7.1 Area of a Region Between Curves

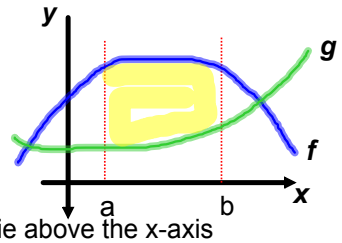
 **What will you learn?** 

- **Find the area of a region between two curves using integration**
- **Find the area of a region between intersecting curves using integration**
- **Describe integration as an accumulation process**

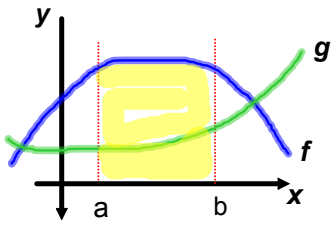
Area of a Region Between Two Curves

You can extend the application of definite integrals from
the area of a region *under a curve*
to the area of a *region between 2 curves*

Given 2 functions *f* and *g* that are *continuous* on the interval [a,b]



- Both curves lie above the x-axis
- *g* is below *f*
- Area between the curves ⟶ area of the region under *g* subtracted from the area of the region under *f*



Area of the region under f

$$\int_a^b f(x)\,dx$$

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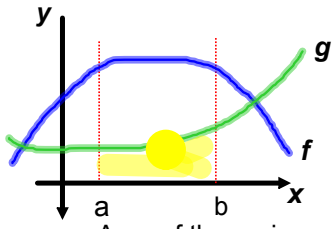
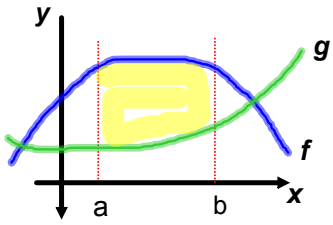


Area of the region under g

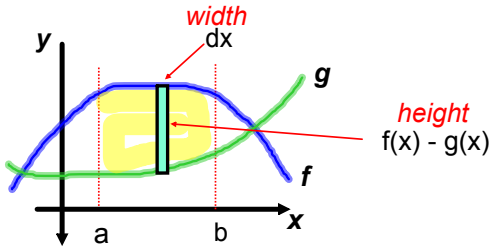
$$\int_a^b g(x)\,dx$$

$=$



Area of the region between *f* & *g*

$$\int_a^b [f(x) - g(x)]\,dx$$



You can partition the interval in n subintervals of <u>width</u> Δx

<u>height</u> is $f(x) - g(x)$

Area of a Region Between Two Curves

If f and g are continuous on [a,b] and $g(x) < f(x)$ for all x in [a,b], then the area of the regions bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int [f(x) - g(x)]\,dx$$

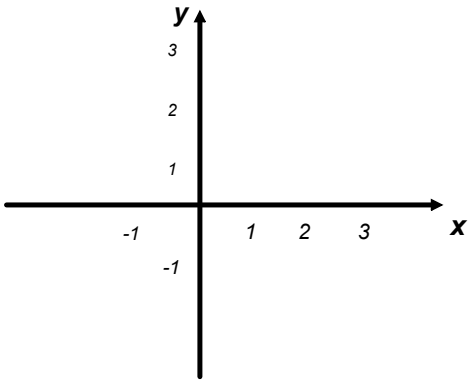
It is NOT necessary for f and g to lie above the x -axis

You can use the same integrand as as long as f and g are continuous AND $g(x) \leq f(x)$

Example 1 - Finding the Area of a Region Between Two Curves

Find the area of the region bounded by the graphs of

$y = x^2 + 2, \quad y = -x, \quad x = 0 \text{ and } x = 1$



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Area of a Region Between Intersecting Curves

In the previous example - the curves did not intersect and
the values of a and b are given explicitly

A more common problem involves the area of a region bounded by two intersecting curves where the values of a and b must be calculated

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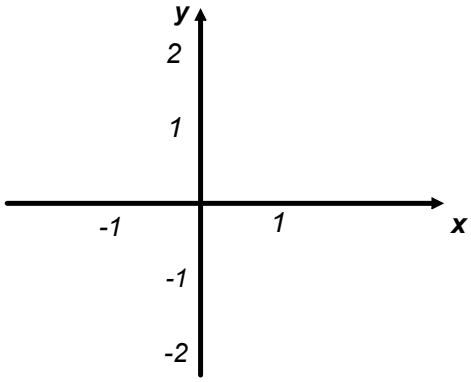
Example 2 - A Region Lying Between Two Intersecting Graphs

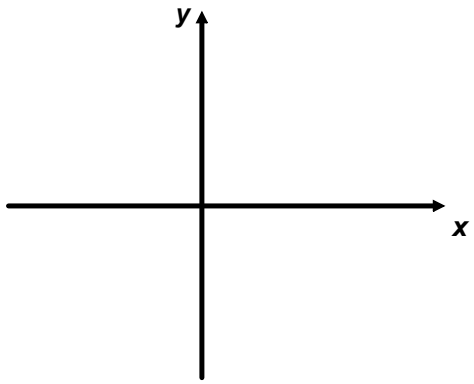
Find the area of the region bounded by the graphs of

$f(x) = 2 - x^2$ *and* $g(x) = x$



Example 3 - A Region Lying Between Two Intersecting Curves

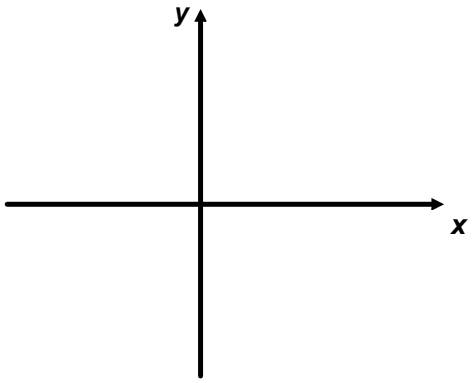
The sine and cosine curves intersect infinitely many times, bounding regions of equal areas.
Find the area of one of these regions.



Example 4 - Curves That Intersect at More Than Two Points

Find the area of the region between the graphs of

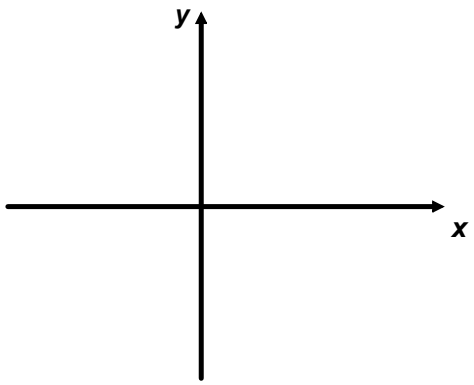
$f(x) = 3x^3 - x^2 - 10x$, and $g(x) = -x^2 + 2x$



Example 5 - Horizontal Representative Rectangles

Find the area of the region bounded by the graphs of

$x = 3 - y^2$ *and* $x = y + 1$



Integration as an Accumulation Process

Representative rectangles will be used in the rest of this chapter -
each integration formula will be based on these representative rectangles

$$A = (\text{height})\ \ (\text{width})$$

$$\Delta A = [\, f(x) - g(x)]\ \ \Delta x$$

$$A = \int [f(x) - g(x)]\, dx$$

Example 6 - Describing Integration as an Accumulation

Find the area of the region bounded by the graph of

$y = 4 - x^2$ and the x-axis.

Describe the integratin process as an accumulation