General & Particular Solutions

Physical phenomenon can be described by differential equations

ie: radioactive decay, population growth, Newton's Law of Cooling

A function $y = f(x)$ is called a solution of a differential equation if the equations is satisfied when y and its derivatives are replaced by $f(x)$ and it derivatives

Example 1 - Verifying Solutions

Determine whether the function is a solution of the differential equation

$$y'' - y = 0$$

a.) $y = \sin x$

b.) $y = 4e^{-x}$

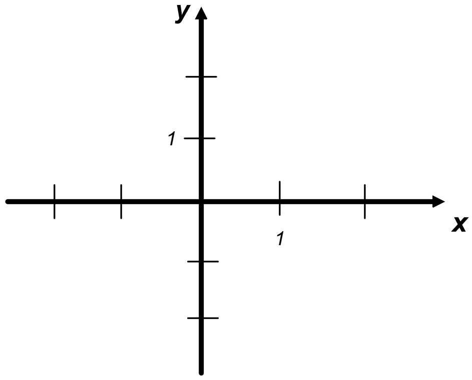
c.) $y = Ce^{x}$

Geometrically, the general solution of a first-order differential equation represents a family of curves known as solution curves one for each value assigned to the arbitrary constant

For example, you can verify that every function of the form

$$y = \frac{C}{x}$$ general solution for $xy' + y = 0$

is a solution of the differential equation $xy' + y = 0$



Remember - particular solutions of differential equations are obtained from initial conditions that give the value of the dependent variable or one of its derivatives for a particular value if the independent variable

Example 2 - Finding a Particular Solution

For the differential equation $xy' - 3y = 0$, verify that $y = Cx^3$ is a solution, and find the particular solution determined by the initial condition $y = 2$ when $x = -3$

Slope Fields

Solving differential equations *analytically* can be difficult or even impossible.
However, there is a graphical approach you can use to learn a lot about the solutions of a differential equation.

Consider a differential equation of the form

$$y' = F(x, y)$$

- ☆ At each point (x, y) in the xy-plane where F is defined, the differential equation determines the slope $y' = F(x, y)$ of the solution at that point

- ☆ If you draw a short line segment with the slope $F(x, y)$ at selected points in the domain of F, then these line segments form a slope field, or a *direction field* for the differential equation $y' = F(x, y)$

- ☆ Each line segment has the same slope as the solution curve through that point

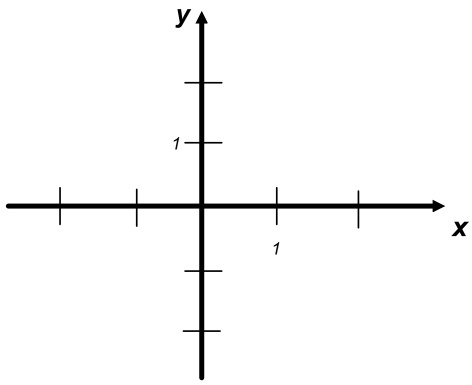
- ☆ A slope field show the general shape of all the solutions

Example 3 - Sketching a Slope Field

Sketch a slope field for the differential equation

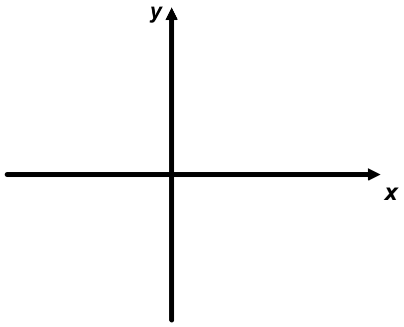
$$y' = x - y$$

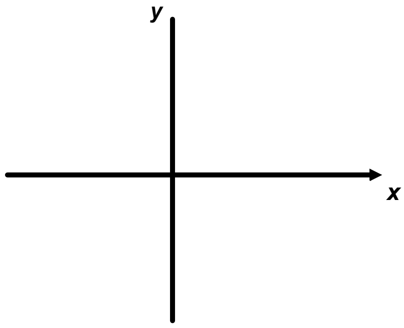
for the points ***(-1, 1) , (0, 1) (1, 1)***

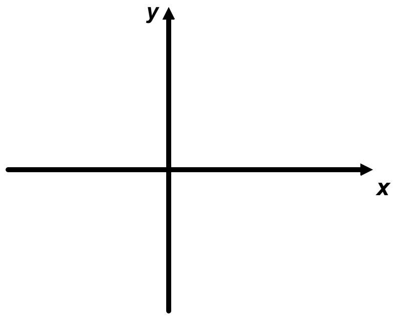


Example 4 - Identifying Slope Fields for Differential Equations

Match each slope field with its differential equation







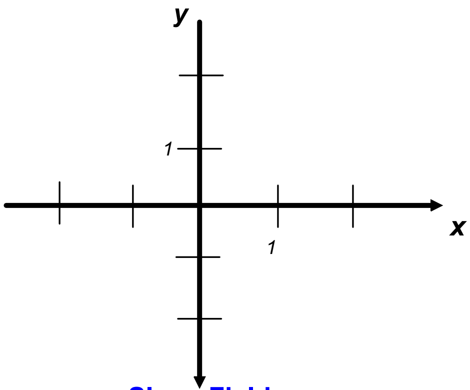
$y' = x + y$ $y' = x$ $y' = y$

Example 5 - Sketching a Solution Using a Slope Field

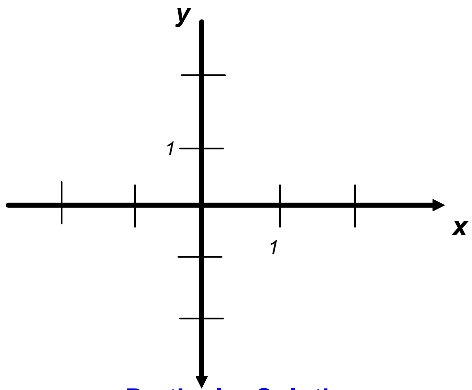
Sketch a slope field for the differential equation

$$y' = 2x + y$$

Use the slope field to sketch the solution that passes through the point ***(1,1)***



Slope Field



Particular Solution

<u>Euler's Method</u>

a numerical approach to approximating the particular solutions of the differential equation $y' = F(x, y)$ that passes through the point (x_0, y_0)

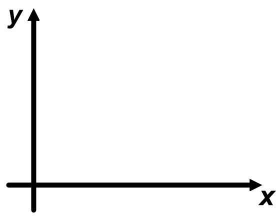
you know that the graph of the solution passes through the point (x_0, y_0) and has a slope of $F(x_0, y_0)$

this gives you a starting point for approximating the solution

From the starting point you can proceed in the direction indicated by the slope

using a small step h, move along the tangent line until you arrive at the point (x,y), where

$$x_1 = x_0 + h \qquad \text{and} \qquad y_1 = y_0 + hF(x_0, y_0)$$



think of (x_1, y_1) as a new starting point, repeat the process

Example 6 - Approximating a Solution Using Euler's Method

Use Euler's Method to approximate the particular solution of the differential equation

$$y' = x - y$$

passing through the point ***(0, 1)***
use a step of ***h = 0.1***