *must* be parallel?

- A** a and c
- B** b and c
- C** b and d
- D** a and d

SSM:

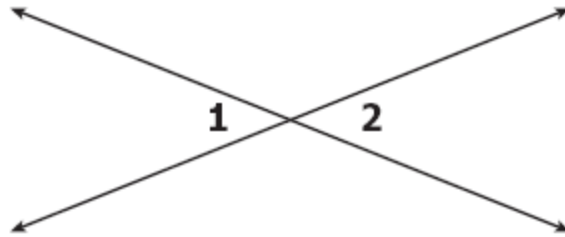
- Use straight-edge tool to continue lines as far as possible
- Parallel never can cross
 b crosses a and c
 d crosses all of them

If lines are parallel, then all acute angles are equal
 and all obtuse angles are equal

Form linear pair combos with given angles

Pick which has same pairs of angles

8 In the diagram, $m\angle 1 = (6x + 12)^\circ$ and $m\angle 2 = (9x - 4)^\circ$.



Which is closest to the value of x ?

- F** 5.3
- G** 5.5
- H** 11.5
- J** 12.5

SSM:

- Our eyes tell us that $\angle 1$ and $\angle 2$ are small acute angles
- Plug in answers:
only F and G give small acute angles

Vertical angles are equal

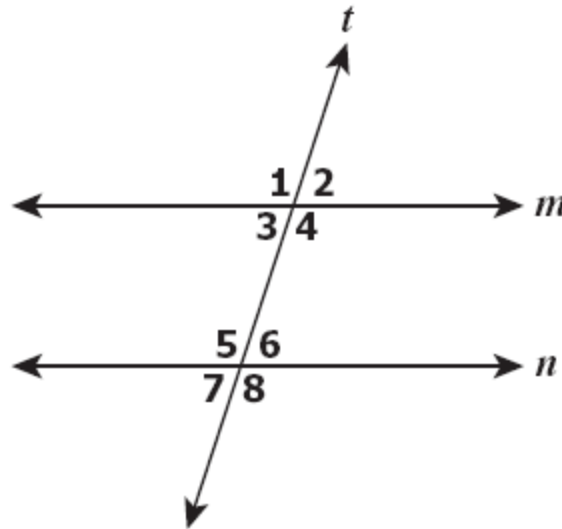
$$6x + 12 = 9x - 4$$

$$6x + 16 = 9x$$

$$16 = 3x$$

$$16/3 = x$$

9 In this figure, line t is a transversal of lines m and n .



SSM:

- Angles must deal with both lines m and n :
 angles 1 and 4 deal with m
 angles 2 and 7 deal with both
 angle 5 is obtuse \rightarrow c is false
 angles 6 and 8 deal with n

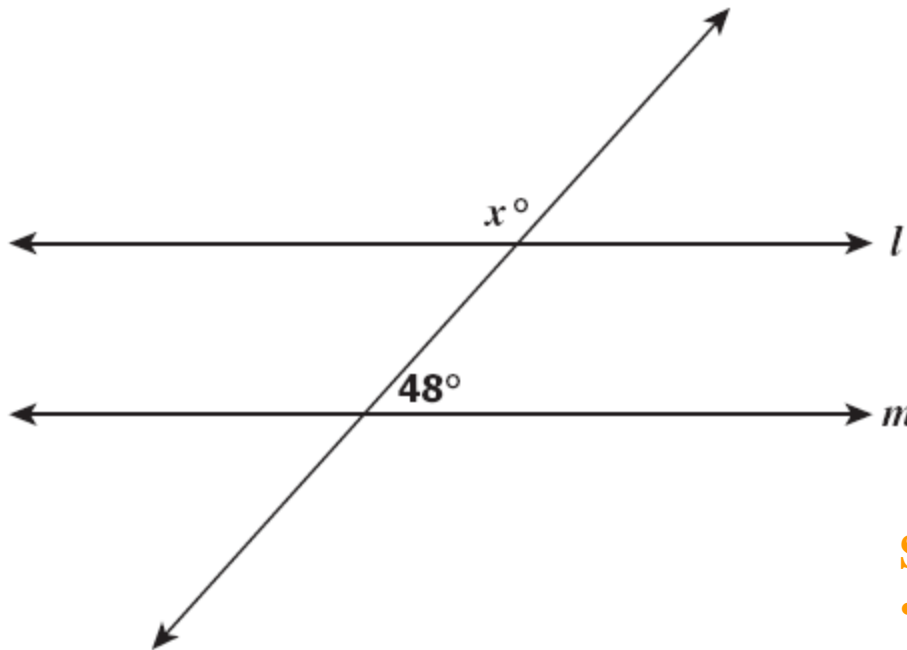
Which of the following statements determines that lines m and n are parallel?

- A $\angle 1 \cong \angle 4$
- B $\angle 2 \cong \angle 7$**
- C $\angle 3$ and $\angle 5$ are complementary
- D $\angle 6$ and $\angle 8$ are supplementary

Parallel lines:

Alternate exterior angles are equal

10 For what value of x is line l parallel to line m in this figure?



SSM:

- x is obtuse (answers F & G are wrong)
- 180 is a magic number

- F 42
G 48
H 132
J 138

Vertical angles are equal

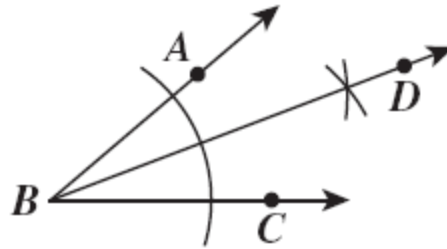
Consecutive interior angles are supplementary

$$48 + x = 180$$

$$x = 180 - 48$$

$$x = 132$$

11 Amber constructed \overrightarrow{BD} as shown.



Which of the following statements *must* be true?

- A $BA = BC$
- B $BD = 2BA$
- C** $m\angle ABD = m\angle CBD$
- D $m\angle CBD = 2m\angle ABC$

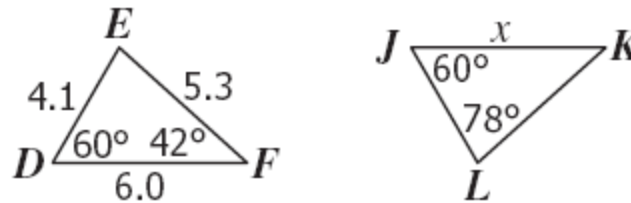
SSM:

- Our eyes tell us that $\angle ABD$ and $\angle DBC$ are equal

Construction is an angle bisector

$\angle ABC$ is the whole and $\angle ABD$ and $\angle CBD$ are the halves

12



What value of x makes $\triangle DEF \cong \triangle JKL$?

- F $x = 9.4$
- G** $x = 6.0$
- H $x = 5.3$
- J $x = 4.1$

SSM:

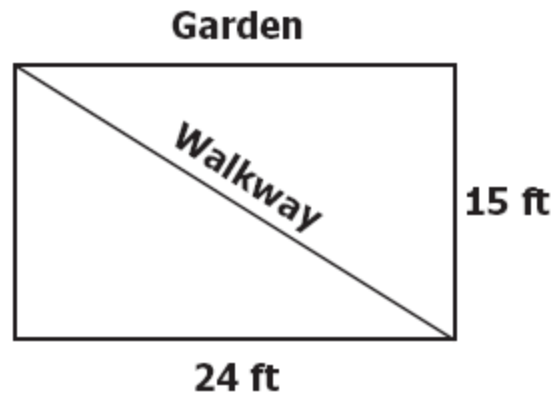
- Our eyes tell us that JK and DF are longest sides
- Congruent means equal so $JK = DF$

ASA provides triangle congruence

$\angle D \cong \angle J$ and $\angle K \cong \angle F$ (after solving for missing angles)

the included side between the two pairs of angles must be equal

- 13 Mr. Ammons is constructing a walkway through his rectangular garden. The walkway runs diagonally as shown in the diagram.



Which is closest to the length of the walkway?

- A 18.7 ft
- B 28.3 ft**
- C 30.0 ft
- D 39.0 ft

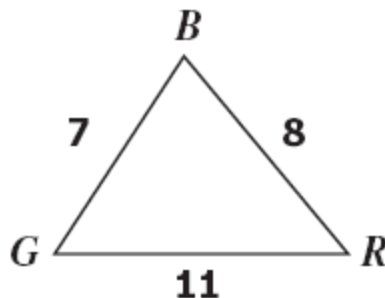
SSM:

- Our eyes tell us that the walkway must be bigger than both sides (A is wrong)
- Answer D does not form a triangle (so it is wrong)

Pythagorean Thrm:

$$\begin{aligned}15^2 + 24^2 &= W^2 \\225 + 576 &= W^2 \\801 &= W^2 \\28.3 &= W\end{aligned}$$

14 In the triangle shown, $GR = 11$, $BR = 8$, and $BG = 7$.



SSM:

• Remember the virtual alligator
largest stick \rightarrow largest mouth

Which statement is true about the angles in $\triangle RGB$?

F $m\angle R$ is the greatest

G $m\angle G$ is the greatest

H $m\angle R$ is the least

J $m\angle G$ is the least

Order the measures of sides from largest to smallest:

$$11 > 8 > 7$$

Replace with the letter of the sides

$$GR > BR > BG$$

Put in the missing letter of the triangle

$$B > G > R$$

Now the angles are ordered from largest to smallest:

$$\angle B > \angle G > \angle R$$

15 Consider the following statement.

If $4x = 8$, then $x = 2$.

Which is the inverse of the statement?

- A If $x = 2$, then $4x = 8$.
- B If $x \neq 2$, then $4x \neq 8$.
- C If $x = 2$, then $4x \neq 8$.
- D** If $4x \neq 8$, then $x \neq 2$.

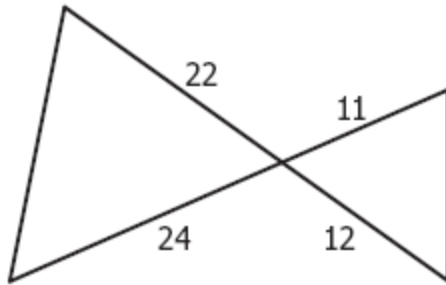
SSM:

- need to memorize the vocabulary

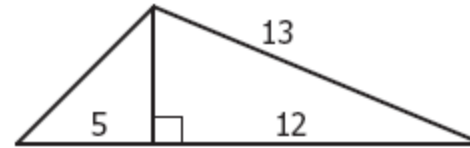
converse:	flip
inverse:	negates
contrapositive:	flips and negates

16 Which drawing contains a pair of similar triangles?

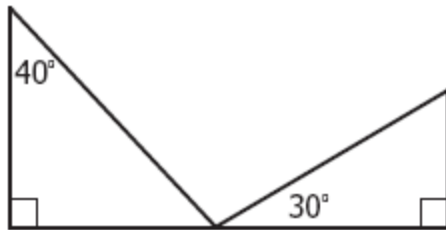
F



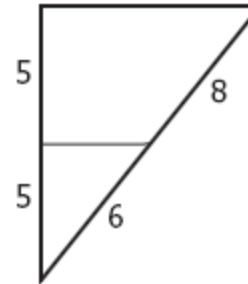
H



G



J



SSM:

• similar triangles → same shape

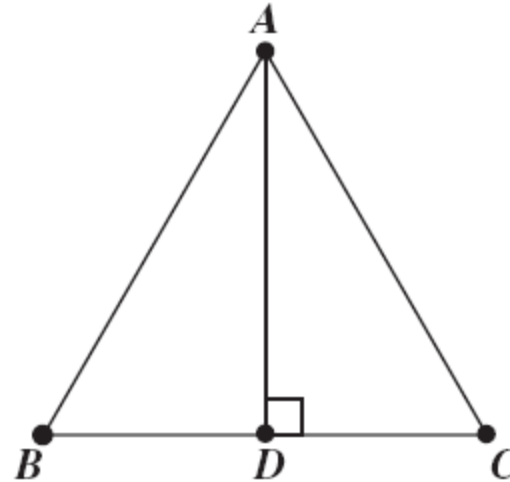
Check the ratios of the sides to see if they are the same

answer H and J don't

Similar triangles must have the same angles

answer G is wrong

17 Triangle ABC is an equilateral triangle with side lengths of 10 inches.



SSM:

- measure AC and compare to AD
- compare with answers

What is the length, in inches, of \overline{AD} ?

A 5

B $\frac{10\sqrt{3}}{3}$

C $5\sqrt{2}$

D $5\sqrt{3}$

Special case right triangle

side opposite 60 is $\frac{1}{2}$ hyp $\sqrt{3}$
 $\frac{1}{2} (10) \sqrt{3}$
 $5 \sqrt{3}$

18 John wants to make a triangular garden. Which of the following are possible dimensions?

F 4 ft by 5 ft by 10 ft

G 6 ft by 6 ft by 12 ft

H 6 ft by 8 ft by 10 ft

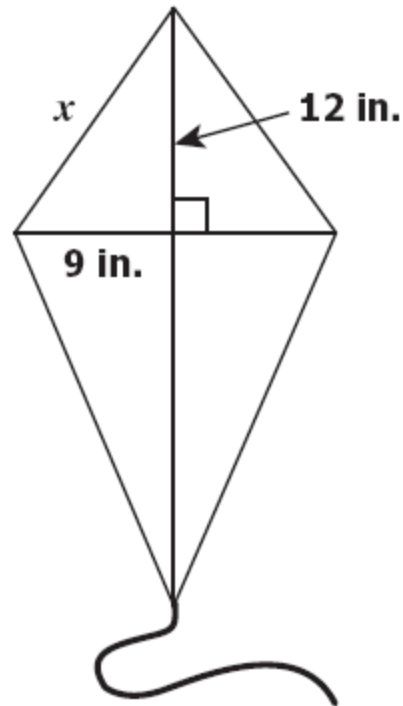
J 8 ft by 12 ft by 20 ft

SSM:

• try and graph it on graph paper

Take the smallest two numbers
add them together
if they are greater than the third,
then we can have a triangle

19 A drawing of Mark's kite is shown below.



SSM:

- $x > 12$
- Pythagorean triple $(3 - 4 - 5) \times 3$

What is the length of the short section of the outer frame indicated by x in the drawing?

- A 16 in.
- B 15 in.
- C 14 in.**
- D 13 in.

Pythagorean Theorem:

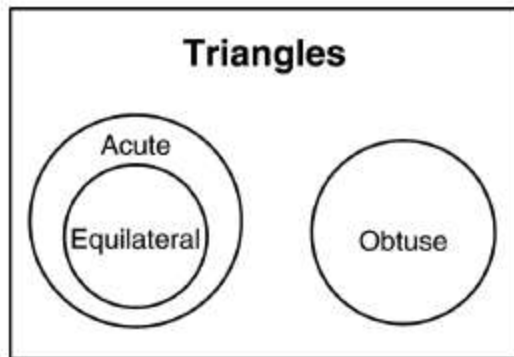
$$\begin{aligned}
 9^2 + 12^2 &= x^2 \\
 81 + 144 &= x^2 \\
 225 &= x^2 \\
 15 &= x
 \end{aligned}$$

20 Which Venn diagram represents all the following set of statements?

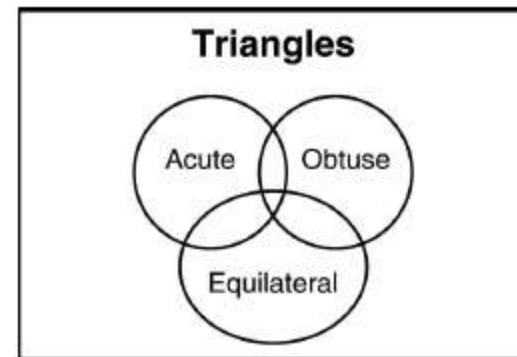
- Some triangles are acute.
- Some triangles are obtuse.
- No triangle is both acute and obtuse.
- Some acute triangles are equilateral.

statement 1 and 2 have no overlap
statement 1 and 4 overlap

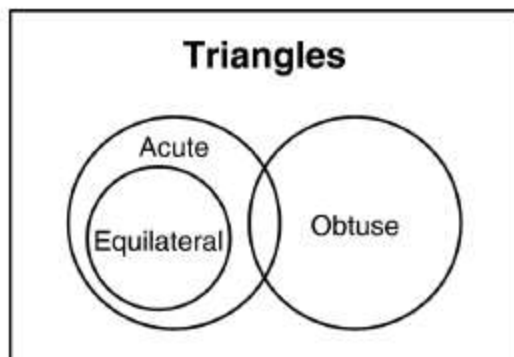
F



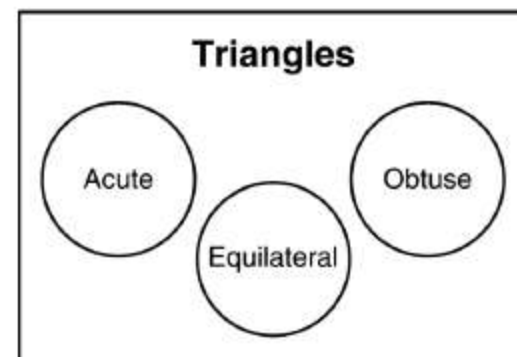
H



G

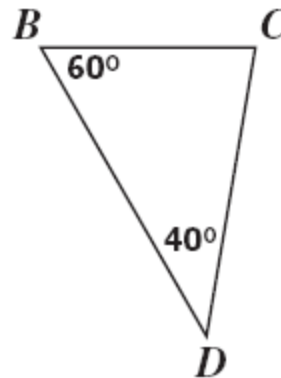


J



SSM:

• no help



SSM:

- Measure the sides with ruler or scrap paper
- BC is the shortest
- BD the longest

Which lists the sides of $\triangle BCD$ in order from shortest to longest?

- A $\overline{CD}, \overline{BD}, \overline{BC}$
- B** $\overline{BC}, \overline{CD}, \overline{BD}$
- C $\overline{BD}, \overline{CD}, \overline{BC}$
- D $\overline{BC}, \overline{BD}, \overline{CD}$

Order the measures of angles from smallest to largest:

$$40 < 60 < 80$$

Replace with the letter of the angles

$$\angle D < \angle B < \angle C$$

Put in the missing letters of the triangle

$$BC < CD < BD$$

Now the sides are ordered from smallest to largest:

$$BC < CD < BD$$

22 With the information given in the drawings, which pair of triangles can be proven congruent by the Side-Angle-Side postulate?

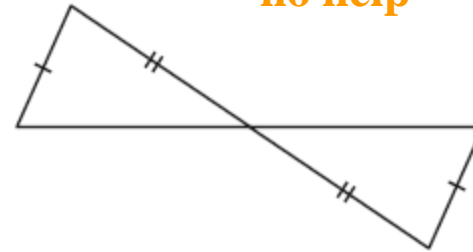
SSM:

• no help

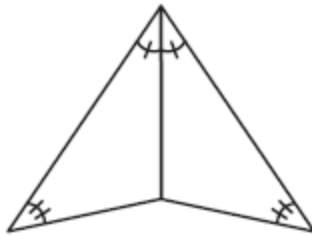
F



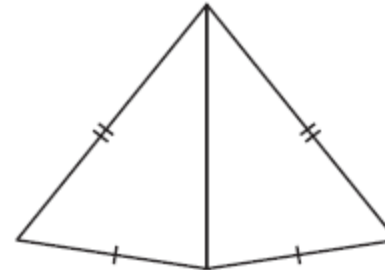
H



G



J



Replace the congruent angles with “A” and the congruent sides with “S”

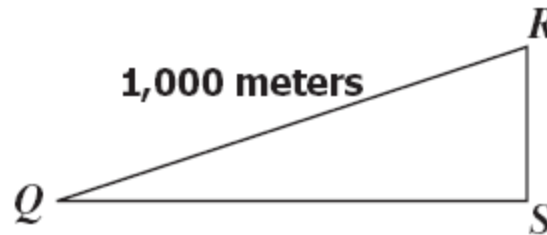
F – SAS

G – AAS (included side)

H – SSA (vertical angle)

J – SSS (included side)

23 Given: $\triangle QRS$ where $m\angle Q = 20^\circ$ and $m\angle S = 90^\circ$



What is the length, to the nearest meter, of \overline{RS} ?

- A** 342 m
- B** 364 m
- C** 500 m
- D** 940 m

SSM:

- RS is smallest side and less than $\frac{1}{2}$ hypotenuse
- answers A or B

Trigonometry problem:

SOH CAH TOA

label the sides of the triangle:

QR (1000) is hyp

RS (x) is opp

QS is adj

must use sin

$$\sin 20^\circ = x / 1000$$

$$1000 (\sin 20^\circ) = x$$

$$342 = x$$

24 Which of the following quadrilaterals is *not* a parallelogram?

F Rectangle

G Rhombus

H Square

J Trapezoid

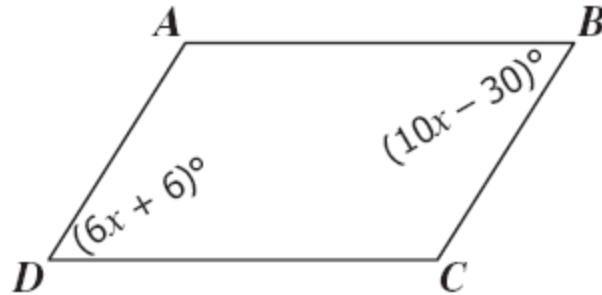
SSM:

- draw the answer and see if it looks like a parallelogram

Remember Quadrilateral hierarchy:

Parallelogram
Rectangle Rhombus
Square

Trapezoid



SSM:

- $\angle C$ is medium obtuse
answers A and B are wrong

In parallelogram $ABCD$, the measure of $\angle C$ is —

- A 82.5°
- B 97.5°
- C 120.0°**
- D 130.0°

Opposite angles are congruent:

$$6x + 6 = 10x - 30$$

$$6 = 4x - 30$$

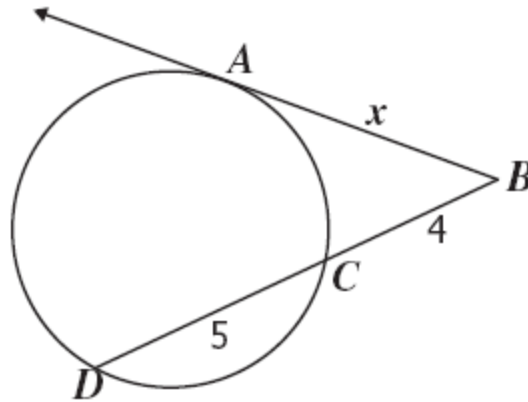
$$36 = 4x$$

$$9 = x$$

Consecutive angles are supplementary:

$$6(9) + 6 = 60 \quad 180 - 60 = 120$$

- 26 In the diagram, \overline{AB} is tangent to the circle at point A , and \overline{BD} intersects the circle at points C and D .



SSM:

- measure AB and compare to DC
- $AB > DC$

What is the value of x ?

- F 3
G 4
H 5
J 6

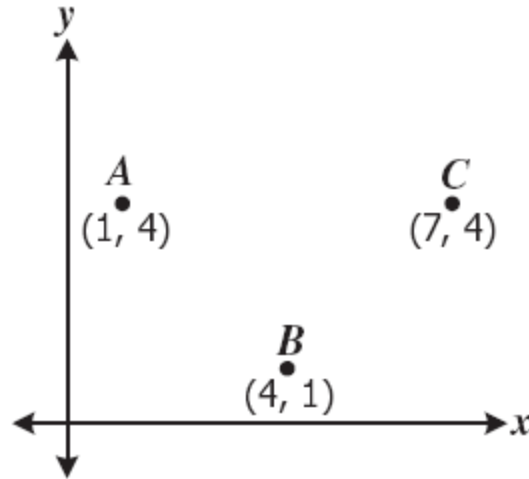
Outside \times whole = Outside \times whole

$$x \times (x + 0) = 4 \times (4 + 5)$$

$$x^2 = 4 \times 9$$

$$x^2 = 36$$

$$x = 6$$



SSM:

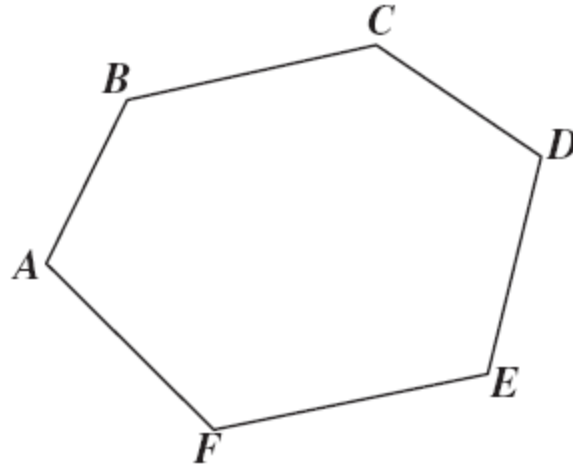
- graph all answers (points)
- pick the one that makes it a square (4,7)

In the drawing above, what must be the coordinates of D to show $ABCD$ is a square?

- A (7, 7)
- B (4, 7)**
- C (4, 5)
- D (4, 4)

Square – all sides equal

up 3 and right 3 to get from B to C, so
up 3 and right 3 to get from A to D (4, 7)

**SSM:**

- start with $n = 3$ and $S = 180$
- add 1 to n and 180 to S
- repeat until $n = 6$

Given the polygon shown above, $m\angle A + m\angle F + m\angle E + m\angle D + m\angle C + m\angle B =$

F 360°

G 540°

H 720°

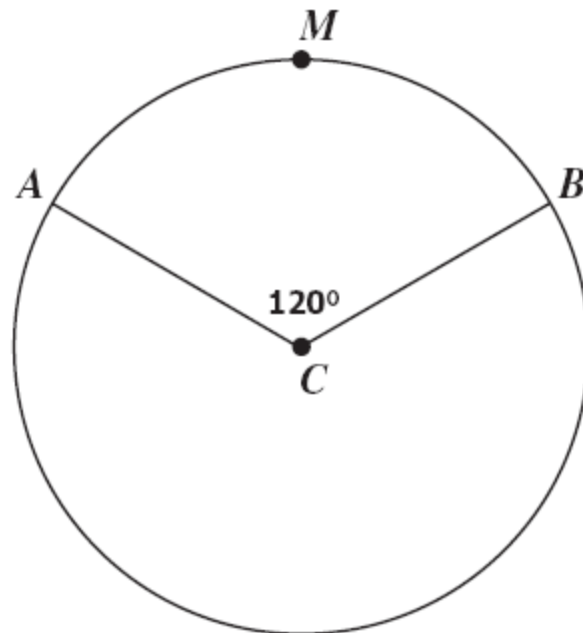
J 900°

$$\text{Sum of } \angle\text{'s} = (n - 2) \times 180$$

$$= (6 - 2) \times 180$$

$$= 720$$

29 The circumference of circle C is 144π .



SSM:

- how many 120's in 360?
- divide 144π by 3

What is the length of \widehat{AMB} ?

- A 8π
- B 16π
- C** 48π
- D 96π

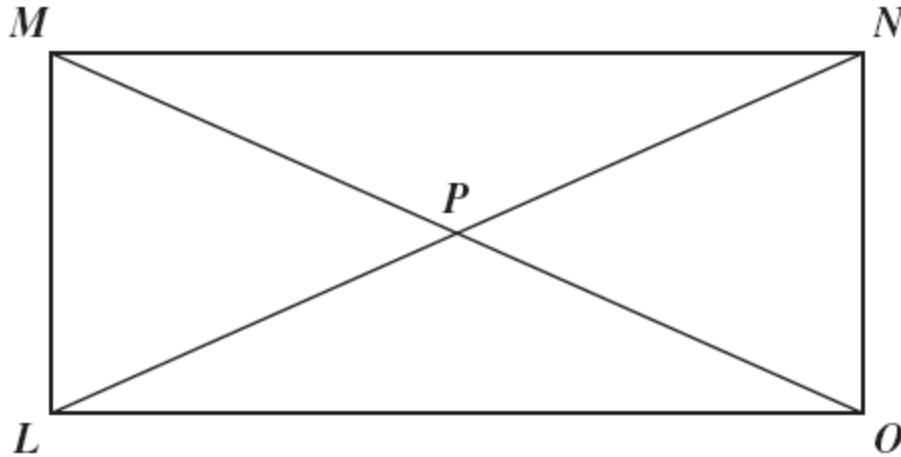
$$\frac{120}{360} = \frac{\text{arc AMB}}{C = 144\pi}$$

$$120 (144\pi) = \text{AMB} (360)$$

$$120 (144\pi) / 360 = \text{AMB}$$

$$48\pi = \text{AMB}$$

- 30 Rectangle $LMNO$ represents a park that has walking paths \overline{LN} and \overline{MO} that intersect at P . The length of \overline{PN} is 195 feet, and the length of \overline{MN} is 360 feet. What is the length of \overline{MO} , one of the walking paths?



- F 150 ft
- G 195 ft
- H 360 ft
- J 390 ft**

SSM:

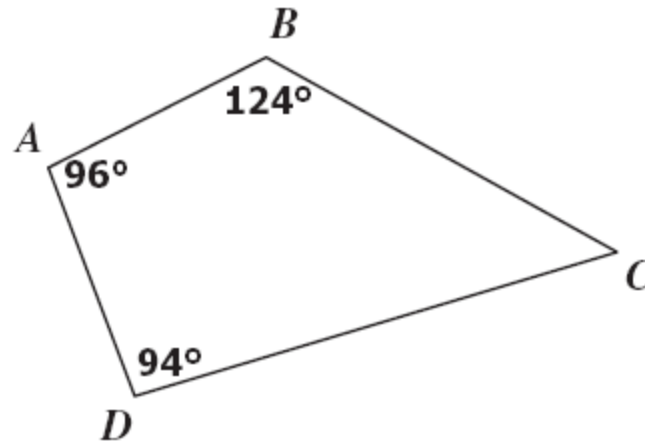
- measure PN
- compare with MP and PO
- add together

Rectangle's diagonals bisect each other and are equal

$$2(PN) = MO$$

$$2(195) = 390 = MO$$

31 What is the measure of $\angle C$ in quadrilateral $ABCD$?



- A** 46°
- B** 56°
- C** 86°
- D** 96°

SSM:

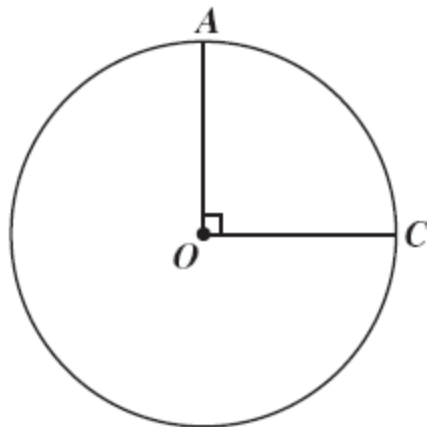
- $\angle C$ is middle acute so answers C and D are wrong
- compare with corner of paper folded in half (45° angle)

Sum of \angle 's = 360

$$360 = 94 + 96 + 124 + x$$

$$360 = 314 + x$$

$$46 = x$$



SSM:

- looks to be a quarter of the circle, so $360 / 4 = 90$

In circle O , the degree measure of \widehat{AC} is —

- F 45°
- G** 90°
- H 135°
- J 180°

central angle (90°) = the measure of its arc

33 When tiles are tessellated in a plane, what angle measure sum is required of the tiles surrounding a single point?

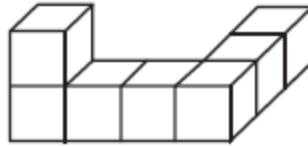
- A 90°
- B 180°
- C 360°**
- D 720°

SSM:

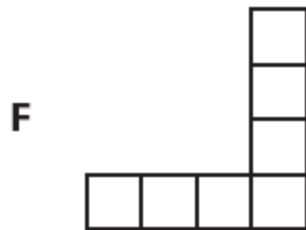
• once around a point is 360

tessellation → no gaps or overlaps
< 360° or 360° >

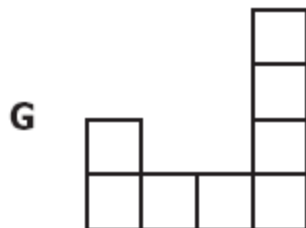
34 This solid figure is constructed with seven cubes.



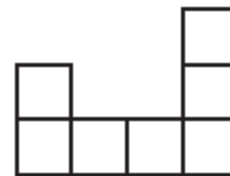
Which shape represents the top view of this three-dimensional solid?



H



J



picture cubes from the top
number outside edges for dimensions

SSM:

- count outside edges
4 across and 3 up

35 Which is closest to the total surface area of a cylinder with a radius of 5 inches and a height that is equal to its diameter?

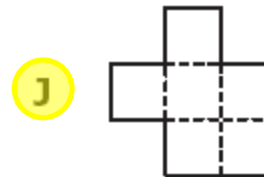
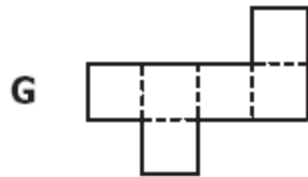
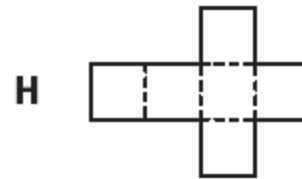
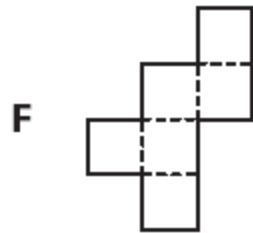
- A 314 sq in.
- B 471 sq in.
- C 596 sq in.
- D 785 sq in.**

$$\begin{aligned}V &= \pi r^2 h \\&= \pi(5)^2(2 \times 5) \\&= \pi(25)(10) \\&= 250\pi \\&\approx 785\end{aligned}$$

SSM:

- Find formula
- find variables
- plug in and solve

36 Which of the following nets could *not* be folded along the dotted lines to form a cube?



fold them up in your mind

SSM:

- Label each part either
F(front)
Bk (back)
S (side)
T (top)
B (bottom)
- one that has a missing ltr

- 37 The radius of Sphere *A* is 2 inches, and the radius of Sphere *B* is 4 inches. How many times larger is the volume of Sphere *B* compared to the volume of Sphere *A* ?

- A 2
B 3
C 4
D 8

SSM:

- Find formula
- find variables
- plug in and solve for each radius
- compare answers

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(2r)^3 \\ &= \frac{4}{3}\pi 8r^3 \end{aligned}$$

$$\begin{aligned} &\frac{4}{3}\pi(2)^3 \\ &\frac{4}{3}\pi(8) \\ &(\frac{32}{3})\pi \end{aligned}$$

$$\begin{aligned} &\frac{4}{3}\pi(4)^3 \\ &\frac{4}{3}\pi(64) \\ &(\frac{256}{3})\pi \end{aligned}$$

8 times larger volume

38 A cylinder has a diameter of 10 inches and a height four times its radius.
What is its volume?

- F** 500π cu in.
G $2,000\pi$ cu in.
H $4,000\pi$ cu in.
J $40,000\pi$ cu in.

SSM:

- Find formula
- find variables
- plug in and solve

$$d = 10 = 2r$$

$$5 = r$$

$$h = 4r = 4(5) = 20$$

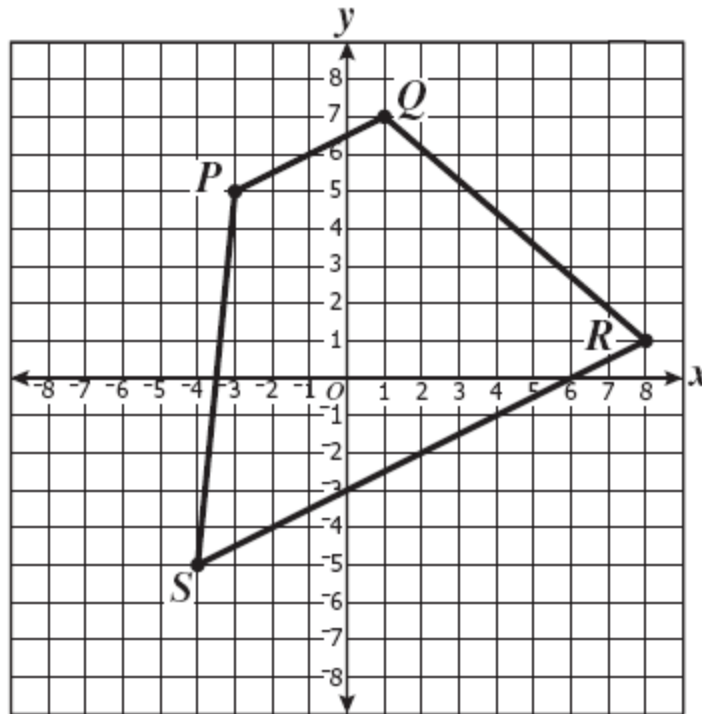
$$V = \pi r^2 h$$

$$= \pi(5)^2(20)$$

$$= \pi(25)(20)$$

$$= 500\pi$$

39 $P(-3, 5)$, $Q(1, 7)$, $R(8, 1)$, and $S(-4, -5)$ are connected to form a trapezoid.



Midpoint formula:

$$\left(\frac{(-4 + 8)}{2}, \frac{(-5 + 1)}{2} \right)$$

$$(2, -2)$$

What is the midpoint of \overline{SR} ?

- A (0, -3)
- B (4, -1)
- C (3, -1.5)
- D (2, -2)**

SSM:

- plot the answers (points)
- which is in the middle and on \overline{SR}

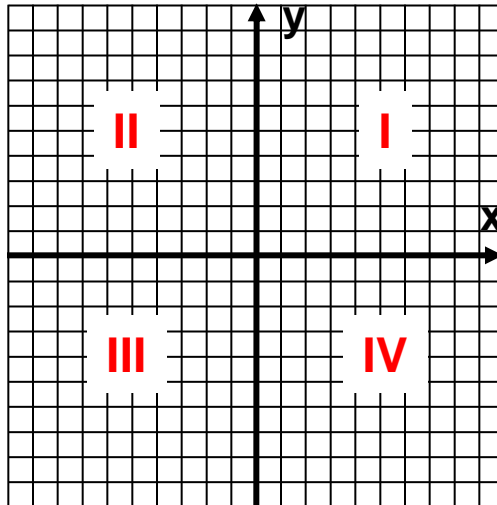
- 40 A trapezoid is located entirely in quadrant II. If this trapezoid is reflected across the x -axis, in which quadrant will the new trapezoid be located?

F I
G II
H III
J IV

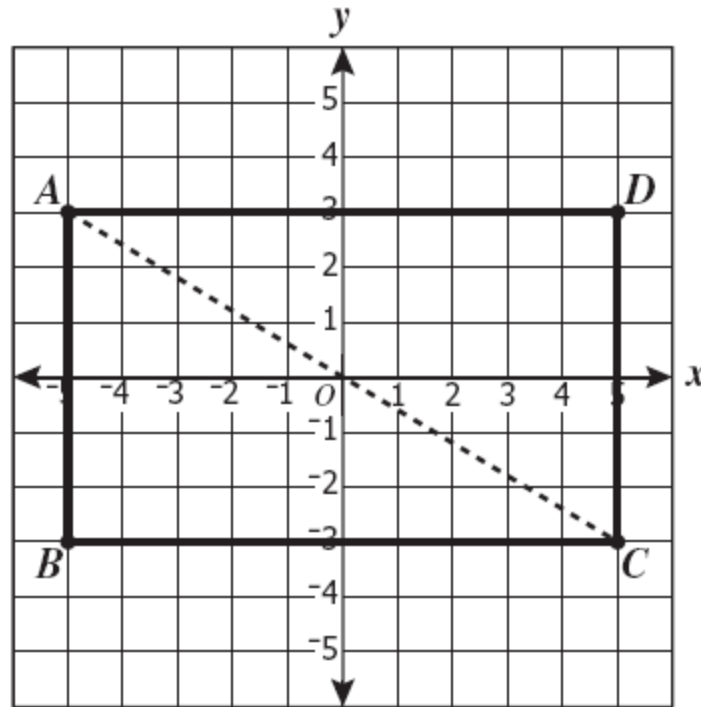
SSM:

- plot an example
- flip

flip it over the x axis and it goes to the 3 quadrant



41 Rectangle $ABCD$ is placed on a grid as shown.



SSM:

- measure AC
- use graph to estimate

Which is *closest* to the length of diagonal \overline{AC} ?

- A 8.0
- B 10.0
- C 11.3
- D 11.7**

Pythagorean Theorem

$$6^2 + 10^2 = AC^2$$

$$36 + 100 = AC^2$$

$$136 = AC^2$$

$$11.67 = AC$$

or

Distance formula

$$\sqrt{(-5 - 5)^2 + (3 - (-3))^2}$$

$$\sqrt{(-10)^2 + (6)^2}$$

$$\sqrt{(100 + 36)}$$

$$\sqrt{136} = 11.67$$

42 Which of the following letters has both line symmetry and point symmetry?

S D M H

F S

G D

H M

J H

SSM:

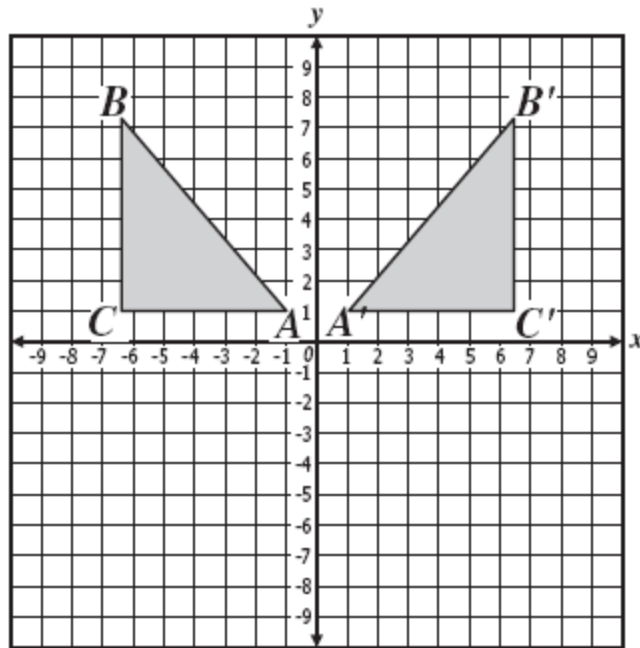
• no help

M has a line of symmetry

S and D do not have a line of symmetry

H has two lines of symmetry and a point of symmetry
(at the intersection of the two lines)

- 43 Triangle ABC was transformed into triangle $A'B'C'$. Which term most accurately describes this transformation?



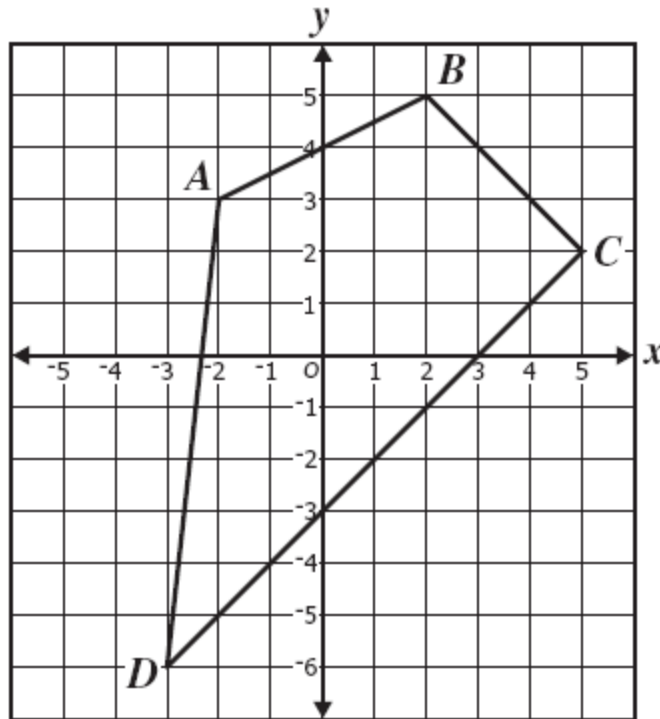
SSM:

- orientation changed
- flip or turn
- folded over y-axis

- A** Tessellation
B Reflection
C Rotation
D Translation

Since A was closest to y-axis and A' is closest to y-axis, a reflection or flip occurred

44 A quadrilateral is placed on a grid as shown.



Midpoint formula:

$$\left(\frac{(-3 + 2)}{2}, \frac{(-6 + 5)}{2} \right)$$

$$(-0.5, -0.5)$$

The apparent midpoint of \overline{BD} is —

- F** $(-0.5, -0.5)$
- G** $(0.5, 3.5)$
- H** $(1.5, 1.5)$
- J** $(1.5, 2.5)$

SSM:

- plot the answers (points)
- draw \overline{BD}
- which is in the middle and on \overline{BD}