

4.3 Riemann Sums & Definite Integrals



What will you learn?



- Understand the definition of a Riemann Sum
- Evaluate a definite integral using limits
- Evaluate a definite integral using properties of definite integrals

Definite Integrals

$$\int_a^b f(x) dx$$

Definite integral of f from a to b

$a \longrightarrow$ Lower Limit

$b \longrightarrow$ Upper Limit

What's the difference between a definite integral and an indefinite integral?

An indefinite integral is a FAMILY OF FUNCTIONS

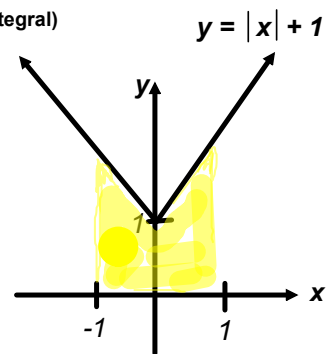
A definite integral is a NUMBER

Theorem 4.4 - Continuity Implies Integrability

If a function f is continuous on the closed interval $[a, b]$,
then f is integrable on $[a, b]$

Example 1

Set up a definite integral that yields the area of the region. (Do not use the integral)



Example

Sketch the region whose area is given by the definite integral.
Then use a geometric formula to evaluate the integral.

$$\int_{-a}^a 4 \, dx$$

$$\int_0^8 (8 - x) \, dx$$

³[Example](#)

Evaluate the integral using the following values.

$$\int_1^3 x^2 dx = 40$$

$$\int_1^3 x dx = 9$$

$$\int_1^3 dx = 3$$

a.) $\int_1^3 5x dx$

b.) $\int_3^1 x dx$

c.) $\int_1^3 (x^2 + 4) dx$

Example

Given $\int_2^6 f(x) dx = 10$ and $\int_2^6 g(x) dx = -2$ evaluate

$$\int_2^6 [f(x) + g(x)] dx$$

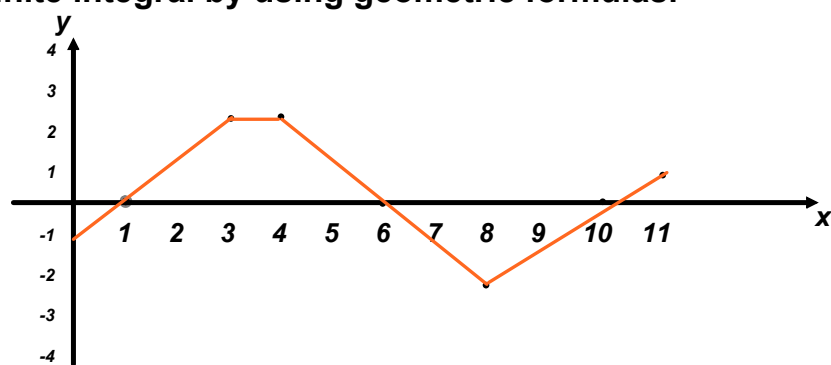
$$\int_2^6 [g(x) - f(x)] dx$$

$$\int_2^6 2g(x) dx$$

$$\int_2^6 3f(x) dx$$

Example

The graph of f consists of line segments as shown in the figure. Evaluate each definite integral by using geometric formulas.



$$\int_0^1 -f(x) \, dx$$

$$\int_5^{11} f(x) \, dx$$

$$\int_3^4 3f(x) \, dx$$

$$\int_0^{11} f(x) \, dx$$

$$\int_0^7 f(x) \, dx$$

$$\int_4^{10} f(x) \, dx$$