Calc

Chapter 2 - Differentiation

2.1 - The Derivative & The Tangent Line Problem

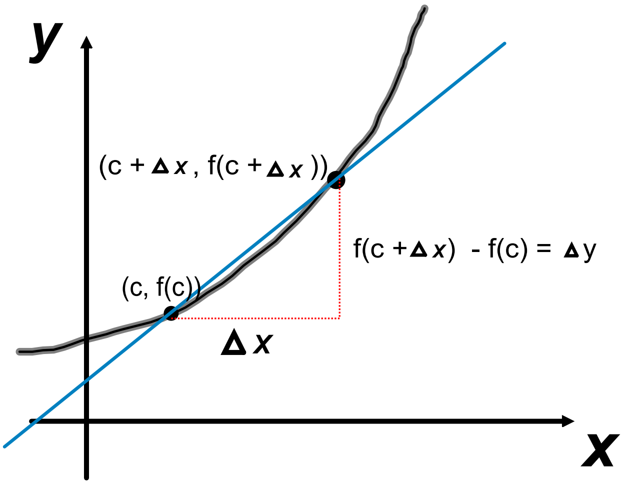
☺ What will you learn??? ☺

- **Find the slope of the tangent line to a curve at a point**
- **Use the limit definition to find the derivative of a function**
- **Understand the relationship between differentiability & continuity**

The Tangent Line Problem

Essentially, the problem of finding the tangent line at a point P boils down to the problem of finding the *slope of the tangent line* at point P.

You can approximate this slope using the secant line through the point of tangency and a second point on the curve.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{sec} = \frac{f(c + \Delta x) - f(c)}{(c + \Delta x) - c}$$

$$m_{sec} = \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

Definition of Tangent Line with Slope m

If f is defined on an open interval containing c, and if the limit

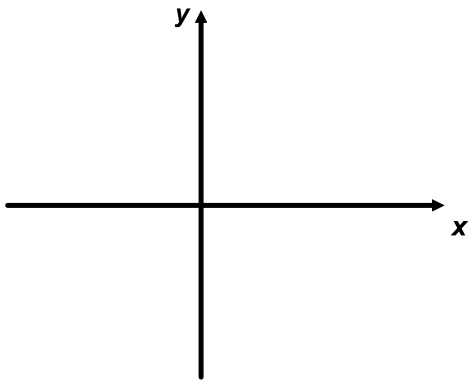
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through $(c, f(c))$ with slope m is the tangent line to the graph of f at the point $(c, f(c))$.

Example 1 - The Slope of the Graph of a Linear Function

Find the slope of the graph of

$$f(x) = 2x - 3$$

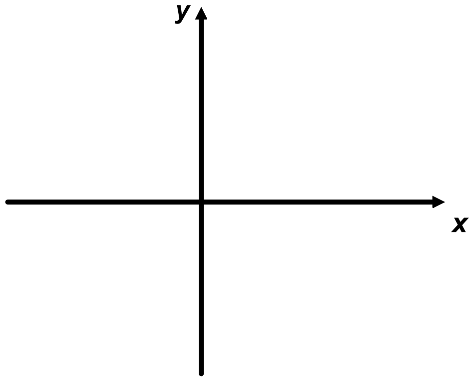


Example 2 - Tangent Lines to the Graph of a Nonlinear Function

Find the slopes of the tangent lines to the graph of

$$f(x) = x^2 + 1$$

at the points $(0, 1)$ **and** $(-1, 2)$



The definition of a tangent line to a curve *does not* cover the possibility of a *vertical tangent line*. For vertical tangent lines, you can use the following definition.

If *f* is continuous at *c* and

$$\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \infty \quad \text{or} \quad \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = -\infty$$

the vertical line *x* = *c* passing through *(c, f(c))* is a vertical tangent line to the graph of *f.*

The Derivative of a Function

The limit used to define the slope of a tangent line is also used to define one of the two fundamental operations of calculus - *differentiation*

Definition of the Derivative of a Function

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists f' is a function of x.

- **This new function is also a *function of x***
- **It gives the *slope of the tangent line* to the graph of f at $(x, f(x))$ - provided that the graph has a tangent line at this point**
- **This process is called *differentiation***
- **A function is differentiable at x if its derivative exists at x**
- **A function is *differentiable on an open interval (a, b)* if it is differentiable at every point in the interval**

Most Common Notations to denote the Derivative of y= f(x)

$f'(x)$	$\frac{dy}{dx}$	y'	$\frac{d}{dx}[f(x)]$	$D_x[y]$

Example 3 - Finding the Derivative by the Limit Process

Find the derivative of $f(x) = x^3 + 2x$

Example 4 - Using the Derivative to Find the Slope at a Point

Find $f'(x)$ for $f(x) = \sqrt{x}$.
Find the slope of the graph of f at the points $(1, 1)$ and $(4, 2)$.
Discuss the behavior of f at $(0, 0)$.

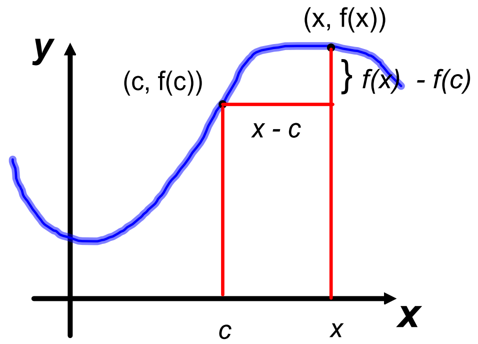
Example 5 - Finding the Derivative of a Function

Find the derivative with respect to *t* for the function $y = 2 / t$

Differentiability & Continuity

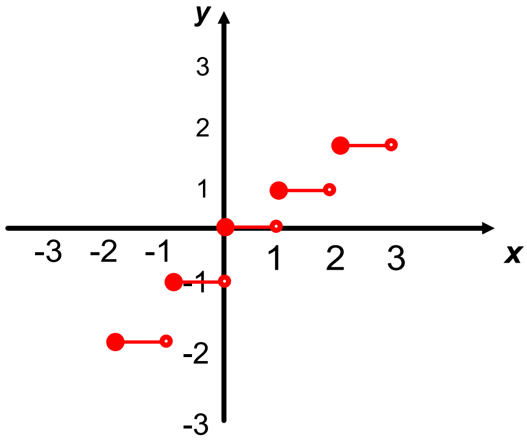
The derivative of *f* at *c*

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$



If a function is <u>not continuous</u> at $x = c$, it is also <u>not differentiable</u> at $x = c$

For example, the greatest integer function



$f(x) = [[x]]$

f(x) is not continuous at x = 0
∴ not differentiable at x = 0

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \frac{[[x]] - 0}{x} = \infty$$

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{[[x]] - 0}{x} = 0$$

Differentiablility implies continuity
Converse is NOT true
It is possible to be continuous at x = c but not diffentiable at x = c

Example 6 - A Graph with a Sharp Turn

$$f(x) = |x - 2|$$

Example 7 - A Graph with a Vertical Tangent Line

$$f(x) = x^{1/3}$$

Theorem 2.1 Differentiability Implies Continuity

If *f* is differentiable at $x = c$, the *f* is continuous at $x = c$.