# Section 2.2 - Basic Differentiation Rules & Rates of Change

## 🙂 What will you learn? 🙂

- Find the derivative using the Constant Rule
- Find the derivative using the Power Rule
- Find the derivative using the Constant Multiple Rule
- Find the derivative using the Sum & Difference Rules
- Find the derivative of the sine and cosine functions
- Use derivatives to find rates of change

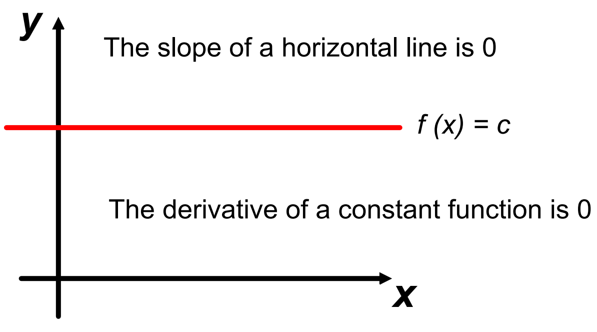
# The Constant Rule

**The derivative of a constant function is *0*.**
**That is, if *c* is a real number, then**

$$\frac{d}{dx}[\,c\,] = 0$$

## Example 1 - Using the Constant Rule

| Function | Derivative |
|---|---|
| a.) $y = 7$ | |
| b.) $f(x) = 0$ | |
| c.) $s(t) = -3$ | |
| d.) $y = k\pi^2$, *k* is a constant | |



# The Power Rule

If $n$ is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}\left[ x^n \right] = n\,x^{n-1}$$

For $f$ to be differentiable at $x = 0$, $n$ must be a number s.t. $x^{n-1}$ is defined on an interval containing $0$.

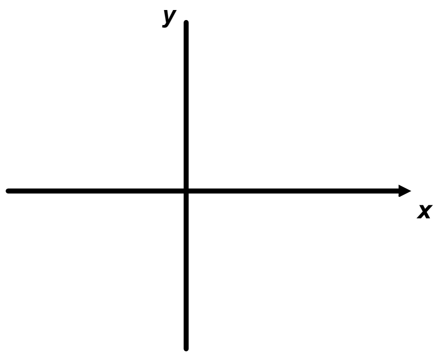
## Example 2 - Using the Power Rule

| Function | Derivative |
| --- | --- |
| a.) $f(x) = x^3$ | |
| b.) $g(x) = \sqrt[3]{x}$ | |
| c.) $y = 1/x^2$ | |

| Given | | Rewrite | | Differentiate | | Simplify |
| --- | --- | --- | --- | --- | --- | --- |
| $y = 1/x^2$ | $\Rightarrow$ | $y = x^{-2}$ | $\Rightarrow$ | $\frac{dy}{dx} = (-2)\,x^{-3}$ | $\Rightarrow$ | $\frac{dy}{dx} = \frac{-2}{x^3}$ |

## Example 3 - Finding the Slope of a Graph

**Find the slope of the graph of** $f(x) = x^4$ **when**



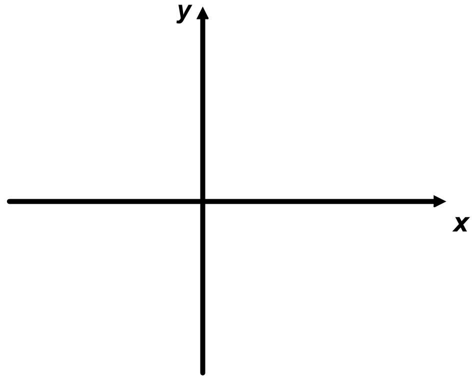

**a.)** $x = -1$

**b.)** $x = 0$

**c.)** $x = 1$

## Example 4 - Finding the Equation of a Tangent Line

**Find an equation of the tangent line to the graph of** $f(x) = x^2$ **when** $x = 2.$



## The Constant Multiple Rule

If $f$ is a differentiable function and $c$ is a real number, then $cf$ is also differentiable and $\frac{d}{dx}[cf(x)] = cf'(x)$.

## Example 5 - Using the Constant Multiple Rule

| Function | Derivative |
|---|---|
| a.) $y = \frac{2}{x}$ | |
| b.) $f(t) = \frac{4t^2}{5}$ | |
| c.) $y = \sqrt{x}$ | |
| d.) $y = \frac{1}{2\sqrt[3]{x^2}}$ | |
| e.) $y = \frac{-3x}{2}$ | |

## Example 6 - Using Parentheses When Differentiating

| Original Function | Rewrite | Differentiate | Simplify |
|---|---|---|---|
| a.) $y = \frac{5}{2x^3}$ | | | |
| b.) $y = \frac{5}{(2x)^3}$ | | | |
| c.) $y = \frac{7}{3x^{-2}}$ | | | |
| d.) $y = \frac{7}{(3x)^{-2}}$ | | | |

## The Sum & Difference Rules

**The sum ( or difference ) of two differentiable functions *f* and *g* is itself differentiable.**
**Moreover, the derivative of *f + g* ( or *f - g* ) is the sum (or difference) of the derivatives of *f* and *g*.**

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$ Sum Rule

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$ Difference Rule

## Example 7 - Using Sum & Difference Rules

| Function | Derivative |
| --- | --- |
| a.) $f(x) = x^3 - 4x + 5$ | |
| b.) $g(x) = -\frac{x^4}{2} + 3x^3 - 2x$ | |

## Derivatives of Sine & Cosine Functions

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

## Example 8 - Derivatives Involving Sines & Cosines

| Function | Derivative |
|---|---|
| a.) $y = 2\sin x$ | |
| b.) $y = \frac{\sin x}{2} = \frac{1}{2}\sin x$ | |
| c.) $y = x + \cos x$ | |

# Rates of Change

## Position Function

**The function *s* that gives the position (relative to the origin) of an object as a function of time *t***

**If, over a period of time $\Delta t$, the object changes its position by the amount $\Delta s = s(t + \Delta t) - s(t)$, then by the familiar formula**

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$$

The **Average Velocity** is $\dfrac{\text{Change in Distance}}{\text{Change in Time}} = \dfrac{\Delta s}{\Delta t}$

## Example 9 - Finding Average Velocity of a Falling Object

**If a billiard ball is dropped from a height of 100 feet, its height *s* at time *t* is given by the position function**

$$s = -16t^2 + 100$$

**where *s* is measured in feet and *t* is measured in seconds. Find the average velocity over each of the following time intervals.**

**a.)** *[1, 2]*

**b.)** *[1, 1.5]*

**c.)** *[1, 1.1]*

Suppose you wanted to find the *instantaneous velocity* (or simply the velocity) of the object when $t = 1$. You can approximate the velocity at $t = 1$ by calculating the average velocity over a small interval $[1, 1+\Delta t]$.

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t} = s'(t)$$

**Velocity Function**

$$v(t) = s'(t)$$

The velocity function is the first derivative of the position function

**Velocity** can be positive, negative or zero

**Speed** is the absolute value of velocity - only positive!

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**Position of a free-falling object under the influence of gravity** (excluding air resistance)

$$s(t) = gt^2 + v_0 t + s_0$$

$s_0$ - initial height

g - acceleration due to gravity

Earth's gravity - $-32$ ft/sec$^2$ or $-9.8$m/sec$^2$

## Example 10 - Using the Derivative to Find Velocity

**At $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is**

$s(t) = -16t^2 + 16t + 32$

**where $s$ is measured in feet and $t$ is measured in seconds**

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**a.) When does the diver hit the water?**

**b.) What is the diver's velocity at impact?**