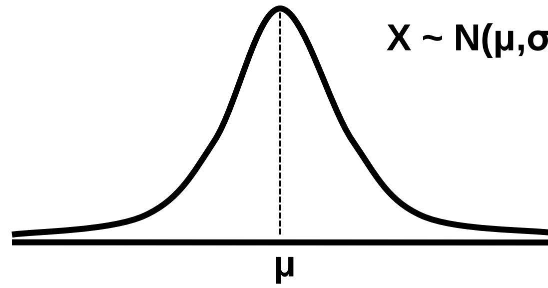
Solving Problems Involving Normal Curves Review Sheet

Normal Curve Characteristics:

	Any Normal	Z (Stnd Normal)
Shape	Symmetric mound	Symmetric mound
Center	Mean of μ	Mean of 0
Spread	Std Dev of σ	Std Dev of 1



Mean = Median = Mode (middle of the graph)
Area to *left* or *right* of mean is 50%; total area under the curve adds to 1

Z-Scores:

Positive values are above the mean and negative values are below

Formula: $z = \dfrac{x - \mu}{\sigma}$ **= number of standard deviations (σ), x is away from mean μ**

When comparing separate events, the smaller of two z scores is worse

Example: If Jon scores a 92 on a test with a mean of 83 and a standard deviation of 6, what is his z-score.

a) $z = \dfrac{x - \mu}{\sigma} = \dfrac{92 - 83}{6} = \dfrac{9}{6} = 1.5$

Z-Table:

Measures the area to the *left* of a value. For example, z = 1.68

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

gives us a value of 0.9535, which mean 95.35% of the area under the curve is to the left of 1.68 (smaller than it)

Empirical Rule: also known as 68-95-99.7 Rule

A normal curve will have the following percentages of its area within set distance from the mean

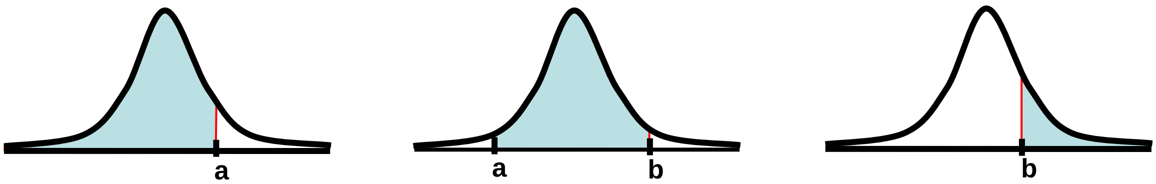
Distance	Within	Area
μ ± σ	1 Stnd Dev	68%
μ ± 2σ	2 Stnd Dev	95%
μ ± 3σ	3 Stnd Dev	99.7%

Example:
If 68% of the scores on the SOL lie between 388 and 432, what is the mean and standard deviation of the SOL scores.

b) 432 – 388 = 44 = 2σ ; so the standard deviation, σ = 22.
The mean, μ, lies halfway between 432 and 388 or 410.

Find probabilities (area under the curve): 2nd Vars 3: normalcdf(LB,UB, μ,σ)

2nd VARS



Using	P(x < a)	P(a < x < b)	P(x > b)
Z-table	Z_a value from table	Z_b - Z_a value from table	1 - Z_b value from table
Calculator	Normcdf(-E99,a,μ,σ)	Normcdf(a,b,μ,σ)	Normcdf(b,E99,μ,σ)

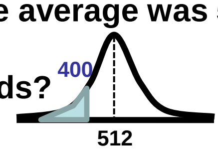
Remember that the mean and standard deviation of a Z distribution is (0,1). Draw the curve and shade in the area that you are looking for. This will help determine which bound (upper or lower) that we have in the problem. If we only have one bound, then if we have an upper bound (figure on the left) we use –E99 as the lower bound. If we have a lower bound (figure on the right), then we use E99 as the upper bound. We get to the E# by using the 2nd "," (comma) key on our calculator.

Word problems finding the (normal) probability

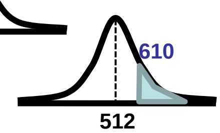
In most word problems the mean and standard deviation are clearly given to us in the problem. We need to figure out which bounds we are given. Pay attention to the words *__less than__* (< - picture on the left above) and *__more than__* (> - picture on the right above). If we are given two bounds, then the smallest is the lower and the largest is the upper.

Example:

In a gym class students have to run a mile. For a 6th grade class the average was 512 seconds with a standard deviation of 68.

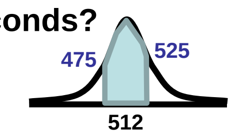


c) What is the probability of a student running less than 400 seconds?
normalcdf(-E99,400,512,68) = 0.0498



d) What is the probability of a student running more than 610 seconds?
normalcdf(610,E99,512,68) = 0.0748

e) What is the probability of a student running between 475 and 525 seconds?
normalcdf(475,525,512,68) = 0.2826



Given the area (percentile) and find a number corresponding

In some word problems they give us the mean and standard deviation and the ask what value corresponds to a certain percentage or a percentile. These problems are the inverse of the probability problems and we use the invnorm function (option 3 from 2nd Vars) of the calculator and the calculator will give us the value corresponding to that percentile. Invnorm (percentile (in decimal form), mean, standard deviation)

Example:

f) On a math test which had a mean of 83 and a standard deviation of 6, what is the 90th percentile score? Invnorm(0.90,83,6) = 90.69

g) On a math test which had a mean of 83 and a standard deviation of 6, what is the 45th percentile score? Invnorm(0.45,83,6) = 82.25

Worksheet Problems:

1. (Ref a) Find the following z-scores with a mean of 25 and a standard deviation of 4:

   a) 20 b) 32 c) 28 d) 25

2. (Ref a) If Sarah scored 78 on her History test which had a mean of 70 and a standard deviation of 3 and she scored 84 on her Math test which had a mean of 80 and a standard deviation of 2, on which test did she score better?

3. (Ref a) Find the following z-scores with a mean of 20 and a standard deviation of 5:

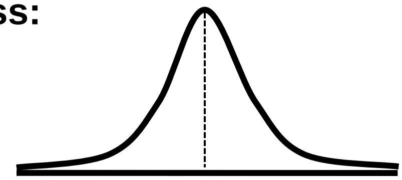
   a) 20 b) 32 c) 28 d) 25

4. (Ref a) Ted Williams was the last player to hit over .400 (.406 in 1941, mean .26648 and standard deviation of 0.051). Since then George Brett has come the closest, hitting .390 in 1980, mean average of .26907 and a standard deviation of 0.036. Who had a better year?

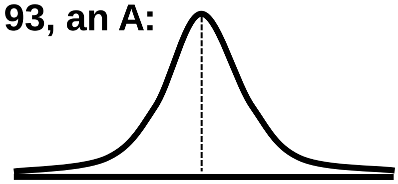
5. (Ref b) If 2.5% of scores on a normally distributed college entrance test were below 60% and 2.5% of the scores were above 84%, what was the mean and the standard deviation of the test? (Hint: empirical rule)

6. (Ref c-e) If Statistics test scores were normally distributed with a mean a 81 and a standard deviation of 5, find the following probabilities

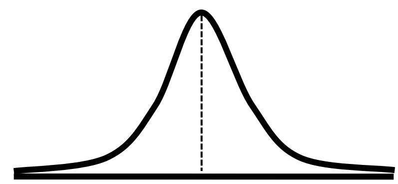
a) That a randomly selected student scored 75 or less:



b) That a randomly selected student scored above a 93, an A:

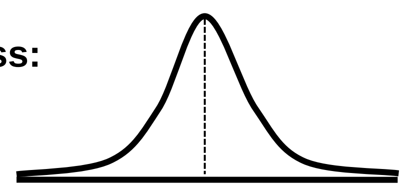


c) That a randomly selected student scored between 77 and 84, a C:

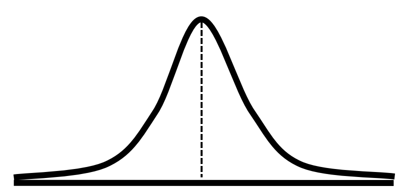


7. (Ref c-e) If Biology test scores were normally distributed with a mean a 78 and a standard deviation of 7, find the following probabilities

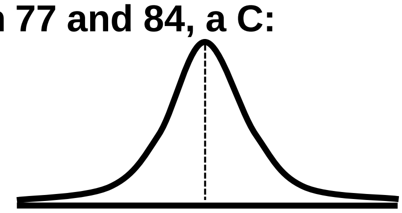
a) That a randomly selected student scored 75 or less:



b) That a randomly selected student scored above a 85, a B or better:



c) That a randomly selected student scored between 77 and 84, a C:



8. (Ref f-g) If Statistics test scores were normally distributed with a mean a 81 and a standard deviation of 5, what score is the 90^{th} percentile?