Chapter 5 - Logarithmic, Exponential & Other Transcendental Functions

5.1 The Natural Logarithmic Function: Differentiation

What will you learn?

- **Develop and use properties of the natural logarithmic function.**
- **Understand the definition of the number *e***
- **Find the derivatives of functions involving the natural logarithmic function.**

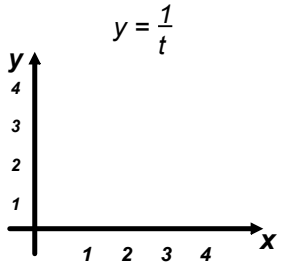
The Natural Logarithmic Function

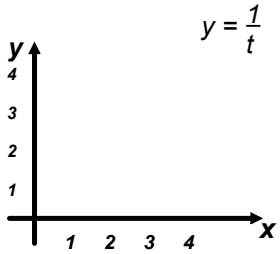
The general power rule for integrals does not work when $n = -1$

Definition of the Natural Logarithmic Function

$$\ln x = \int_1^x \frac{1}{t}\,dt, \qquad x > 0$$

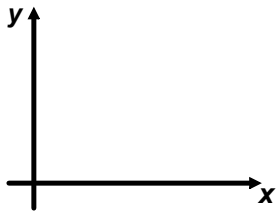
$\ln x$ is positive for $x > 1$ and negative for $0 < x < 1$
$\ln 1 = 0$, because the upper and lower limits of integration are equal when $x = 1$





Do you remember what the graph of $y = \ln x$ looks like?

From Precal!



Properties of the Natural Log

1. The domain is (0, ∞) and the range is ($-\infty$, ∞)
2. The function is continuous, increasing and one-to-one
3. The graph is concave downward

Logarithmic Properties

If a and b are positive numbers and n is rational, then the following properties are true.

1. $\ln 1 = 0$
2. $\ln ab = \ln a + \ln b$
3. $\ln a^n = n \ln a$
4. $\ln \frac{a}{b} = \ln a - \ln b$

Example 1 - Expanding Log Expressions

a. $\ln \frac{10}{9}$

b. $\ln \sqrt{3x + 2}$

c. $\ln \frac{6x}{5}$

d. $\ln \frac{(x^2 + 3)^2}{x \sqrt[3]{(x^2 + 1)}}$

The Number *e*

Base for the natural logarithm

$e \approx 2.71828182846$

Definition of *e*

The letter *e* denotes the positive real number s.t.

$$\ln e = \int_1^e \frac{1}{t}\, dt = 1$$

Example 2- Evaluating Natural Logarithmic Expressions

a. ln 2

b. ln 32

c. ln 0.1

e

The Derivative of the Natural Log

Let u be a differentiable function of x

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$

2. $\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u} \quad u > 0$

Example 2 - Differentiation of Logs

a.) $\frac{d}{dx}[\ln (2x)]$

b.) $\frac{d}{dx}[\ln (x^2 + 1)]$

c.) $\frac{d}{dx}[x \ln x]$

d.) $\frac{d}{dx}[(\ln x)^3]$

Example 4 - Log Properties as Aids to Differentiation

Differentiate

$$f(x) = \ln \sqrt{x+1}$$

Example 5 - Log Properties as Aids to Differentiation

Differentiate

$$f(x) = \ln \frac{x(x^2 + 1)}{\sqrt{2x^3 - 1}}$$

Example 6 - Log Differentiation

Find the derivative of

$$y = \frac{(x-2)^2}{\sqrt{x^2+1}} \qquad x \neq 2$$

Theorem 5.4 - Derivative Involving Absolute Value

If u is a differentiable function of x s.t. $u \neq 0$, then

$$\frac{d}{dx}\left[\ln|u|\right] = \frac{u'}{u}$$

Example 7 - Derivative Involving Absolute Value

Find the Derivative of

$$f(x) = \ln|\cos x|$$

Example 8 - Finding Relative Extrema

Locate the relative extrema of

$$y = \ln (x^2 + 2x + 3)$$