6.2 Differential Equations : Growth & Decay

 **What will you learn?** 

- **Use separation of variables to solve a simple differential equation**
- **Use exponential functions to model growth and decay in applied problems**

Differential Equations

Strategy - rewrite the equation so that each variable occurs on only one side of the equation

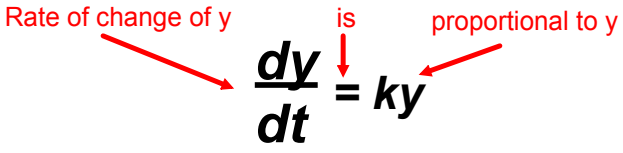
Example 1 - Solving Differential Equations

Solve the differential equation

$$y' = 2x / y$$

Growth & Decay Models

In many application, the rate of change of a variable *y* is proportional to the value *y*
If *y* is a function of time *t*, the proportion can be written as shown



Theorem 6.1 - Exponential Growth & Decay Model

If *y* is a differentiable function of t *s.t.* *y > 0* and *y ' = ky*, for some constant *k*, then

$$y = C\,e^{kt}$$

C = initial value of *y*
k = the proportionality constant

k > 0 GROWTH
k < 0 DECAY

Proof

$$y' = ky$$

$$\frac{y'}{y} = k$$

$$\int \frac{y'}{y}\,dt = \int k\,dt$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln y = kt + C_1$$

$$y = e^{kt}\,e^{c_1}$$

$$y = C\,e^{kt}$$

Example 2 - Using an Exponential Growth Model

The rate of change of y is proportional to y.
When $t = 0$, $y = 2$
When $t = 2$, $y = 4$
What is the value of y when $t = 3$?

Example 3 - Radioactive Decay

Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident.
How long will it take for the 10 grams to decay to 1 gram?

Example 4 - Population Growth

Suppose an experimental population of fruit flies increases according to the law of exponential growth.
There were 100 flies after the second day of the experiment and 300 flies after the fourth day.
Approximately how many flies were in the original population?

Example 5 - Declining Sales

Four months after it stops advertising, a manufacturing company notices that its sales have dropped from 100,000 units per month to 80,000 units per month.
If the sales follow an exponential pattern of decline, what will they be after another 2 months?

Example 6 - Newton's Law of Cooling

Let y represent the temperature (in ^{0}F) of an object in a room whose temperature is kept at a constant 60^0.
If the object colls form 100^0 to 90^0 in 10 minutes, how much longer will it take for its temperature to decrease to 80^0?