5.6 Inverse Trig Functions : Differentiation

What will you learn?

- Develop properties of the six trig functions
- Differentiate an inverse trig function
- Review the basic differentiation rules for elementary functions

Inverse Trig Functions

The six trig functions are NOT 1:1, therefore, they do NOT have inverses

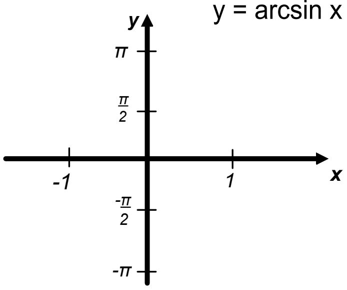
In order to have an inverse we must RESTRICT THE DOMAIN

Definition of Inverse Trig Functions

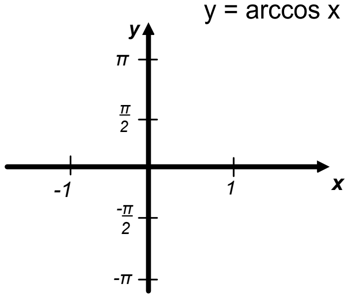
Function	Domain	Range
$y = \arcsin x$ iff $\sin y = x$	$-1 \leq x \leq 1$	$\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x$ iff $\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arccot}\, x$ iff $\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \text{arcsec}\, x$ iff $\sec y = x$	$\lvert x \rvert \geq 1$	$0 \leq y \leq \pi$ $\quad y \neq \frac{\pi}{2}$
$y = \text{arccsc}\, x$ iff $\csc y = x$	$\lvert x \rvert \geq 1$	$\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

The Graphs of the 6 Inverse Trig Functions

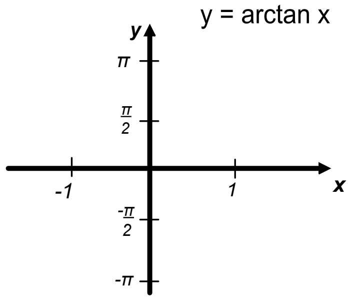
y = arcsin x



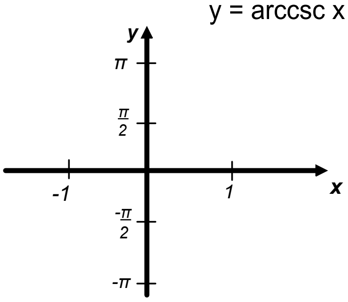
y = arccos x



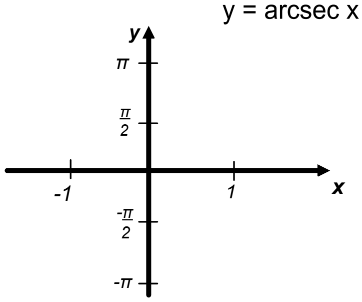
y = arctan x



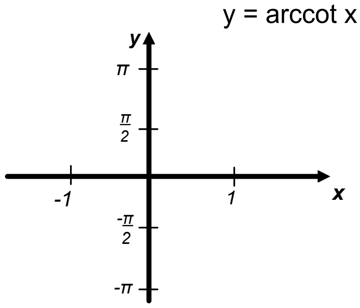
y = arccsc x



y = arcsec x



y = arccot x



Example 1 - Evaluating Inverse Trig Functions

Evaluate each function

a.) arcsin (-1/2) =

b.) arccos 0 =

c.) arctan $\sqrt{3}$

d.) arcsin (0.3)

Properties of Inverse Trig Functions

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, $\longrightarrow$ sin(arcsin x) = x and arcsin (sin y) = y

If $-\pi/2 < y < \pi/2$ $\longrightarrow$ tan(arctan x) = x and arctan (tan y) = y

If $|x| \ge 1$ and $0 \le y \le \pi/2$ or $\pi/2 < y < \pi$ $\longrightarrow$ sec(arcsec x) = x and arcsec (sec y) = y

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Example 2 - Solving an Equation

$$\arctan(2x - 3) = \frac{\pi}{4}$$

Example 3- Using Right Triangles

a.) Given $y = \arcsin x$, where $0 < y < \pi/2$, *find* cos y

b.) Given $y = \text{arcsec}\,(\sqrt{5}/2)$, *find* tan y

Derivatives of Inverse Trig Functions

Let u be a differentiable function of x

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{1-u^2}} \qquad \frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{1-u^2}}$$

Example 4 - Differentiating Inverse Trig Functions

a.) $\frac{d}{dx}[\arcsin(2x)]$

b.) $\frac{d}{dx}[\arctan(3x)]$

c.) $\frac{d}{dx}[\arcsin(\sqrt{x})]$

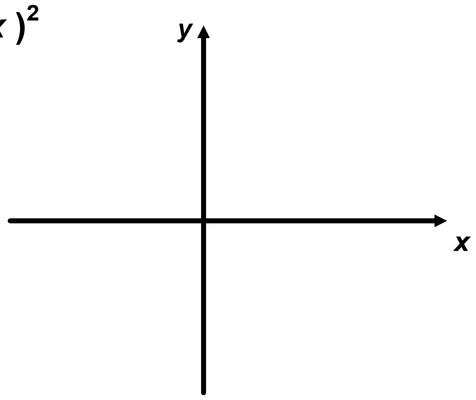
d.) $\frac{d}{dx}[\operatorname{arcsec}(e^{2x})]$

Example 5 - A Derivative That Can Be Simplified

Differentiate $y = \arcsin x + x\sqrt{1 - x^2}$

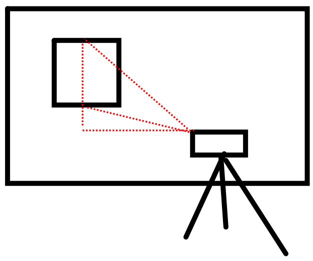
Example 6 - Analyzing an Inverse Trig Graph

Analyze the graph of $y = (\arctan x)^2$



Example 7 - Maximizing an Angle

A photographer is taking a picture of a four-foot painting hung in an art gallery.
The camera lens is 1 foot below the lower edge or the painting.
How far should the camera be from the painting to maximize the angle subtended by the camera lens?



ALL your Basic Differentiation Rules
are listed on P. 376
& in the front cover of your book!