5.4 - Exponential Functions : Differentiation & Integration

 **What will you learn?** 

- **Develop properties of the natural exponential function**
- **Differentiate natural exponential functions**
- **Integrate natural exponential functions**

The Natural Exponential Function

The inverse of the natural logarithmic function $f(x) = \ln x$ is called the natural exponential function and is denoted by

$$f^{-1}(x) = e^{x}$$

That is,

$$y = e^{x} \quad \text{iff} \quad x = \ln y$$

Inverse relationship:
Remember?

$$\ln e^{x} = x \quad \text{and} \quad e \ln x = x$$

Example 1 - Solving Exponential Equations

Solve $7 = e^{x+1}$

Example 2 - Solving a Logarithmic Equation

Solve $\ln(2x - 3) = 5$

Theorem 5.10 - Operations with Exponential Functions

Let a and b be any real numbers.

1. $e^a e^b = e^{a+b}$

2. $\frac{e^a}{e^b} = e^{a-b}$

Properties of the Natural Exponential Function

1. Domain : $(-\infty,\infty)$
   Range : $(0, \infty)$

2. Continuous
   Increasing
   1 : 1

3. Concave UP

4. $\lim_{x \to -\infty} e^x = 0$ and $\lim_{x \to \infty} e^x = \infty$

Derivatives of Exponential Functions

¡¡¡¡¡¡¡ IT IS ITS OWN DERIVATIVE !!!!!!!

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

Example 3 - Differentiating Exponential Functions

a.) $\frac{d}{dx}[e^{2x-1}]$

b.) $\frac{d}{dx}[e^{-3/x}]$

Example 4 - Locating Relative Extrema

Find the relative extrema of $f(x) = x e^{x}$

Example 5 - The Standard Normal Probability Density Function

Show that the standard normal probability density function has points of inflections when x = ± 1

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Example 6 - Shares Traded

The number y of shares traded (in millions) on the NYSE from 1990-2002 can be modeled by

$$y = 36{,}663\, e^{0.1902t}$$

where t represents the year, with $t = 0$ corresponding to 1990.
At what rate was the number of shares traded changing in 1998?

Integrals of Exponential Functions

1. $\int e^{x}\,dx = e^{x} + c$

2. $\int e^{u}\,du = e^{u} + c$

Example 7 - Integrating Exponential Functions

Find $\int e^{3x+1}\,dx$

Example 8 - Integrating Exponential Functions

Find $\int 5x\, e^{-x^2}\, dx$

Example 9 - Integrating Exponential Functions

a.) $\int \frac{e^{1/x}}{x^2}\,dx$

b.) $\int \sin x\, e^{\cos x}\,dx$

Example 10 - Finding Areas Bounded by Exponential Functions

a.) $\int_0^1 e^{-x}\,dx$

b.) $\int_0^1 \frac{e^x}{1+e^x}\,dx$

c.) $\int_{-1}^0 [\, e^x \cos(e^x)\,]\,dx$