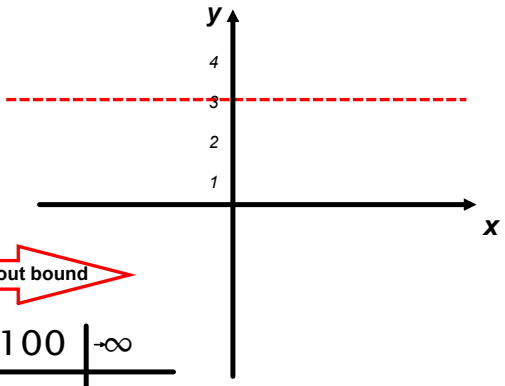
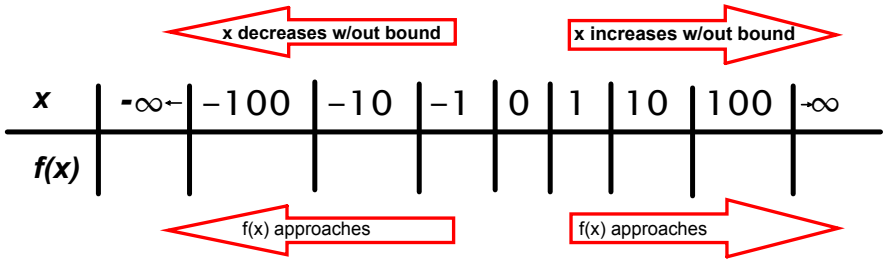
# 3.5 Limits at Infinity

Concerned with END BEHAVIOR on an infinite interval

**Given :** $f(x) = \dfrac{3x^2}{x^2+1}$




$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to \infty} f(x) =$

## Definition of Limits at Infinity

Let L be a real number.

1. The statement

# Horizontal Asymptotes

## Definition of a Horizontal Asymptote

**The line $y = L$ is a HA of the graph of $f$ if**

## Theorem - Limits at Infinity

**If $r$ is a positive rational number and $c$ is any real number, then**

**Furthermore, if $x^r$ is defined when $x < 0$**

## Example 1 - Finding a Limit at Infinity

**Find the limit:** $\lim_{x \to \infty} \left( 5 - \frac{2}{x^2} \right)$

## Example 2 - Finding a Limit at Infinity

**Find the limit:** $\lim_{x \to \infty} \frac{2x - 1}{x + 1}$

## Example 3 - A Comparison of Three Rational Functions

a. ) $\lim_{x \to \infty} \frac{2x + 5}{3x^2 + 1}$

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b.) $\lim_{x \to \infty} \frac{2x^2 + 5}{3x^2 + 1}$

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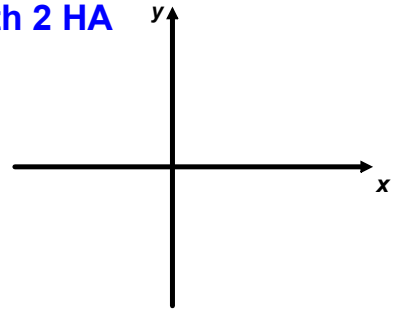
c.) $\lim_{x \to \infty} \frac{2x^3 + 5}{3x^2 + 1}$

## Guidelines for Finding Limits at $\pm\infty$ of a Rational Function

1. If the degree of numerator < degree of denominator then the limit is 0
2. If the degree of numerator = degree of denominator then the limit is the ratio of the lead coefficients
3. If the degree of numerator > degree of denominator then the limit DNE

## Example 4 - A Function with 2 HA

Find each limit.




a.) $\lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$

b.) $\lim_{x \to -\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$

## Example 5 - Limits Involving Trig Functions

Find each limit.

a.) $\lim_{x \to \infty} \sin x$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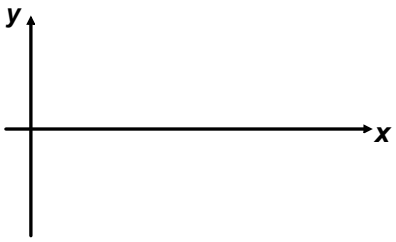
b.) $\lim_{x \to \infty} \frac{\sin x}{x}$



**Example 6 - Oxygen Level in a Pond**

Suppose that $f(t)$ measures the level of oxygen in a pond where $f(t) = 1$ is the normal (unpolluted) level and the time $t$ is measured in weeks.

When $t = 0$, organic waste is dumped into the pond, and as the waste material oxidizes, the level of oxygen in the pond is

$$f(t) = \frac{t^2 - t + 1}{t^2 + 1}$$

What percent of the normal level of oxygen exists in the pond after 1 week?
after 2 weeks?
after 10 weeks?

What is the limit as $t$ approaches infinity?

## Infinite Limits at Infinity

Many functions do not approach a finite limit as x increases(or decreases) without bound.

For example, NO polynomial function has a finite limit at infinity.

### Definition of Infinite Limits at Infinity

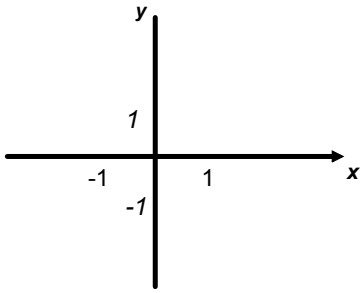
**Let *f* be a function defined on the interval *(a,* $\infty$)**

1. **The statement $\lim_{x \to \infty} f(x) = \infty$ means that for each positive number M, there exists a corresponding number *N > 0*, s.t. f(x) > M whenever x > N.**

2. **The statement $\lim_{x \to \infty} f(x) = -\infty$ means that for each negative number M, there exists a corresponding number *N > 0*, s.t. f(x) < M whenever x > N.**

## Example 7 - Finding Limits at Infinity

Find each limit.

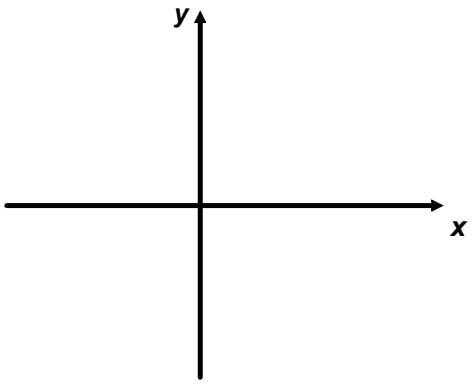



$$\lim_{x \to \infty} x^3$$

$$\lim_{x \to -\infty} x^3$$

## Example 8 - Finding Limits at Infinity

Find each limit.




$$\lim_{x \to \infty} \frac{2x^2 - 4x}{x + 1}$$

$$\lim_{x \to -\infty} \frac{2x^2 - 4x}{x + 1}$$