5-2 The Natural Logarithmic Function : Integration

What will you learn?

- **Use Log Rule for Integration to integrate a rational function**
- **Integrate Trig functions**

Theorem 5.5 - Log Rule for Integration

Let u be a function of x

1. $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$

2. $\displaystyle\int \frac{1}{u}\ du = \ln|u| + C$

Example 1 - Using the Log Rule for Differentiation

Find

$$\int \frac{2}{x}\ dx$$

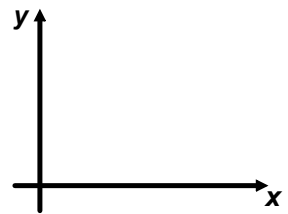**Example 2 - Using the Log Rule with Change of Variables**

Find

$$\int \frac{1}{4x-1}\ dx$$

Example 3 - Finding Area with Log Rule

Find the area of the region bounded by the graph of

$$y = \frac{x}{x^2 + 1}$$

the x-axis, and the line $x = 3$

Example 4 - Recognizing Quotient Forms of the Log Rule

a.) $\displaystyle\int \frac{3x^2 +1}{x^3 +x}\, dx$

b.) $\displaystyle\int \frac{\sec^2 x}{\tan x}\, dx$

c.) $\displaystyle\int \frac{x + 1}{x^2 + 2x}\, dx$

d.) $\displaystyle\int \frac{1}{3x + 2}\, dx$

Example 5 - Using Long Division Before Integrating

Find $\displaystyle\int \frac{x^2 + x + 1}{x^2 + 1}\, dx$

Example 6 - Change of Variables with Log Rule

Find

$$\int \frac{2x}{(x+1)^2} \, dx$$

Think of integration in this way....

Here's the answer: What is the question?

<u>Guidelines for Integration</u>

1. **Learn a basic list of integration formulas.** (You have 15 right now!)

2. **Find a formula that resembles all or part of the integrand, and by trial and error, find a choice of *u* that will make the integrand conform to the formula.**

3. **If you cannot find a *u* - sub that works, try altering the integrand.**

4. **If you have access to computer software that will find antiderivatives symbolically, use it.**

Example 7 - *u* - Sub and Log Rule

Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1}{x \ln x}$

Integrals and Trig Functions

Example 8 - Using a Trig Identity

Find

$$\int \tan x \, dx$$

Example 9 - Derivation of the Secant Formula

Find

$$\int \sec x \, dx$$

Integrals of the 6 Basic Trig Functions

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

Example 10 - Integrating Trig Functions

Evaluate

$$\int_{0}^{\pi/4} \sqrt{1 + \tan^2 x}\ \ dx$$

Example 11 - Finding an Average Value

Find the average value of $f(x) = \tan x$ on the interval $[\ 0,\ \pi/4]$