4.2 Area

 **What will you learn?** 

- **Use sigma notation to write and evaluate a sum**
- **Understand the concept of area**
- **Approximate the area of a plane region**
- **Find the area of a plane region using limits**

Sigma Notation

The sum of n terms $a_1, a_2, a_3, \ldots a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

where i is the index of summation,
a_i is the *ith* term of the sum, and
the upper and lower bounds are n and 1

Area

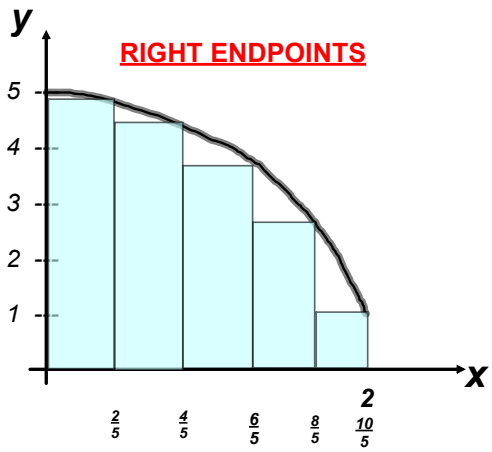
Area of a Plane Region

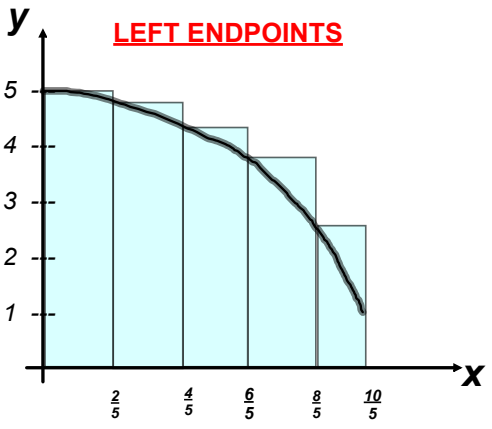
Example 3 - Approximating the area of a Plane Region

Use 5 the rectangles in the figure to find 2 approximating of the area of the region lying between the graph of

$$f(x) = -x^2 + 5$$

and the x-axis between $x = 0$ **and** $x = 2$





Upper and Lower Sums

The sum of the areas of the inscribed rectangles - LOWER SUM

The sum of the areas of the circumscribed rectangles - UPPER SUM

Lower Sums

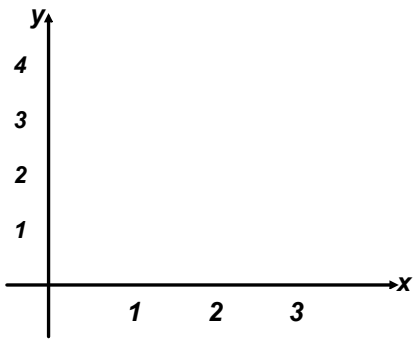
Area of inscribed rectangles is LESS than the area of the region

Upper Sums

Area of circumscribed rectangles is GREATER than the area of the region

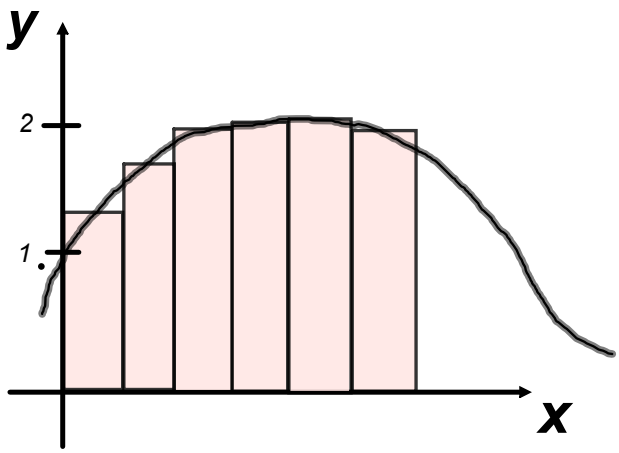
Finding Upper and Lower Sums for a Region

Find the upper and lower sums fof the region bounded by the graph of $f(x) = x^2$ and the x - axis between $x = 0$ and $x = 2$



Midpoint Rule

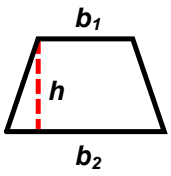
Area = Sum of the areas of rectangles ------ $f\left(\frac{x_1 + x_2}{2}\right)\Delta x$



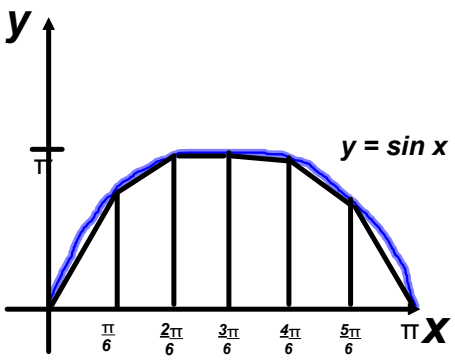
Trapezoidal Rule

Add the areas of trapezoids

Remember????



$$A = \frac{h}{2}(b_1 + b_2)$$



$\int_0^2 x^2\,dx$ $\quad n = 4$