

4. Find the indefinite integral.

$$\int \csc(8x) dx$$

A) $\frac{-1}{8} \ln |\csc(8x) + \cot(8x)| + C$

B) $\frac{-1}{8} \ln |\csc(8x)| + C$

C) $8 \ln |\csc(8x) + \cot(8x)| + C$

D) $\frac{-1}{8} \ln |\csc x + \cot x| + C$

E) Both A and D

$$u = 8x$$

$$du = 8 dx$$

$$\frac{1}{8} \int \csc u du$$

$$\frac{1}{8} (-\ln |\csc u + \cot u|) + C$$

$$= -\frac{1}{8} \ln |\csc 8x + \cot 8x| + C$$

5. Find $f^{-1}(x)$ if $f(x) = \sqrt{2-x^2}$ ($0 \leq x \leq \sqrt{2}$).

A) $f^{-1}(x) = x + \sqrt{2}$

B) $f^{-1}(x) = (2-x^2)^2$

C) $f^{-1}(x) = \sqrt{x^2-2}$

D) $f^{-1}(x) = \frac{1}{\sqrt{2-x^2}}$

E) $f^{-1}(x) = \sqrt{2-x^2}$

$$y = \sqrt{2-x^2}$$

$$x = \sqrt{2-y^2}$$

$$x^2 = 2-y^2$$

$$y^2 = 2-x^2$$

$$y = \sqrt{2-x^2}$$

6. Find $\frac{dy}{dx}$ if $y = x^7 e^{x^6}$.

A) $\frac{dy}{dx} = e^{x^6} \left(\frac{x^8}{8} + \frac{x^7}{7} \right)$

B) $\frac{dy}{dx} = e^{x^6} (x^7 + x^6)$

C) $\frac{dy}{dx} = e^{x^6} (7x^6 + 6x^{12})$

D) $\frac{dy}{dx} = 7x^6 e^{x^6-1}$

E) $\frac{dy}{dx} = 7x^6 e^{x^6}$

$$y' = x^7 e^{x^6} (6x^5) + e^{x^6} (7x^6)$$

$$e^{x^6} (6x^{12} + 7x^6)$$

7) Find the slope of the tangent line to the graph of $y = \ln(e^{-x^2})$ at the point $(2, -4)$

$$y = -x^2 \ln e$$

$$y = -x^2$$

$$y' = -2x$$

$$y'(2) = -4$$

$$y + 4 = -4(x - 2)$$

$$y + 4 = -4x + 8$$

$$y = -4x + 4 \leftarrow \text{not nec.}$$

8) Find the integral: $\int_1^3 \frac{e^x}{e^x - 1} dx$

$$u = e^x - 1 \quad x = 3 \quad u = e^3 - 1$$

$$du = e^x dx \quad x = 1 \quad u = e - 1$$

$$\int_{e-1}^{e^3-1} \frac{1}{u} du$$

$$\ln|u| \Big|_{e-1}^{e^3-1}$$

$$\ln|e^3 - 1| - \ln|e - 1|$$

$$\approx 2.4076$$

9) Differentiate: $f(x) = \ln \frac{e^{3x}}{e^x - 3}$

$$f(x) = \ln e^{3x} - \ln(e^x - 3)$$

$$f(x) = 3x \ln e - \ln(e^x - 3)$$

$$f(x) = 3x - \ln(e^x - 3)$$

$$f'(x) = 3 - \frac{e^x}{e^x - 3}$$

1. Find an equation of the tangent line to the graph of $y = \ln(x^{13})$ at the point (1,0).

- A) $y = x - 13$
B) $y = 13(x+1)$
C) $y = 13(x-1)$
D) $y = x - 1$
E) $y = 13(x-1) + 1$

$$y = 13 \ln x$$

$$y' = \frac{13}{x}$$

$$y'(1) = 13$$

$$y - 0 = 13(x - 1)$$

$$y = 13x - 13$$

2. Find the derivative of the function $y = \ln \sqrt{x^2 - 11}$.

- A) $\frac{dy}{dx} = \frac{1}{2x-11}$
B) $\frac{dy}{dx} = \frac{x}{2x-11}$
C) $\frac{dy}{dx} = \frac{1}{x^2-11}$
D) $\frac{dy}{dx} = \frac{x}{x^2-11}$
E) $\frac{dy}{dx} = \frac{2x}{\sqrt{x^2-11}}$

$$\frac{1}{2} \ln(x^2 - 11)$$

$$\frac{1}{2} \left(\frac{2x}{x^2 - 11} \right)$$

$$\frac{x}{x^2 - 11}$$

3. Find the indefinite integral.

- $\int \frac{(\ln x)^4}{x} dx$
- A) $\frac{(\ln x)^4}{4} + C$
B) $4(\ln x)^3 + C$
C) $\frac{5 \ln x}{x} + C$
D) $\frac{(\ln x)^5}{5} + C$
E) None of the above

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u^4 du$$

$$\frac{1}{5} u^5 + C$$

$$\frac{1}{5} (\ln x)^5 + C$$

10) Find $\frac{dy}{dx}$: $3xe^y - 4y = 2xy$

$$3xe^y y' + e^y(3) - 4y' = 2xy' + y(2)$$

$$3xe^y y' - 4y' - 2xy' = 2y - 3e^y$$

$$y'(3xe^y - 4 - 2x) = 2y - 3e^y$$

$$y' = \frac{2y - 3e^y}{3xe^y - 4 - 2x}$$

11) Use logarithmic differentiation to find $\frac{dy}{dx}$: $y = \frac{(4x^3 - 5)^4 \sqrt{3 - 2x^2}}{(x^2 + 1)^3}$

(No LCD needed in answer)

$$\ln y = 4 \ln(4x^3 - 5) + \frac{1}{2} \ln(3 - 2x^2) - 3 \ln(x^2 + 1)$$

$$\frac{y'}{y} = 4 \left(\frac{12x^2}{4x^3 - 5} \right) + \frac{1}{2} \left(\frac{-4x}{3 - 2x^2} \right) - 3 \left(\frac{2x}{x^2 + 1} \right)$$

$$y' = y \left(\frac{48x^2}{4x^3 - 5} - \frac{2x}{3 - 2x^2} - \frac{6x}{x^2 + 1} \right)$$

$$y' = \frac{(4x^3 - 5)^4 \sqrt{3 - 2x^2}}{(x^2 + 1)^3} \left(\frac{48x^2}{4x^3 - 5} - \frac{2x}{3 - 2x^2} - \frac{6x}{x^2 + 1} \right)$$

12) Find the integral: $\int \frac{e^{4x} - e^{2x} + 1}{e^x} dx$

$$\int (e^{3x} - e^x + e^{-x}) dx$$

$$\boxed{\frac{1}{3}e^{3x} - e^x - e^{-x} + C}$$

13) Find the integral: $\int \frac{x^2 + 4x - 1}{x - 2} dx$

$$\int (x + 6 + \frac{11}{x-2}) dx$$

$$\begin{array}{r} 2 \overline{) 1 \quad 4 \quad -1} \\ \underline{ 2 \quad 12} \\ 1 \quad 6 \quad 11 \end{array}$$

$$\boxed{\frac{1}{2}x^2 + 6x + 11 \ln|x-2| + C}$$

14) Calculate the area of the region bounded by $y = xe^{-x^2}$, $y = 0$, $x = 0$, $x = 4$

$$-\frac{1}{2} \int_0^4 2x e^{-x^2} dx$$

$$\begin{aligned} u &= -x^2 & x=4 & u=-16 \\ du &= -2x dx & x=0 & u=0 \end{aligned}$$

$$-\frac{1}{2} \int_0^{-16} e^u du$$

$$+\frac{1}{2} \int_{-16}^0 e^u du$$

$$\frac{1}{2} e^u \Big|_{-16}^0$$

$$\frac{1}{2} [e^0 - e^{-16}]$$

$$\frac{1}{2} [1 - e^{-16}]$$

12

- 15) A population of bacteria, with an initial population of 1000, is changing at the rate given by the differential equation

$$\frac{dP}{dt} = \frac{2000}{1+0.2t} \quad \text{where } t \text{ is the time in days}$$

- a) Write the equation that gives the population at any time t . 4

$$P = \int \frac{2000}{1+0.2t} dt$$

$$P = 2000 \int \frac{1}{1+0.2t} dt \quad \begin{array}{l} u = 1+0.2t \\ du = .2 dt \quad \frac{2}{10} \end{array}$$

$$P = 2000 \left(\frac{10}{2} \right) \int \frac{1}{1+0.2t} \left(\frac{2}{10} \right) dt$$

$$P = 10,000 \int \frac{1}{u} du$$

$$P = 10,000 \ln |1+0.2t| + C \quad \leftarrow C = 1000$$

$$P = 10,000 \ln |1+0.2t| + 1000$$

- b) Find the population after 7 days.

$$P(7) = 10,000 \ln |1+0.2(7)| + 1000 \quad \checkmark$$

$$\approx 9754.687$$

≈ 9755 Bacteria

5