# 7.3 Volume : The Shell Method

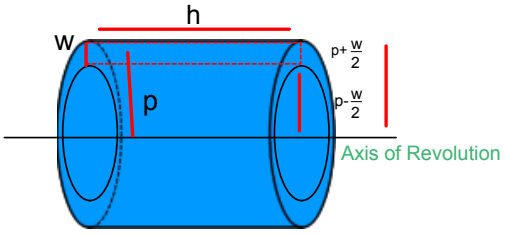
## What will you learn?

- Find the volume of a solid of revolution using the shell method.
- Compare the uses of the disk method and shell method.

# The Shell Method

Alternative method for finding the volume of a solid of revolution

**Using cylindrical shells**




**Representative Rectangle**

w= width of the rectangle
h = length of the rectangle
p = distance from axis of rev. to *center* of rectangle

When the rectangle is revolved around the axis of revolution, it forms a *cylindrical shell* (or tube) of thickness w

To find the volume of the shell: consider the 2 cylinders

The radius of the larger corresponds to the outer radius
The radius of the smaller corresponds to the inner radius

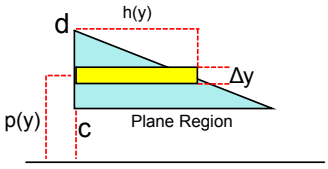
p is the *average radius* of the shell → outer radius $= p + \frac{w}{2}$
p is the *average radius* of the shell → inner radius $= p - \frac{w}{2}$

**Volume of the shell = volume of cylinder - volume of hole**

$$= \pi \left(p + \frac{w}{2}\right)^2 h - \pi \left(p - \frac{w}{2}\right)^2 h$$

$$= 2\pi p h w$$

$$= 2\pi \text{ ( average radius)(height)(thickness)}$$




Horizontal Rectangle: width $\Delta y$

As the plane figure is revolved around a line parallel to the x-axis, the rectangle generates a *representative shell* whose volume is:

$$\Delta V = 2\pi[\ p(y)\ h(y)\ ]\Delta y$$

You can approximate the volume of the solid by $n$ such shells of *thickness* $\Delta y$,

*and*

*us* $p(y_i)$

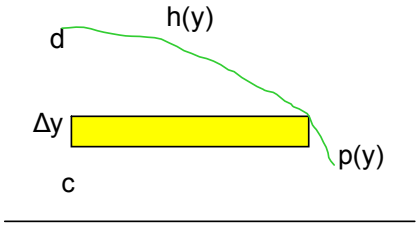
$$\textbf{Volume of a Solid} = 2\pi \int_c^d [\ p(y)\ h(y)\ ]dy$$

# The Shell Method

To find the volume of a solid of revolution with the shell method, use one of the following:

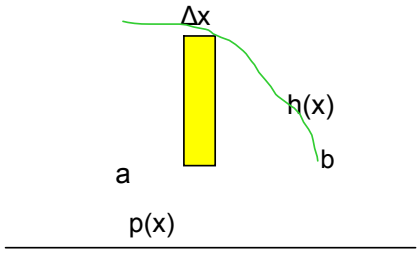
**Horizontal Axis of Revolution**

$$V = 2\pi \int_c^d p(y)\, h(y)\, dy$$




**Vertical Axis of Revolution**

$$V = 2\pi \int_a^b p(x)\, h(x)\, dx$$



## Example 1 - Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = x - x^3$$

*and the x-axis (0* $\leq x \leq 1$) about the y-axis

## Example 2 - Using the Shell Method to Find Volume

Find the volume of the solid of revolution formed by revolving the region bounded by the graph of

$$x = e^{(-y^2)}$$

**and the y -axis (0**$\leq y \leq 1$) about the x-axis

## Comparison of Disk and Shell Methods

## Example 3 - Shell Method Preferable

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad x = 1$$

about the y-axis

## Example 4 - Volume of a Pontoon

A pontoon is to be made in the shape shown. The pontoon is designed by rotating the graph of

$$y = 1 - \frac{x^2}{16} \qquad -4 \le x \le 4$$

about the x-axis, where x and y are measured in feet.
Find the volume of the pontoon

## Example 5 - Shell Method Necessary

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^3 + x + 1, \quad y = 1, \quad x = 1$$

about the line x = 2