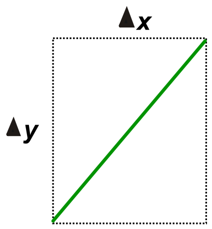
4.4 The Fundamental Theorem of Calculus

☺ **What will you learn?** ☺

- **Evaluate a definite integral using the Fundamental Theorem of Calculus**
- **Understand and the use the Mean Value Theorem for Integrals**
- **Find the average value of a function on a closed interval**
- **Understand and use the Second Fundamental Theorem of Calculus**

The Fundamental Theorem of Calculus

Informally - Differentiation and Definite Integration
are inverse operations



Theorem 4.9 - The Fundamental Theorem of Calculus

If a function *f* is continuous on the closed interval *[a, b]* and *F* is an antiderivative of *f* on the interval *[a, b]*, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Guidelines for using the FTC

1. *Provided you can find* an antiderivative of f, you now have a way to evaluate a definite integral without having to use the limit of a sum.

2. When applying FTC, the following notation is convenient.

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b$$
$$= F(b) - F(a)$$

Eg)

$$\int_1^3 x^3\,dx = \frac{x^4}{4}\Big]_1^3 = \frac{3^4}{4} - \frac{1^4}{4} = \frac{81}{4} - \frac{1}{4} = 20$$

3. It is not necessary to include a constant of integration C in the antiderivative because

$$\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b$$
$$= [F(b) + C] - [F(a) + c]$$
$$= F(b) - F(a)$$

Example 1 - Evaluating a Definite Integral

Evaluate each definite integral.

a.) $\int_{1}^{2} (x^2 - 3)\, dx$

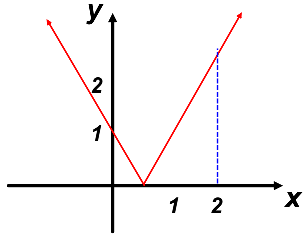
b.) $\int_{1}^{4} 3\sqrt{x}\, dx$

c.) $\int_{0}^{\pi/4} \sec^2 x\, dx$

Example 2 - A Definite Integral Involving Absolute Value

Evaluate

$$\int_0^2 |2x - 1|\, dx$$



Example 3 - Using the FTC to Find Area

Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, **the** x **- axis and the vertical lines** $x = 0$ **and** $x = 2$

The Mean Value Theorem for Integrals

You already learned that the area of a region under a curve is greater than the area of an inscribed rectangle and less than the area of a circumscribed rectangle.

The MVT for Integrals states that somewhere "between" the inscribed and circumscribed rectangles there is a rectangle whose area is precisely equal to the area of the region under the curve.

Theorem 4.10 MVT for Integrals

If *f* is continuous on the closed interval *[a,b]*, then there exists a number *c* in the closed interval [a,b] s.t.

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Average Value of a Function

The value *f (c)* given in the MVT for Integrals
is called the average value of *f* on the interval *[a,b]*.

Definition of the Average Value of a Function on an Interval

If *f* is integrable on the closed interval [a,b], then the average value of *f* on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Example 4 - Finding the Average Value of a Function

Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1,4]$

The Second Fundamental Theorem of Calculus

We already learned how to integrate when
b is the upper limit and *x* is the variable of integration

constant

$$\int_a^b f(x)\,dx$$

constant

f is a function of x

What happens when x is used as the upper limit?

F is a function of x

$$F(x) = \int_a^x f(t)\,dt$$

constant

f is a function of t

Example 6 - The Definite Integral as a Function

Evaluate the function $F(x) = \int_0^x \cos t\,dt$ at $x = 0, \pi/6, \pi/4, \pi/3, \pi/2$

Theorem 4.11 - The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Example 7 - Using the Second Fundamental Theorem of Calculus

Evaluate

$$\frac{d}{dx}\left[\int_0^x \sqrt{t^2+1}\,dt\right]$$

Example 8 - Using the 2nd FTC

Find the derivative of $\boldsymbol{F(x) = \int_{\frac{\pi}{2}}^{x^3} \cos t \, dt}$