5.3 Inverse Functions

What will you learn?

- **Verify that one function is the inverse of another function**
- **Determine whether a function has an inverse function**
- **Find the derivative of an inverse function**

What do you remember about inverse functions?

Precal !!!!!!!

Definition of Inverse Function

A function g is the inverse of the function f if

$$f(g(x)) = x \quad \text{for all } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \quad \text{for each } x \text{ in the domain of } f$$

The function g is denoted by f^{-1} (read " f inverse")

1. If g is the inverse of f, then f is the inverse of g
2. The domain of f^{-1} is equal to the range of f, the range of f^{-1} is equal to the domain of f
3. A function need not have an inverse function, but if it does, the inverse function is unique

Example 1 - Verifying Inverse Functions

Show that the functions are inverse functions of each other

$$f(x) = 2x^3 - 1 \qquad \text{and} \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

Theorem 5.6 - Reflective Property of Inverse Functions

The graph of *f* contains the point *(a, b) iff*
the graph of f^{-1} contains the point *(b, a)*

Existence of an Inverse Function

Not every function has an inverse function!

Remember....

Horizontal Line Test

Theorem 5.7 The Existence of an Inverse Function

1. **A function has an inverse function *iff* it is *1:1***
2. **If f is strictly monotonic on its entire domain, then it is *1:1* and therefore has an inverse function**

Example 2 - The Existence of an Inverse Function

Which of these functions has an inverse function?

a.) $f(x) = x^3 + x - 1$

b.) $f(x) = x^3 - x + 1$

Guidelines for Finding an Inverse Function

1. Use Theorem 5.7 to determine if a function has an inverse

2. Solve for x as a function of y: $x = g(y) = f^{-1}(y)$

3. Interchange x and y. The resulting equation is $y = f^{-1}(x)$

4. Define the domain of f^{-1} to be the range of f

5. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Example 3 - Finding an Inverse Function

Find the inverse function of

$$f(x) = \sqrt{2x - 3}$$

Example 4 - Testing Whether a Function is 1:1

Show that the sine function

f (x) = sin x

is not 1:1 on the entire real line

Derivative of an Inverse Function

Theorem 5.8 Continuity and Differentiability of Inverse Functions

Let f be a function whose domain is an interval I.
If f has an inverse function then the following statements are true.

1. If f is continuous on its domain, then f^{-1} is continuous in its domain
2. If f is increasing on its domain, then f^{-1} is increasing in its domain
3. If f is decreasing on its domain, then f^{-1} is decreasing in its domain
4. If f is differentiable on an interval containing c and $f'(c) \neq 0$, then f^{1} is differentiable at $f(c)$

Theorem 5.9 The Derivative of an Inverse Function

Let f be a function that is differentiable on an interval I.
If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$.

$$g'(x) = \frac{1}{f'(g(x))}$$

Example 5 - Evaluating the Derivative of an Inverse Function

Let $f(x) = \frac{1}{4}x^3 + x - 1$

a.) What is the value of $f^{-1}(x)$ when $x = 3$

b .) What is the value of $(f^{-1})'(x)$ when $x = 3$

Example 6 - Graphs of Inverse Functions Have Reciprocal Slopes

Let $f(x) = x^2$ (for $x \geq 0$) and let $f^{-1}(x) = \sqrt{x}$.

Show that the slopes of the graphs of f and f^{1} are reciprocals of each of the following points.

a.) (2,4) and (4,2)

b.) (3, 9) and (9,3)