7.2 Volume - The Disk Method

What will you learn?

- **Find the volume of a solid of revolution using the disk method**
- **Find the volume of a solid of revolution using the washer method**
- **Find the volume of a solid with know cross sections**

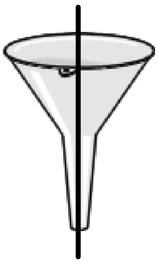
The Disk Method

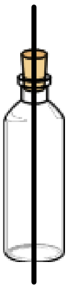
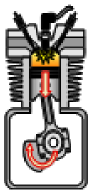
Area is only on of the MANY applications of the definite integral.
Another is finding the volume of a 3-dimensional solid.

Common Uses: engineering & manufacturing

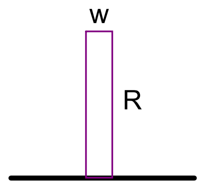
In this section we will study volumes of solids with SIMILAR CROSS- SECTIONS

Examples : axles, funnels, pills, bottles, pistons

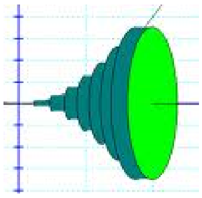



If a region in the plane is revolved about a line, the resulting solid is a <u>SOLID OF REVOLUTION</u>



The line is called the <u>AXIS OF REVOLUTION</u>

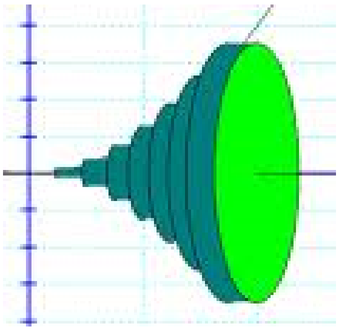
The simplest solid is a *right circular cylinder* or <u>DISK</u>, which is formed by revolving a rectangle about an axis adjacent to one side of the rectangle



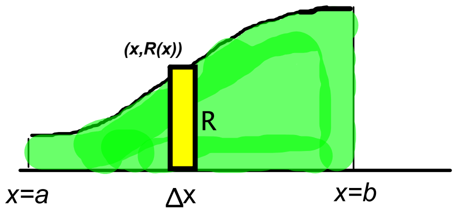
Volume of Disk = (area of disk)(width of disk)

$$= \pi R^2 w$$

where R = radius
w = width



To determine the volume of this solid, consider a *representative rectangle*



Volume of disk

$$\Delta V = \pi R^2 \Delta x$$

$$\Delta V = \pi [R(x)]^2 \Delta x$$

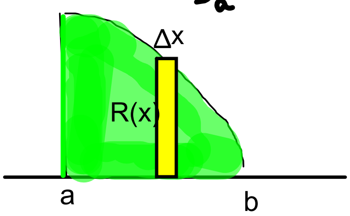
$$V = \pi \int_a^b [R(x)]^2 dx$$

The Disk Method

To find the volume of a solid of revolution with the **disk method,** use the following

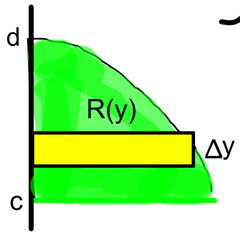
Horizontal Axis of Revolution

$$Volume = V = \pi\int_a^b [R(x)]^2 \, dx$$



Vertical Axis of Revolution

$$Volume = V = \pi\int_c^d [R(y)]^2 \, dy$$

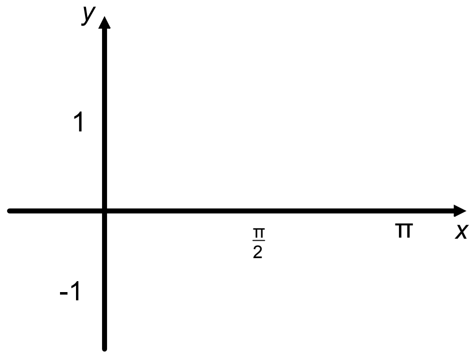


Example 1 - Using the Disk Method

Find the volume of the solid formed by revolving the region bounded by the graph of

$$f(x) = \sqrt{\sin x}$$

and the x-axis ($1 \leq x \leq \pi$) about the x-axis

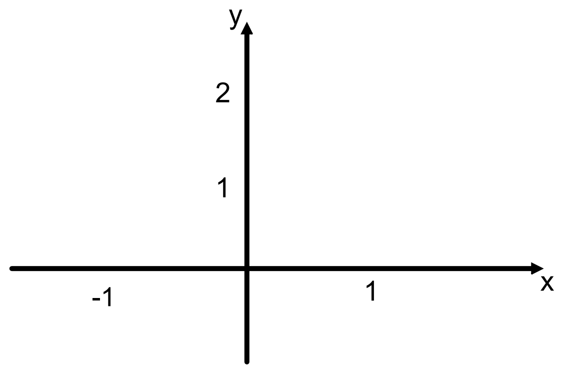


Example 2 - Revolving About a Line That is Not a Coordinate Axis

Find the volume of the solid formed by revolving the region bounded by

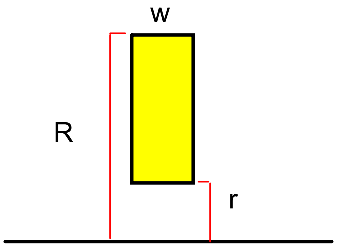
$$f(x) = 2 - x^2$$

and $g(x) = 1$ about the line $y = 1$



The Washer Method

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a washer



The washer is formed by revolving a rectangle about an axis

If r = inner radius
R = outer radius

Volume of washer = $\pi(R^2 - r^2)w$

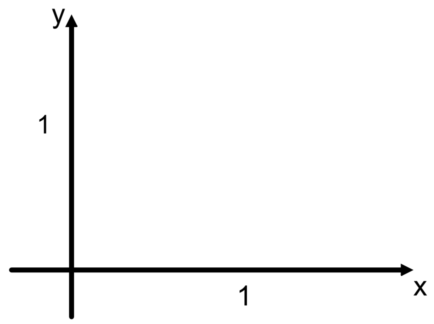
$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx$$

Example 3 - Using the Washer Method

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = \sqrt{x} \text{ and } y = x^2$$

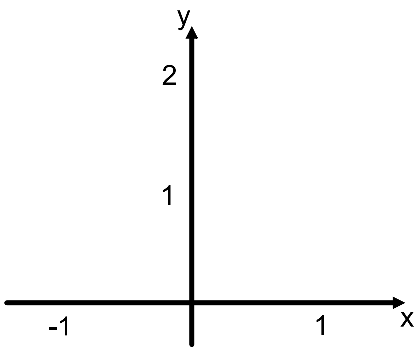
about the x-axis



Example 4 - Integrating with Respect to *y*, Two - Integral Case

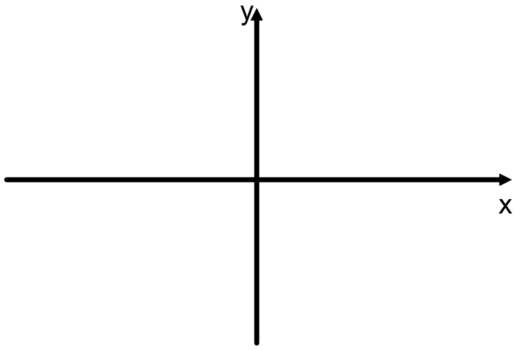
Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0 \quad \text{and} \quad x = 1$$



Example 5 - Manufacturing

A manufacturer drills a hole through the center of a metal sphere of radius 5 inches.
The hole has a radius if 3 inches.
What is the volume of the resulting metal ring?



Solids with Known Cross Sections

With the disk method, you can find the volume of a solid having a circular cross section shose area is $A = \pi r^2$

Cross Sections

$\perp$ to x-axis

$$\int A(x)dx$$

$\perp$ to y-axis

$$\int A(y)dy$$

Square Based Pyramid

A pyramid 3 m high has congruent triangular sides and a s square base that is 3 m on each side.
Each cross section of the pyramid parallel to the base is a square.
Find the volume of the pyramid.

Math Paperweight

The base is the shape of the region between the x- axis and the arch of the equation $y = 2\sin x$.

Each cross section perpendicular to the x-axis is a semi-circle whose diameter runs from the x-axis to the curve.

Find the volume of the paper weight

Find the volume of the solid whose base is the area bounded by the lines

$y = 1 - x/2$ $y = -1 + x/2$ and $x = 0$

whose cross sections are perpendicular to the x-axis and are equilateral triangles.